FLOW ANALYSIS OF A LOW-PRESSURE IMPACTOR BY COMPRESSIBLE NAVIER-STOKES EQUATION

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Key Words: Fluid Mechanics, Computational Fluid Dynamics, TVD Scheme, Separation Efficiency

A TVD (Total Variation Diminishing) scheme based on a compressible Navier-Stokes equation was applied to axisymmetric flow in a low pressure impactor with a circular nozzle. This TVD scheme was a type combined to a two-step MacCormack scheme. The shock was clearly observed at the half distance between the outlet of jet and the impaction plate. The critical stagnation pressure ratio of the impactor designed by Hering *et al.*¹⁾ was found to be about 0.40. The separation efficiency curve of the impactor was not dependent on the stagnation pressure ratio, nor the distance between the jet and the plate when the stagnation pressure ratio was less than 0.70.

Introduction

The flow and the separation efficiency of a low pressure impactor were theoretically investigated by Flagan³⁾, Jurcik *et al.*⁵⁾ and Koga *et al.*^{7, 8)}. Flow reported by Flagan and Koga *et al.*, was obtained by adding an additional dissipation term to the Lax-Wendoroff¹⁰⁾ or MacCormack⁹⁾ scheme of the Euler equation. Jurcik *et al.* analyzed the Euler equation by the use of the flux corrected transfer scheme of a finite element method. In their analysis, the additional dissipation term was empirically determined. Their flow analysis is not sufficient since the additional dissipation term tends to increase with a decrease in the ratio of the outlet pressure to inlet pressure.

The main difficulty in solving nonlinear hyperbolic partial differential equations such as the Euler equation is the need to allow for discontinuous solutions, i.e., shock, even when the initial data are smooth¹⁰⁾. The development of numerical methods for compressible flow has been a rapidly growing area in the last twenty years. Major results of this work have been the development of total variation diminishing (TVD) finite difference schemes. These schemes have a number of attractive features for solving equations having a discontinuous solution, and for damping the spurious oscillation that occurs near discontinuities. Unfortunately, the apparent complexity of these TVD schemes has discouraged their widespread adoption. Recently, Yee¹²⁾, Roe¹¹⁾ and Davis²⁾ have developed a family of TVD schemes which are easily applicable to practical use.

The TVD scheme of an axisymmetric compressible Navier-Stokes equation was designed in order to determine flow in a low pressure impactor with a circular nozzle. This scheme was based on the MacCormack TVD scheme of compressible inviscid flow proposed by Yee¹²⁾, Roe¹¹⁾ and Davis²⁾. The flow and separation efficiency curves determined by this scheme were discussed by comparison with

previous ones^{7, 8)} obtained by adding an additional dissipation term to the MacCormack scheme of the Euler equation.

1. MacCormack TVD Variant of Yee-Roe-Davis Scheme of Inviscid Flow

First consider a scalar nonlinear homogeneous hyperbolic equation,

$$\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial z} = 0. \quad (1)$$

The MacCormack TVD variant of the Yee-Roe-Davis Scheme¹²⁾ for resolving Eq. (1) is given as,

Predictor:

$$w_j^{(1)} = w_j^n - \frac{\Delta t}{\Delta z} (f_{j+1}^n - f_j^n), \quad (2)$$

Corrector:

$$w_j^{(2)} = \frac{1}{2} \left\{ w_j^{(1)} + w_j^n - \frac{\Delta t}{\Delta z} (f_j^{(1)} - f_{j-1}^{(1)}) \right\}, \quad (3)$$

TVD:

$$w_j^{n+1} = w_j^{(2)} - \left[\phi_{j+\frac{1}{2}}^{(2)} - \phi_{j-\frac{1}{2}}^{(2)} \right], \quad (4)$$

here the superscripts (1) and (2) denote the values of the function evaluated at the intermediate solutions $w_j^{(1)}$ and $w_j^{(2)}$. Also, $\phi_{j+\frac{1}{2}}$ is

$$\phi_{j+\frac{1}{2}} = \frac{1}{2} \left[\varphi \left(v_{j+\frac{1}{2}} \right) - v_{j+\frac{1}{2}}^2 \right] \left(\Delta_{j+\frac{1}{2}} - Q_{j+\frac{1}{2}} \right), \quad (5)$$

