GAS-LIQUID TURBULENT TWO PHASE FLOW ALONG VERTICAL FLAT PLATE WITH GAS EVOLUTION

AKIHIRO MATSUURA AND HAJIME NAKAMURA

Department of Mechanical Engineering, Daido Institute of Technology, Nagoya 457

SETSURO HIRAOKA, YUTAKA TADA,

YOSHIHITO KATO AND KOH SEUNG-TAE

Department of Applied Chemistry, Nagoya Institute of Technology, Nagoya 466

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Gas-liquid turbulent two phase flow along a vertical flat plate with gas evolution was numerically simulated with a two-fluid model and Plandtl's mixing length theory using the SIMPLER method. The calculation domain was assumed to be two-dimensional and gas bubbles to be evolved uniformly from the plate surface. The computed velocity profiles were affected by void fraction on the wall and the bubble diameters, but not by the bubble departure velocity from the plate surface. Both the gas and liquid velocity profiles and the computed wall shear stress with the present model were qualitatively and quantitatively coincident with those measured using LDV for the vertical flat plate electrode with H_2 or O_2 gas evolution.

Introduction

Gas-liquid two-phase flows in chemical plants and BWR type nuclear power units have been studied by many investigators, both experimentally and numerically^{1-6, 8, 9, 12-14, 17-19, 21, 22}. With rapid progress in the performance of computer in recent years, some investigators have tried to analyze transport phenomena in two-phase flows numerically. Sato *et al.*¹⁹ studied turbulent momentum and heat transfer in two-phase bubble flow with a known time-averaged velocity profile. Okazaki¹⁴, and Kataoka and Serizawa⁶ suggested derivation of basic equations and a turbulent model for two-phase flow. Sami¹⁸ reported a numerical solution of annular two-phase flow. Minato and Kawabe¹² reported two-dimensional two-phase flow in a pipe using a two-velocity and two-temperature model.

In an analysis of bubble column reactors, Sperling *et al.*²¹ published a simulation of gas-bubble flow using mixing length theory and a one-fluid model. Ranade¹⁷ conducted numerical experiments of two-phase flow in a bubble column for a given gas-liquid slip velocity using the $k-\epsilon$ model. Neti and Mohamed¹³ calculated two-phase turbulent jet and vertical pipe flow using a combined Eulerian and Lagrangian method. de Jong and Sabnis¹ simulated cryogenic liquid flows with vapor bubbles using CELMINT code and the $k-\epsilon$ model. Doi and Murakami² used a combination of a one-fluid model and LES for calculation of turbulent flow in a square duct.

So far, most papers related to calculations of two-phase turbulent flow used a one-fluid model or combined Eulerian and Lagrangian methods, because of the difficulty

of application of turbulent models to two-phase flow, and the need for extended calculation time. However, both liquid and gas velocities and void fraction profile are important for industrial applications. Then, to certify the two-phase flow more clearly, we have to calculate liquid and gas velocities and void fraction profile separately. The two-fluid model can derive the liquid and gas velocities and the void fraction profile without the information of void fraction profile or drift-flux parameter, but the homogeneous fluid model and the drift-flux model can not do this without this information. We proposed the calculation results of laminar and turbulent flow along a vertical plate with an inflow using the two-fluid model^{10, 11}.

In this paper, we analyzed the two-dimensional velocity profiles in a gas-liquid two-phase flow along a vertical flat plate with gas evolution using a two-fluid model and the SIMPLER method^{10, 15, 16, 20}. The numerical results were compared with experimental results under the same conditions.

1. Numerical Analysis

Figure 1 indicates a model of two-dimensional turbulent bubble flow. The vertical flat plate was immersed in a semi-infinite liquid. In the liquid phase, fluid is continuous but gas bubbles are discrete particle in the liquid. Exact two-fluid model equations need Heaviside function for all terms. These equations are thus not continuous. For the bubble flow, bubbles are small and disperse uniformly in a controlled volume. Then, the averaged governing equations can be considered to be quasi-continuous. We introduced a two-fluid model described below.

* Received August 8, 1994. Correspondence concerning this article should be addressed to A. Matsuura.

