EXPERIMENTAL STUDY ON SHAPE SEPARATION OF PARTICLES BY USING RADIAL SETTLING CHARACTERISTICS THROUGH STATIONARY LIQUID IN ROTATING VESSEL

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For the purpose of wet separation of particles differing in shape, particle settling characteristics have been investigated in a stationary liquid simultaneously subjected to centrifugal and gravitational forces. Several kinds of irregularly shaped particles were settled through viscous silicone oil at rest in a closed rectangular vessel rotated at low speeds. The settling trajectories of the particles in the liquid were measured in the laminar flow region. When the initial settling orientation of each particle was set in a given and appropriate direction, the radial settling displacement of the particle depended approximately on the particle shape, regardless of the size. By devising a simple feeding method, it was possible to make the initial settling orientation of particles automatically uniform. Continuous shape separation of particles in the liquid could be expected by applying a feeding method to the separation field receiving both centrifugal and gravitational forces as used in this paper.

Introduction

Recently, separation of fine particles based on shape in liquid has received considerable attention (Endoh, *et al.*, 1992; Moro, *et al.*, 1989), because even comparatively adhesive particles in the air are more easily dispersed in liquid. The settling velocity of a non-isometric particle in liquid depends not only on the particle shape, but on the size or the settling orientation (Carmichael, 1982; Heiss and Coull, 1952). Conventional wet classifications have been performed using differences in the settling velocity of particles. However, it is very difficult, for example, to prepare particles of precisely the same size from mixtures. Therefore, these methods can not be so easily applied to the separation of differently shaped particles.

In the present paper, for the purpose of shape separation of particles in liquid, the settling behavior of irregularly shaped particles has been investigated experimentally. The settling trajectories of particles differing in shape and size were measured in the laminar flow region in a stationary viscous liquid subjected to both centrifugal and gravitational forces, simultaneously. Furthermore, a simple feeding method was discussed to make the initial settling orientation of particles uniform automatically.

1. Experimental

1.1 Tri-axial shape analysis of a particle

As listed in Table 1, twenty glass particles differing in shape and size were used as experimental materials. Each of six irregularly shaped particles in every group L, M, S,

has a similar value of d_s , the diameter of a sphere of volume equivalent to the particle, which is given as

$$d_s = \sqrt[3]{6m / \pi \rho_p} \quad (1)$$

where m and ρ_p are the mass and density of a particle. The values of $\bar{d}_s$ in Table 1 show the average of d_s for about 6 particles in every group.

As shown in Fig. 1, a stable orientation for each particle was observed on a vibrating flat baseplate with a side wall along which the particle could be transported. Then the directions of the particle tri-axes t , u , and v were defined (Yamamoto, *et al.*, 1991). The axes t and u are parallel and normal to the side wall, while the axis v is perpendicular to the baseplate. To identify the direction of axes, two kinds of small circles were marked on the particle surface along the t -axis. The projections in Fig. 1 were obtained by viewing each of four representative particles in group L parallel to the axes t , u , and v .

Furthermore, the corresponding shape indices $\psi = (\psi_{(t)}, \psi_{(u)}, \psi_{(v)})$ defined by Eq.(2) were determined by Image Analyzer (Nireco, Luzex II) for a contour of the particle image.

$$\Psi = s/l (\leq 1.0) \quad (2)$$

where l and s are the maximum length and breadth of the contour. In this paper, as used in Fig. 1, subscripts (t) , (u) , and (v) denote values measured in the direction of axes t , u , and v for a particle, respectively.

A shape index $\psi_{(tuv)}$ defined by the following expression was used to evaluate the three-dimensional shape of

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Table 1 Properties of glass particles used

Particle	Shape	d_s ($\bar{d}_s$) [mm]	$\psi_{(uv)}$ [-]	ρ_p [kg·m ⁻³]
SP1	spherical	2.97	0.992	2490
SP2	spherical	5.03	0.986	2490
Group L (L1-L6)	irregular	4.48-4.66 (4.58)	0.505-0.919	2490
Group M (M1-M6)	irregular	3.86-3.94 (3.91)	0.455-0.936	2490
Group S (S1-S6)	irregular	3.01-3.16 (3.11)	0.403-0.893	2490

Fig. 1 Representative images of four particles viewed from three directions

a single particle (Yamamoto, *et al.*, 1991).

$$\psi_{(tuv)} = 1 - \sqrt{\{(1 - \psi_{(t)})^2 + (1 - \psi_{(u)})^2 + (1 - \psi_{(v)})^2\} / 3} \quad (\leq 1.0) \quad (3)$$

It can be seen in Fig.1 that the shapes of particles become more spherical as $\psi_{(tuv)}$ is closer to unity, and that the directions of the axes t , u , and v of a particle approximately correspond to ones of the length, breadth, and thickness, respectively.