where $\varphi(v)$ is sometimes known as the coefficient of the numerical viscosity term and is also an entropy correction for $|v|$ as;

$$\varphi(v) = \begin{cases} |v| & |v| \geq \delta_1 \\ (v^2 + \delta_1^2) / 2\delta_1 & |v| < \delta_1 \end{cases}, \quad (6)$$

where $\delta_1 = \max(0, v_{j+1/2} - v_p, v_j - v_{j-1/2})$,

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$$v_{j+\frac{1}{2}} = \frac{\Delta t}{\Delta z} c_{j+\frac{1}{2}}, \quad (7)$$

$$c_{j+\frac{1}{2}} = (f_{j+1} - f_j) / \Delta_{j+\frac{1}{2}}, \quad (8)$$

and $\Delta_{j+1/2} = w_{j+1} - w_j$, $Q_{j+1/2}$ is

$$Q_{j+\frac{1}{2}} = \minmod \left[2\Delta_{j-\frac{1}{2}}, 2\Delta_{j+\frac{1}{2}}, 2\Delta_{j+\frac{3}{2}}, \frac{1}{2} (\Delta_{j-\frac{1}{2}} + \Delta_{j+\frac{3}{2}}) \right], \quad (9)$$

where,

$\minmod(x_1, x_2, \dots) = \text{sgn}(x_1) \max(0, \min(|x_1|, \text{sgn}(x_1) x_2, \dots))$, i.e., $\minmod(x_1, x_2, \dots)$ is zero when signs of $x_2, \dots$ are different from that of x_1 and x_1 ($i = 1, 2, \dots$), of which absolute is minimum when signs of $x_1, x_2, \dots$ are the same.

2. Scheme for Axisymmetric Compressible Viscous Flow

The Navier-Stokes equation for a compressible viscous flow in a z-r coordinate system is given as:

$$\frac{\partial \mathbf{E}}{\partial \bar{t}} + \frac{\partial \mathbf{F}}{\partial \bar{z}} + \frac{\partial \mathbf{G}}{\partial \bar{r}} + \frac{\mathbf{H}}{\bar{r}} = \frac{\partial \mathbf{F}_1}{\partial \bar{z}} + \frac{\partial \mathbf{G}_1}{\partial \bar{r}} + \frac{\mathbf{H}_1}{\bar{r}}, \quad (10)$$

where $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{F}_1, \mathbf{G}_1$ and $\mathbf{H}_1$ represent the vectors as:

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} \bar{p} \\ m \\ n \\ q \end{bmatrix}, \mathbf{F}(\mathbf{E}) = \begin{bmatrix} m \\ m^2 / \bar{p} + \bar{P} \\ mn / \bar{p} \\ (q + \bar{P}) m / \bar{p} \end{bmatrix}, \\ \mathbf{G}(\mathbf{E}) &= \begin{bmatrix} n \\ mn / \bar{p} \\ n^2 / \bar{p} + \bar{P} \\ (q + \bar{P}) n / \bar{p} \end{bmatrix}, \mathbf{H}(\mathbf{E}) = \begin{bmatrix} n \\ mn / \bar{p} \\ n^2 / \bar{p} \\ (q + \bar{P}) n / \bar{p} \end{bmatrix}, \\ \mathbf{F}_1(\mathbf{E}) &= \begin{bmatrix} 0 \\ \tau_{zz} \\ \tau_{zr} \\ \frac{k}{a_i \rho_i D} \frac{\partial \bar{T}}{\partial \bar{z}} + m \tau_{zz} / \bar{p} + n \tau_{zr} / \bar{p} \end{bmatrix}, \\ \mathbf{G}_1(\mathbf{E}) &= \begin{bmatrix} 0 \\ \tau_{zr} \\ \tau_{rr} \\ \frac{k}{a_i \rho_i D} \frac{\partial \bar{T}}{\partial \bar{r}} + m \tau_{zr} / \bar{p} + n \tau_{rr} / \bar{p} \end{bmatrix}, \\ \mathbf{H}_1(\mathbf{E}) &= \begin{bmatrix} 0 \\ \tau_{zr} \\ \tau_{rr} - \tau_{\theta\theta} \\ \frac{k}{a_i \rho_i D} \frac{\partial \bar{T}}{\partial \bar{r}} + m \tau_{zr} / \bar{p} + n \tau_{rr} / \bar{p} \end{bmatrix}, \end{aligned} \quad (11)$$