Fig. 1 Model of two-dimensional flow

Mass conservation equation

$$\frac{\partial \rho_k \theta_k}{\partial t} + \frac{\partial \rho_k \theta_k u_{i,k}}{\partial x_i} = 0 \quad (1)$$

Momentum equation

$$\begin{aligned} & \frac{\partial}{\partial t} (\rho_k \theta_k u_{i,k}) + \frac{\partial}{\partial x_j} (\rho_k \theta_k u_{i,k} u_{j,k}) \\ &= -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left\{ \frac{1}{\rho_k} (\nu_k + \nu_{ik}) \left(\frac{\partial \theta_k u_{i,k}}{\partial x_j} + \frac{\partial \theta_k u_{j,k}}{\partial x_i} \right) \right\} + S_{j,k} \end{aligned} \quad (2)$$

Phase equation

$$\theta_k = 1 \quad (3)$$

where k is phase, $k = 1$ denote liquid phase, $k = 2$ denote gas phase, i and j are dimensions of flow field, and S is a source term. The source term in Eq. (2) includes several body forces¹⁰.

For the liquid phase

$$\tilde{S}_1 = F_{drag} \quad (4)$$

For the gas phase

$$\tilde{S}_2 = -F_{drag} + F_{bovy} + F_{hydro} \quad (5)$$

In Eqs. (4) and (5), the first term in the right hand side is a cell drag force. In Eq. (5), the second term expressed a cell buoyancy force and the third term a hydrodynamic force exerted on a bubble, which was introduced to account for the effect of pressure difference due to the liquid velocity gradient across the bubble. To apply a turbulent model to the two-fluid model, we used van Driest's model with Plandtl's mixing length theory.

Mixing length theory

$$\nu_t = l^2 \left| \frac{\partial u}{\partial y} \right| \quad (6)$$

van Driest model

$$l = \kappa y \left\{ 1 - \exp \left(\frac{-y^+}{A} \right) \right\} \quad (7)$$

Fig. 2 Experimental apparatus ① thermostant both ② vessel ③ counter electrode ④ gas evolving electrode ⑤ galvanostat ⑥ He-Ne Laser ⑦ photo-multiplier ⑧ shifter ⑨ tracker

We used the following boundary conditions. At the wall, the liquid phase exhibits non-slip conditions and the gas phase slip conditions. At the upper side boundary of calculation domain X , both liquid and gas leave smoothly. At the boundary far from the gas evolving wall Y , liquid and gas velocities and void fraction are zero.

$$\begin{aligned} y = 0 : u_{1,1} &= \frac{\partial u_{1,2}}{\partial y} = u_{2,1} = 0, u_{2,2} = u_w, \theta_2 = \theta_w \\ x = X : u_{1,1} &= \frac{\partial u_{1,2}}{\partial x} = \frac{\partial u_{2,1}}{\partial x} = \frac{\partial u_{2,2}}{\partial x} = \frac{\partial \theta_k}{\partial x} = 0 \\ x = 0 : u_{1,1} &= u_{1,2} = u_{2,1} = u_{2,2} = \theta_2 = 0 \\ y = Y : u_{1,1} &= u_{1,2} = u_{2,1} = u_{2,2} = \theta_2 = 0 \\ t = 0 : u_{1,1} &= u_{1,2} = u_{2,1} = u_{2,2} = \theta_2 = 0 \end{aligned}$$

We assumed that gas was evolved uniformly from the vertical flat plate surface and its bubble diameter was less than 1 mm. Stokes's law for a solid sphere is thus applicable to calculate the drag force for a bubble. Wall void fractions of gas θ_w were assumed to be 0.37, 0.7, 1.28, 2 and 5 %. Departure velocity of bubbles from the plate surface and bubble diameters⁵ have been reported for both electrolysis and boiling, but the observed values were not consistent. Thus, the departure velocities of bubbles from the plate surface were assumed to be $v_w = 0.15, 0.1$ and 0.05 cm/s and the bubble diameters d to be 0.03, 0.05 and 0.1 mm.