where

$$m = \bar{p} \bar{u},$$

$$n = \bar{p} \bar{v},$$

$$q = \frac{\bar{P}}{\gamma - 1} + \frac{\bar{p} (\bar{u}^2 + \bar{v}^2)}{2},$$

$$\tau_{zz} = \frac{2\mu}{3a_i \rho_i D} \left(2 \frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\partial \bar{v}}{\partial \bar{r}} - \frac{\bar{v}}{\bar{r}} \right),$$

$$\tau_{rr} = \frac{2\mu}{3a_i \rho_i D} \left(2 \frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\partial \bar{v}}{\partial \bar{z}} - \frac{\bar{v}}{\bar{r}} \right),$$

$$\tau_{zr} = \frac{\mu}{a_i \rho_i D} \left(\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{z}} \right),$$

$$\tau_{\theta\theta} = \frac{2\mu}{3a_i \rho_i D} \left(2 \frac{\bar{v}}{\bar{r}} - \frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\partial \bar{v}}{\partial \bar{r}} \right).$$

The variables, viz., gas density ρ , velocity u in the z direction, velocity v in the r direction, static pressure P , temperature T and time t , can be made nondimensional using diameter D of the impactor inlet, inlet gas density ρ_i , and velocity of sound a_i , thus,

$$\bar{p} = \rho / \rho_i, \bar{u} = u / a_i, \bar{v} = v / a_i, \bar{P} = P / \rho_i a_i^2, \\ \bar{T} = T \gamma R / a_i^2, \bar{t} = t a_i / D, \bar{z} = z / D, \bar{r} = r / D,$$

where γ is the heat capacity ratio and R is the gas constant.

The MacCormack TVD variant of the Yee-Roe-Davis scheme was applied to solve Eq. (10). In the application, $\mathbf{H}/\bar{r}$ and all terms in the right-hand side are considered as additional terms of

$$\frac{\partial \mathbf{E}}{\partial \bar{t}} + \frac{\partial \mathbf{F}}{\partial \bar{z}} + \frac{\partial \mathbf{G}}{\partial \bar{r}}.$$

Then, the left-hand side of Eq. (10) is rewritten as;

$$\frac{\partial \mathbf{E}}{\partial \bar{t}} + \frac{\partial \mathbf{F}}{\partial \bar{z}} + \frac{\partial \mathbf{G}}{\partial \bar{r}} = \frac{\partial \mathbf{E}}{\partial \bar{t}} + \mathbf{A} \frac{\partial \mathbf{E}}{\partial \bar{z}} + \mathbf{B} \frac{\partial \mathbf{E}}{\partial \bar{r}},$$

where

$$\mathbf{A} = \partial \mathbf{F} / \partial \mathbf{E}, \mathbf{B} = \partial \mathbf{G} / \partial \mathbf{E}.$$

The Jacobian matrices $\mathbf{A}$ and $\mathbf{B}$ are diagonalized by the matrices $\mathbf{S}_z, \mathbf{S}_r$ and their inverses as;

$$\mathbf{A} = \mathbf{S}_z^{-1} \mathbf{\Lambda}_A \mathbf{S}_z, \mathbf{B} = \mathbf{S}_r^{-1} \mathbf{\Lambda}_B \mathbf{S}_r,$$

where

$$\begin{aligned} \mathbf{S}_z &= \begin{bmatrix} 1 & 0 & 0 & -1/\bar{a}^2 \\ 0 & \bar{\rho}\bar{a} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -\bar{\rho}\bar{a} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\bar{u}/\bar{p} - 1/\bar{p} & 0 & 0 & 0 \\ -\bar{v}/\bar{p} & 0 & -1/\bar{p} & 0 \\ \alpha\beta & -\bar{u}\beta & -\bar{v}\beta & \beta \end{bmatrix}, \\ \mathbf{S}_r &= \begin{bmatrix} 1 & 0 & 0 & -1/\bar{a}^2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \bar{\rho}\bar{a} & 1 \\ 0 & 0 & -\bar{\rho}\bar{a} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\bar{u}/\bar{p} - 1/\bar{p} & 0 & 0 & 0 \\ -\bar{v}/\bar{p} & 0 & -1/\bar{p} & 0 \\ \alpha\beta & -\bar{u}\beta & -\bar{v}\beta & \beta \end{bmatrix}, \\ \mathbf{\Lambda}_A &= \begin{bmatrix} \bar{a} & 0 & 0 & 0 \\ 0 & \bar{u} + \bar{a} & 0 & 0 \\ 0 & 0 & \bar{a} & 0 \\ 0 & 0 & 0 & \bar{u} - \bar{a} \end{bmatrix}, \\ \mathbf{\Lambda}_B &= \begin{bmatrix} \bar{v} & 0 & 0 & 0 \\ 0 & \bar{v} & 0 & 0 \\ 0 & 0 & \bar{v} + \bar{a} & 0 \\ 0 & 0 & 0 & \bar{v} - \bar{a} \end{bmatrix}, \end{aligned}$$

where $\bar{a}$ is the velocity of sound normalized with the inlet

Fig. 1 Computational grids used in flow field calculation

velocity of sound, $\alpha = 1/2 (\bar{u}^2 + \bar{v}^2)$ and $\beta = \gamma - 1$.

The matrices S_z and S_r are each given above as the product of two matrices. For each, the right matrix represents a transformation from conservative to nonconservative variables, for example, from $(\delta\bar{p}, \delta\bar{m}, \delta\bar{n}, \delta\bar{q})$ to $(\delta\bar{p}, \delta\bar{m}, \delta\bar{n}, \delta\bar{P})$, where δ represents the differential respect to $\bar{t}$, $\bar{z}$ or $\bar{r}$. The left matrix transforms from nonconservative to the characteristics forms $(\delta\bar{p} - \delta\bar{P}/\bar{a}^2, \rho\bar{a}\delta\bar{u} + \delta\bar{P}, \delta\bar{n}, -\rho\bar{a}\delta\bar{v} + \delta\bar{P})$ and $(\delta\bar{p} - \delta\bar{P}/\bar{a}^2, \delta\bar{u}, \rho\bar{a}\delta\bar{v} + \delta\bar{P}, -\rho\bar{a}\delta\bar{v} + \delta\bar{P})$ for the S_z and S_r matrices, respectively.

Equation (10) is rewritten in a finite difference form applied to the MacCormack TVD scheme as:

Predictor:

$$\Delta E_{jk}^{(1)} = -\Delta \bar{t} \left(\frac{\Delta_+ E_{jk}^n}{\Delta \bar{z}} + \frac{\Delta_+ G_{jk}^n}{\Delta \bar{r}} + \frac{H_{jk}^n}{\bar{r}} - \frac{\Delta_+ F_{1jk}^n}{\Delta \bar{z}} - \frac{\Delta_+ G_{1jk}^n}{\Delta \bar{r}} - \frac{H_{1jk}^n}{\bar{r}} \right), \quad (12)$$

$$E_{jk}^{(1)} = E_{jk}^n + \Delta E_{jk}^{(1)}, \quad (13)$$

Corrector:

Table 1 Boundary conditions used in flow field calculation

Bc	Bw1	Bw2	Bw3	Bi	Bo
$\frac{\partial \bar{p}}{\partial \bar{r}} = 0$	$\frac{\partial \bar{p}}{\partial \bar{r}} = 0$	$\frac{\partial \bar{p}}{\partial \bar{z}} = 0$	$\frac{\partial \bar{p}}{\partial \bar{z}} = 0$	$\bar{p} = 1$	$\frac{\partial \bar{p}}{\partial \bar{r}} = 0$
$\frac{\partial \bar{u}}{\partial \bar{r}} = 0$	$\bar{u} = 0$	$\bar{u} = 0$	$\bar{u} = 0$	$\frac{\partial \bar{u}}{\partial \bar{z}} = 0$	$\frac{\partial \bar{u}}{\partial \bar{r}} = 0$
$\bar{v} = 0$	$\bar{v} = 0$	$\bar{v} = 0$	$\bar{v} = 0$	$\bar{v} = 0$	$\frac{\partial \bar{v}}{\partial \bar{r}} = 0$
$\frac{\partial \bar{P}}{\partial \bar{r}} = 0$	$\frac{\partial \bar{P}}{\partial \bar{r}} = 0$	$\frac{\partial \bar{P}}{\partial \bar{z}} = 0$	$\frac{\partial \bar{P}}{\partial \bar{z}} = 0$	$\bar{P} = \bar{P}_i$	$\bar{P} = \bar{P}_o$