2. Experimental

Figure 2 shows the experimental apparatus for measuring the liquid and gas velocity profiles near the vertical flat plate. The plate, i.e., the gas evolving electrode, was 50 mm wide, 100 mm long, 0.1 mm thick and was made from nickel Nikkel. The vessel was 80 mm wide, 180 mm long and 160 mm high and it was immersed in a water bath to maintain its temperature at 303 K. The electrode was immersed in the center of the vessel, and water was electrolyzed with a constant current in 1N-H₂SO₄ solution.

Both liquid and gas velocities were measured by a tracker type He-Ne Laser Doppler Velocimeter controlled by a personal computer. The seeding particle to measure the liquid velocity was a 30 μ m glass bubble whose specific gravity was 0.997 - 1.003.

Fig. 3 Calculated flow pattern with vector arrows

The total sampling number of measured velocity data was 5000, and the liquid velocity was separated from the gas velocity by using a histogram⁹⁾. After separating the velocities, both the liquid and gas velocities were evaluated by averaging.

To measure the average shear stress on the electrode's surface, a flat plate electrodes of 50 mm width and 30, 50 and 80 mm length were used. The electrode was hung on a phosphor-bronze arm to measure its weight with a strain gauge³⁾. Since the decrement of weight was proportional to the shear force, the average shear stress was determined from it.

3. Result and Discussion

3.1 Numerical results

Velocity Profiles Figure 3 shows liquid velocity profiles using vector arrows where the bubble diameter d was 0.05 mm and the wall void fraction θ_w was 0.0128. The liquid velocities were almost parallel to the plate, except near the leading edge. Gas velocities were only indicated in the region where bubbles exist. The gas velocity indicated departure from the plate near the leading edge since the gas bubble departure velocity from the plate was assumed to be 0.1 cm/s. Far from the leading edge, both the liquid and gas velocities were slightly toward the plate because of acceleration of liquid. For a vertical distance from the leading edge x larger than 3 cm, the velocity profiles were almost similar.

Figures 4 and 5 show the upward liquid and gas velocity profiles as parameters of the distance from the leading edge, x , the bubble diameter, d , and the wall void fraction, θ_w . In Fig. 4 the liquid computed velocity profiles were similar to those for natural convection flow

Fig. 4. Calculated upward liquid velocity profiles

Fig. 5 Calculated upward gas velocity profiler

or free jets. The maximum upward liquid velocity increased with increasing x , but it seemed to approach a certain value in the region of $x > 2.04$ cm. At $x = 0.52$ cm, the liquid velocity far from the wall indicated a negative value because of sudden acceleration at the leading edge. The maximum upward liquid velocity increased with increase of the bubble diameter and the wall void fraction. The boundary layer thickness of liquid flow was extended with increasing x and d , but it was extended only slightly with decreasing θ_w . The position at which the liquid velocity profiles took a maximum value was $y = 0.4$ mm.

In Fig. 5 the gas velocity profiles were also affected by x , d and θ_w in the same manner as the liquid ones. The upward gas velocity was 40 - 50 % faster than the upward liquid velocity.

Figure 6 shows the computed liquid velocity profiles at vertical distances of $x = 0.98$ and 4.02 cm with bubble

Fig. 6 Calculated upward liquid velocity profiles at $x = 4.02$ cm with wall void fraction $\theta_w = 0.00128$

Fig. 7 Calculated void fraction profiles

departure velocities of $v_w = 0.05, 0.1$ and 0.15 cm/s, and a wall void fraction of $\theta_w = 0.0128$. The velocity profiles at $x = 1.02$ cm were coincident with each other, while those at $x = 4.02$ cm were almost coincident within a difference of 5%. The effect of the bubble departure velocity on the liquid velocity profile was thus shown to be negligible.

Void Fraction Distribution Figure 7 shows void fraction profiles using the same parameters as used in Figs. 4 and 5. The void fraction profiles were quite similar to each other. The position at which the void fraction profile took a maximum value was $y = 0.25$ mm, and the width of the region where bubbles existed was not thicker than 0.7 mm. This phenomenon depends on the hydrodynamic force exerted on bubbles.