$$\Delta E_{jk}^{(2)} = -\Delta \bar{t} \left(\frac{\Delta_+ F_{jk}^{(1)}}{\Delta \bar{z}} + \frac{\Delta_+ G_{jk}^{(1)}}{\Delta \bar{r}} + \frac{H_{jk}^{(1)}}{\bar{r}} - \frac{\Delta_+ F_{1jk}^{(1)}}{\Delta \bar{z}} - \frac{\Delta_+ G_{1jk}^{(1)}}{\Delta \bar{r}} - \frac{H_{1jk}^{(1)}}{\bar{r}} \right), \quad (14)$$

$$E_{jk}^{(2)} = \frac{1}{2} (E_{jk}^n + E_{jk}^{(1)} + \Delta E_{jk}^{(2)}), \quad (15)$$

TVD:

$$E_{jk}^{n+1} = E_{jk}^{(2)} + \left[S_{j+\frac{1}{2}}^{-1(2)} \Phi_{j+\frac{1}{2}}^{(2)} - S_{j-\frac{1}{2}}^{-1(2)} \Phi_{j-\frac{1}{2}}^{(2)} \right] + \left[S_{k+\frac{1}{2}}^{-1(2)} \Phi_{k+\frac{1}{2}}^{(2)} - S_{k-\frac{1}{2}}^{-1(2)} \Phi_{k-\frac{1}{2}}^{(2)} \right]. \quad (16)$$

Δ_+ and Δ_- represent the forward and backward spatial differences and Δ represents the forward time difference. The elements of F_1 , G_1 and H_1 , $\bar{\tau}_{zz}$, $\bar{\tau}_{rr}$, $\bar{\tau}_{zr}$ and $\bar{\tau}_{\theta\theta}$ are rewritten in finite difference forms as:

$$\begin{aligned} \bar{\tau}_{zz} &= \frac{2\mu}{3a_i\rho_i D} \left(2\frac{\Delta_+ \bar{u}}{\Delta \bar{z}} - \frac{\Delta_+ \bar{v}}{\Delta \bar{r}} - \frac{\bar{v}}{\bar{r}} \right), \\ \bar{\tau}_{rr} &= \frac{2\mu}{3a_i\rho_i D} \left(2\frac{\Delta_+ \bar{u}}{\Delta \bar{r}} - \frac{\Delta_+ \bar{v}}{\Delta \bar{z}} - \frac{\bar{v}}{\bar{r}} \right), \\ \bar{\tau}_{zr} &= \frac{\mu}{a_i\rho_i D} \left(\frac{\Delta_+ \bar{u}}{\Delta \bar{r}} + \frac{\Delta_+ \bar{v}}{\Delta \bar{z}} \right), \\ \bar{\tau}_{\theta\theta} &= \frac{2\mu}{3a_i\rho_i D} \left(2\frac{\bar{v}}{\bar{r}} - \frac{\Delta_+ \bar{u}}{\Delta \bar{z}} - \frac{\Delta_+ \bar{v}}{\Delta \bar{r}} \right). \end{aligned}$$

The Δ_+ and Δ_- of $\Delta_\pm$ are used in the corrector and predictor step respectively in this study, and the $\bar{\tau}_{zz}$, $\bar{\tau}_{rr}$, $\bar{\tau}_{zr}$ and $\bar{\tau}_{\theta\theta}$ are represented by the central difference form as a result of this scheme. Also $S_{j+1/2}$ and $S_{k+1/2}$ are the difference forms of matrices S_z and S_r , respectively. The inverses S_z^{-1} and S_r^{-1} are easily derived. The elements of the vector $\Phi_{j+1/2}$ in the z-coordinate are the same as Eqs. (4) and (5) with $c_{j+1/2}$ replaced by the diagonal element of Λ_A and $\Delta_{j+1/2}$ replaced by the element of the vector $S_{j+1/2} (E_{j+1,k} - E_{j,k})$. The element of $\Phi_{k+1/2}$ in the r-coordinate is also similar to that of $\Phi_{j+1/2}$.

3. Results and Discussion

This scheme was applied to the analysis of the flow in a low pressure impactor designed by Hering *et al.*⁽¹⁾. Figure 1 shows the computational grid used in the flow field calculation. Table 1 shows the boundary conditions. The characteristic theory⁽⁴⁾ provides guidance as to the number and form of the inlet-outlet boundary conditions,

Fig. 2 Mach number and static pressure contours at ratio of jet-plate distance to jet diameter of $S/D = 0.5$ and static pressure ratio of $P_o/P_i = 0.93$

Fig. 3 Mach number and static pressure contours at ratio of jet-plate distance to jet diameter of $S/D = 0.5$ and static pressure ratio of $P_o/P_i = 0.76$

Bi and Bo. Physically, information is carried along the characteristics of which slopes are the diagonal elements of Λ_A and Λ_B . Therefore characteristics entering the computational domain require boundary conditions to be specified.

A subsonic inflow requires boundary conditions on three variables and requires the fourth variable to be computed from the boundary values and interior solution. At subsonic outflow, three characteristics are outside of the computational domain, and one enters the domain at the outlet. Consequently one boundary condition must be set at subsonic outflow.

For supersonic inflow, all characteristics enter the domain at the inlet, therefore boundary conditions are required on all variables. Conversely, at a supersonic outflow all characteristics point out of the domain so that

Fig. 4 Relationships of static pressure P_o/P_i with stagnation pressure ratio P_o^*/P_i^* and inlet Mach number M_i of impactor at a ratio of jet-plate distance to jet diameter of $S/D = 0.5$

no boundary conditions may be specified.

Figure 2 shows the calculated contours of Mach number and static pressure in the impactor at the ratio of outlet static pressure to inlet static pressure, $P_o/P_i = 0.93$. The flow in the impactor is subsonic as shown in Fig. 2(a). Since the velocity distribution of bulk fluid in the throat is uniform, the effect of viscosity is not observed. The velocity distribution in the throat reported in the previous paper⁷⁾ was parabolic since the artificial viscosity was too large. Figure. 2(b) shows the static pressure contours in the impactor corresponding to Fig. 2(a). The isobars are plotted as the ratio of the local static pressure to the inlet stagnation pressure. The static pressure of the subsonic jet impinging on the plate is equal to the inlet stagnation pressure at the centerline. The static pressure is not changed in the impactor, except in the region near the plate, as shown in Fig. 2 (b). The outlet static pressure is equal to the outlet stagnation pressure.

Figure 3 shows the calculated contours of Mach numbers and static pressure in the impactor at $P_o/P_i = 0.76$. The Mach number of the inlet of the throat is equal to one. The flow expands and accelerates to the supersonic in the throat. It leaves the jet throat and rapidly decelerates near the plate. The shock is clearly observed at the half distance between the outlet of jet and the impaction plate. The shock could be more accurately simulated by this scheme than previous ones^{7, 8)}. The static pressure at the jet outlet is equal to the outlet pressure of the impactor, and pressure recovery to the stagnation pressure is very nearly complete on the centerline, as shown in fig. 3(b).

The flow pattern in the impactor when its dimensions are the same was determined only by the static pressure ratio and was not changed with the values of inlet static pressure in the range from 0.27 kPa to 6.67 kPa, as was shown in the previous simulated results⁸⁾.

Figure 4 shows the relationship between the static pressure ratio, P_o/P_i and the stagnation pressure ratio,

Fig. 5 Comparison between calculated separation efficiencies and those measured by Hering *et al.*¹⁾ at a ratio of jet-plate distance to jet diameter of $S/D = 0.5$

P_o^*/P_i^* . Since the outlet velocity is negligible, it is assumed that $P_o = P_o^*$. The stagnation pressure ratio P_o^*/P_i^* can thus be determined from the inlet velocity of gas and the static pressure ratio by,

$$\frac{P_o^*}{P_i^*} = \left(\frac{P_o}{P_i} \right) \left(1 + \frac{\gamma-1}{2} M_i^2 \right)^{-\frac{\gamma}{\gamma-1}} \quad (17)$$

where M_i is Mach number at the inlet, which is also shown in Fig. 4. The stagnation pressure ratio at which the Mach number of the inlet of the throat equals $M_i = 1.0$ is defined as the critical one. The critical stagnation pressure ratio of the impactor designed by Hering *et al.*¹⁾ could be estimated to be about 0.4 from the results, as shown in Fig. 4. In flow with a value less than 0.4, the pressure might oscillate vigorously due to the physical nature of the flow itself, and the flow could not be determined by the computational scheme.