The maximum value of θ in the void fraction profile was 2.3 - 2.8 times larger than the wall void fraction θ_w .

Fig. 8 Calculated wall shear stress versus the distance from the leading edge

Fig. 9 Calculated average wall shear stress versus wall void fraction

Wall Shear Stress Figure 8 shows the local and average shear stresses at the wall obtained from a velocity gradient of the computed liquid velocity profile. Both local and average shear stresses increased gradually with distance from the leading edge, and they approached asymptotic values. With decreasing bubble diameter, the local shear stress approached an asymptotic value at a shorter distance from the leading edge. The asymptotic value of the local shear stress was nearly proportional to the bubble diameter, because the buoyancy force was proportional to the square of bubble diameter in two-dimensional flow.

To evaluate the effect of wall void fraction on the shear stress, the average shear stress over the plate τ_{av} was correlated with the wall void fraction θ_w in Fig. 9. τ_{av}/μ increased with increase of θ_w , independently of the plate length L , except at $\theta_w = 0.05$. The average shear stress showed a simple functional relation, except the result at $\theta_w = 0.0037$. The correlation was

Fig. 10 Measured upward liquid velocity profiles for H_2 gas evolution

Fig. 11 Measured upward liquid velocity profiles for O_2 gas evolution

$$\tau_{av} / \mu = 1360 \theta_w^{0.59} \quad (8)$$

3.2 Experimental results

Velocity Profiles The liquid velocity profiles near the vertical flat plate electrode with hydrogen and oxygen gas evolution are showed in Figs. 10 and 11. Figure 10 shows the results for hydrogen gas evolution. The experimental data were slightly scattered, but the upward liquid velocities increased with both increasing electrolysis current I_e and increasing vertical distance x , because of the large buoyancy force caused by the high gas evolution rate and the higher void fraction, which increased with x for uniform gas evolution. The positions at which the profiles took maximum values were close to $y = 0.5$ mm. The boundary layer thickness at $x = 4$ cm for some values

Fig. 12 Comparison of measured upward liquid velocity to calculated one

Fig. 13 Measured average shear stress versus the electrolysis current

of I_e was about $y = 3$ mm but that at $I_e = 2.5$ A extended with increasing x . The increase of boundary layer thickness with increase of x seemed to depend on viscous effect. While no dependence of boundary layer thickness on I_e was observed, the bubbles, which were the driving force of two-phase flow, collected at the position at which the profile took a maximum value and did not exist in the outer region of the boundary layer. The maximum upward velocity for $I_e = 3.5$ A was about 3.5 cm/s.

Figure 11 shows the results for oxygen gas evolution. The effects of I_e and x on the velocity profiles were very similar to those for hydrogen gas evolution.

Figure 12 shows a comparison of the experimental results of liquid velocity profile to the computed ones, where the calculated results were $x = 4.02$ cm, $\theta_w = 0.0128$

Fig. 14 Comparison of measured average shear stress to calculated one

and $d = 0.03, 0.05$, and 0.1 mm. Correspondingly, the experimental results were $x = 4$ cm and $I_e = 1.5$ A for hydrogen gas evolution. The value of I_e was selected as it closely corresponded to the calculated gas evolving rate. The computed maximum upward velocity and its position were roughly coincident with the measured ones when it was assumed $d = 0.05$ or 0.1 mm. The calculated results of liquid velocity profiles also agreed quantitatively with the experimental ones as $d = 0.1$ mm.

Wall Shear Stress Figure 13 show the measured average shear stress τ_{av} versus the electric current density I_g of electrolysis with the electrode length L as a parameter, where the ordinate was empirically corrected by a factor of the 0.11th power of the electrode length. The experimental results were empirically correlated with the following equations,

for hydrogen gas evolution;

$$\tau_{av} / \mu L^{0.11} = 1277 \cdot I_g^{0.60} \quad (9)$$

for oxygen gas evolution;

$$\tau_{av} / \mu L^{0.11} = 920 \cdot I_g^{0.60} \quad (10)$$

For both equations, the exponents of the current density took the same value of 0.6. In the previous results in potassium hydroxide aqueous solution³⁾, the exponents were 0.64 for oxygen gas evolution and 0.43 for hydrogen gas evolution. Then, the present experimental result agreed approximately with the result for oxygen.