The trajectory of a particle in the impactor can be calculated easily if the flow field, $u(z, r)$, $v(z, r)$, and fluid properties are known. the equations of motion of particles^{3, 7, 8)} were nondimensionalized by the same manner as the equation of fluid motion:

$$\frac{d^2 \bar{z}}{d\bar{t}^2} = \frac{1}{Stk_i} \left(\frac{\mu}{\mu_i} \right) \left(\frac{C_i}{C} \right) \left(\bar{w} - \frac{d\bar{z}}{d\bar{t}} \right), \quad (18)$$

$$\frac{d^2 \bar{r}}{d\bar{t}^2} = \frac{1}{Stk_i} \left(\frac{\mu}{\mu_i} \right) \left(\frac{C_i}{C} \right) \left(\bar{v} - \frac{d\bar{r}}{d\bar{t}} \right), \quad (19)$$

where Stk_i is the Stokes number, which is defined in terms of the physical properties of the fluid at the inlet as:

$$Stk_i = \frac{\rho_p a_i D_p^2 C_i}{9\mu_i D}, \quad (20)$$

The Cunningham slip correction factor is assumed to be given by,

$$C_i = 1 + 2\lambda_i / D_p \{ 1.257 + 0.4 \exp(-0.55D_p / \lambda_i) \}, \quad (21)$$

where D_p is the particle diameter and λ is the mean free

Table 2 Diameter of inlet and stagnation pressures of stage of impactor¹⁾

Stage no.	D [mm]	P_i^* [Torr]	P_o^* [Torr]	P_o^*/P_i^*
5	1.10	150	140	0.93
6	0.99	140	106	0.76
7	0.99	106	50	0.47
8	1.40	50	8	0.16

(a) $P_o / P_i = 0.93$

(b) $P_o / P_i = 0.96$

Fig. 6 Separation efficiency curves of impactor at static pressure ratio (a) $P_o/P_i = 0.93$ and (b) $P_o/P_i = 0.96$

path of gas molecules. This Stokes number, defined by Eq. (20), was also used to describe the separation efficiency curve of a low-pressure virtual impactor⁶⁾. The value of the diameter for the particles that just impact the plate was evaluated for different initial radial positions in order to determine the separation efficiency.

Figure 5 shows the calculated separation efficiency curves and the experimental results. **Table 2** lists the diam-

Fig. 7 Relationships between stagnation pressure ratio P_o^*/P_i^* and inlet Mach number M_i of impactor

eter of inlet, the inlet and outlet stagnation pressure, and stagnation pressure ratio at each stage of the impactor designed by Hering *et al.*¹⁾. It is possible to compare the calculated efficiency curves with the measurements since the stagnation pressure ratio can be determined from the inlet velocity of gas and the static pressure ratio as shown in Fig. 4. The high efficiency portion of the calculated separation efficiency curve displays an arc as shown in the figure. While the low efficiency portion is sharper than was found experimentally. The calculated separation efficiency curve was largely changed with a static pressure ratio in the range from 0.93 ($P_o^*/P_i^* = 0.70$) to 0.97 (0.97). The separation efficiency of stage 5 coincides with the curves at the static pressure ratio 0.96 (0.93). The separation efficiency of stage 6 was coincident with the curves at the static pressure ratio 0.95 (0.89). The separation efficiency curves calculated at static pressure ratios 0.76 (0.41) and 0.85 (0.46) were in good agreement with the measured values of stages 7 and 8.

Figure 6 shown the separation efficiency curves of the impactor, for which the distance between the jet outlet and the plate is from 0.5 to 3.0 times the diameter of the throat. The separation efficiency curve was changed little with S/D at $P_o/P_i = 0.93$, while efficiency curves changed at $P_o/P_i = 0.96$. The range of stagnation pressure ratios at $P_o/P_i = 0.93$ is from 0.51 ($S/D = 3$) to 0.70 ($S/D = 0.5$), while the range of stagnation pressure ratios at $P_o/P_i = 0.96$ is from 0.60 ($S/D = 3$) to 0.93 ($S/D = 0.5$).

Figure 7 shows the relationships between the stagnation pressure ratio P_o^*/P_i^* and the inlet Mach number M_i . The inlet Mach number is determined by the stagnation pressure ratio and is changed little with S/D . **Figure 8** shows the relationship between the stagnation pressure ratio P_o^*/P_i^* and the square root of 50% cut stokes number Stk_i . From Fig. 8, we can conclude that the sepa-

Fig. 8 Relationships between stagnation pressure ratio P_o^*/P_i^* and 50% cut Stokes number Stk_{i50}

Fig. 9 Relationships between stagnation pressure ratio P_o^*/P_i^* and 50% cut Stokes number Stk_{jet50}

ration efficiency curve is dependent on neither the stagnation pressure ratio, nor the S/D of the impactor when the stagnation pressure ratio is less than 0.70. **Figure 9** also shows the relationship between the stagnation pressure ratio P_o^*/P_i^* and the square root of 50% cut Stokes number Stk_{jet} , which is defined in terms of physical properties of the fluid at the jet outlet as:

$$Stk_{jet} = \frac{\rho_p u_{jet} D_p^2 C_{jet}}{9 \mu_{jet} D} \quad (22)$$

where u_{jet} is the velocity in the z direction at the jet outlet.

from Figs. 8 and 9, we can conclude that Stk_i is almost equal to Stk_{jet} numerically.

Conclusion

A TVD scheme based on a compressible Navier-Stokes equation was applied to axisymmetric flow in a low-pressure impactor. This TVD scheme was a type combined with a two-step MacCormack scheme.

Through numerical study of the impactor by use of this scheme, it was possible to make the following conclusions:

- 1) the critical stagnation pressure ratio of the impactor designed by Hering *et al*⁽¹⁾ is about 0.40;
- 2) the shock is clearly observed at the half distance between the outlet of the jet and the impaction plate;
- 3) the separation efficiency curve is dependent neither on the stagnation pressure ratio, nor the S/D of impactor when the stagnation pressure ratio is less than 0.70.

Nomenclature

A, B	= Jacobian matrix	[-]
a	= velocity of sound	[m/s]
C	= cunningham slip correction factor	[-]
c	= velocity defined by Eq. (8)	[m/s]
D	= diameter of inlet	[m]
D_p	= diameter of particle	[m]
E, F, G, H, F₁, G₁, H₁	= vector defined by Eq. (11)	[-]
E	= separation efficiency	[-]
$f(w)$	= flux function of w used in Eq. (1)	[kg/(m ² ·s)]
M	= Mach number	[-]
Q	= function defined by Eq. (9)	[-]
q	= dimensionless t particle total energy	[-]
p	= static pressure	[kPa]
R	= gas constant	[J/(kg·K)]
r	= r-coordinate	[m]
Δr	= step size in r direction	[m]
S	= distance between outlet of jet and impaction plate	[m]
Stk	= Stokes number	[-]
T	= temperature	[K]
t	= time	[s]
Δt	= time step	[s]
u	= z component of velocity	[m/s]
v	= r component of velocity	[m/s]

w	= scalar used in Eq. (1)	[kg/m ³]
z	= z-coordinate	[m]
Δz	= step size in z direction	[m]
Δ	= difference of w used in Eqs. (5), (8) and (9)	[-]
δ_i	= parameter	[-]
ϕ	= TVD flux	[-]
γ	= heat capacity ratio	[-]
λ	= mean free path of fluid	[m]
ν	= $(c\Delta t/\Delta z)$ defined by Eq. (7)	[-]
μ	= viscosity of fluid	[pa·s]
ρ	= density of fluid	[kg/m ³]
ρ_p	= density of particle	[kg/m ³]
τ	= viscous stress	[N/m ²]

Superscript

-	= dimensionless quantity
*	= stagnation
(1), (2)	= intermediate solution

subscripts

i	= inlet
j	= grid number in z direction
jet	= jet outlet
k	= grid number in r direction
o	= outlet
r	= r-coordinate
z	= z-coordinate
θ	= θ -coordinate

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