Figure 14 shows comparison of the measured average shear stress with the calculated one. The solid line indicated the empirical correlation for hydrogen gas evolution. The wall void fraction for the experimental results was evaluated from the gas evolution rate per unit area Q using Faraday's law in electrolysis, as $\theta_w = Q / v_w = 24862 I_g / n F v_w$. The dashed line indicates the calculated values for $\theta_w = 0.0128$ and $d = 0.1, 0.05$, mm which are almost coincident with each other. The correlations were obtained as,

for the calculated results;

$$\tau_{av} / \mu = 1360 \cdot \theta_w^{0.59} \quad (11)$$

and for the experimental ones;

$$\tau_{av} / \mu = 1093 \cdot \theta_w^{0.55} \quad (12)$$

The exponents of the wall void fraction in the correlations agreed with each other to a high degree of accuracy within experimental and calculated errors. The difference between these correlations does not prevent practical usage.

Conclusion

Numerical simulations of gas-liquid two-phase flow along a vertical flat plate with gas evolution were conducted by using a two-fluid model and the SIMPLER method, and the calculated results were compared to the experimental ones for electrolysis on a flat plate with hydrogen and oxygen gas evolution.

The computed liquid velocities were almost parallel to the gas evolving plate, except near the leading edge, and the maximum upward liquid velocity increased with increasing distance from the leading edge, bubble diameter and wall void fraction. The upward gas velocities computed were 40 - 50 % higher than the liquid ones. The effect of bubble departure velocity on the liquid velocity was negligible. The positions at which the void fraction profiles took peak values were all at $y = 0.25$ mm, and the width of the regions where bubbles existed was not thicker than 0.7 mm. The local shear stress increased gradually with increase of the distance from the leading edge, and it approached an asymptotic value. The computed average shear stress over the plate was correlated with a simple equation. For the experimental results, the position at which the upward liquid velocity profiles took a maximum value was at $y = 0.5$ mm and that at which the velocity became zero was near $y = 3$ mm. The computed maximum upward liquid velocity and its peak position coincided well with the measured ones. The computed average shear stress was also coincident with the measured one with high accuracy, and they were correlated using the empirical equation.

Nomenclature

A	= damping coefficient	[-]
d	= diameter of a bubble	[cm]
F	= Faraday constant	[C/mol]
F_{drag}	= drag force hanging on a bubble	[dyne]
F_{bouy}	= buoyancy force hanging on a bubble	[dyne]
F_{hydro}	= hydrodynamic pressure difference	[dyne]
g	= gravitational acceleration	[cm/s ²]
I_e	= electric current of electrolysis	[A]
I_g	= electric current density of electrolysis	[A/cm ²]
L	= length of flat plate	[cm]
l	= mixing length	[cm]
n	= number of charge transfer	[-]
P	= pressure	[g/(cm·s ²)]
Q	= gas evolution rate per unit area	[cm ³ /(s·cm ²)]
S	= source term	[g/(cm ² ·s ²)]

u	= velocity components	[cm/s]
v_w	= bubble departure velocity	[cm/s]
X	= upper side boundary of calculation domain	[cm]
x	= distance from leading edge of flat plate	[cm]
Y	= boundary far from the gas evolving plate	[cm]
y	= distance normal to flat plate wall	[cm]
y^+	= dimensionless distance	[-]
θ	= volume ratio	[-]
θ_w	= volume ratio on the gas evolving plate	[-]
κ	= von Karman's constant	[-]
μ	= viscosity	[g/(cm·s)]
ν	= kinematic viscosity	[cm ² /s]
ν_t	= eddy viscosity	[cm ² /s]
ρ	= density	[g/cm ³]
τ_w	= local shear stress	[g/(cm·s ²)]
τ_{ave}	= average shear stress	[g/(cm·s ²)]

<Subscripts>

i, j	= coordinate direction
w	= at the gas evolving wall
k	= phase

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