1.2 Measurements of shape correction factor K

The shape correction factor, K , was measured by using a falling-sphere method for every irregularly shaped particle in the following specified settling orientations. The factor K shows the effects of shape and orientation only (Heiss and Coull, 1952).

A transparent cylindrical acrylic container (inside diameter $D = 63.5$ mm, height = 300mm) was filled with a transparent viscous silicone oil (Shin-Etsu Chemical, KF-96-1000CS, density $\rho_f = 966$ kg·m⁻³, viscosity $\mu_f = 962$ mPa·s, at 25°C, Newtonian fluid), as shown in Fig. 2. The temperature of the liquid was kept at 25°C $\pm$ 0.05°C all through the experiment.

Fig. 2 Measuring method of shape correction factor K

First, the particles were set manually in the prescribed orientations at point S in the center of a guide located at the top of the cylinder, as shown in Fig. 2. The particles were released from tweezers through the guide, and were timed between two specified graduations (G_U and G_L) with a digital stop watch. The measurements were restricted to the cases where the settling positions between G_U and G_L were almost identical to the initial orientations set at point S. The settling velocity, u_p , between the graduations G_U and G_L was measured three times for each orientation. It was confirmed that these experiments were conducted in the Stokes flow region, $Re (= d_s u_i \rho_f / \mu_f) < 0.10$.

From the actual settling velocity measured in the finite liquid inside the cylindrical container, u_p , the terminal velocity of an irregularly shaped particle in an infinite medium, u_{ic} , was estimated by the following equation.

$$u_{ic} = u_i / f_w \quad (4)$$

where f_w is the correction factor for wall effect of the container, and is expressed as a function of d_s and D , as follows.

$$f_w = 1 - 2.104 (d_s / D) + 2.09 (d_s / D)^3 - 0.95 (d_s / D)^5 \quad (< 1.0) \quad (5)$$

The above equation was proposed by Faxen for a sphere of d_s in diameter (JIS Z8803). It was suggested that the equation was approximately applicable to regularly shaped, non-spherical particles as well (Heiss and Coull, 1955). In this paper, therefore, the equation was adopted to estimate the wall effect for irregularly shaped particles used.

On the other hand, the terminal velocity of a sphere of volume equivalent to an irregularly shaped particle in an infinite medium, u_s , is given by Stokes equation in the laminar flow region.

$$u_s = g d_s^2 (\rho_p - \rho_f) / 18 \mu_f \quad (6)$$

where g is acceleration due to gravity. From these procedures, the shape correction factor K defined below was obtained for every orientation.

$$K = u_{ic} / u_s \quad (7)$$

The value of K for each orientation was expressed, as shown in Fig.2, based on the direction of the particle axis corresponding to the falling direction ($-z$). The notations $K_{(+t)}$ and $K_{(-t)}$, for example, denote the values of K

Fig. 5 Relations between shape correction factor ratio $K_{(t)}/K_{(v)}$ or $K_{(u)}/K_{(v)}$ and shape index $\psi_{(tuv)}$ for each particle

2. Results

The values of shape correction factor, $K_{(t)}$, $K_{(u)}$ and $K_{(v)}$, obtained experimentally (cf. Fig. 2) ranged from 0.71 to 1.01, in the order $K_{(t)} \geq K_{(u)} \geq K_{(v)}$ for all irregularly shaped particles used.

Figure 5 shows the relationships between a shape correction factor ratio $K_{(t)}/K_{(v)}$ or $K_{(u)}/K_{(v)}$ and the shape index $\psi_{(tuv)}$ for each particle. As shown in Figs. 5a and 5b, the ratios, $K_{(t)}/K_{(v)}$ and $K_{(u)}/K_{(v)}$, can be approximated by the following equations.

$$K_{(t)}/K_{(v)} = \psi_{(tuv)}^{-0.23} \quad (8-1)$$

$$K_{(u)}/K_{(v)} = \psi_{(tuv)}^{-0.09} \quad (8-2)$$

It can be seen from the above equations that the degree of increase of the shape correction factor ratio with decreasing $\psi_{(tuv)}$ is larger in $K_{(t)}/K_{(v)}$ than $K_{(u)}/K_{(v)}$.

Figure 6 shows an example of settling trajectories and orientations about the particle L6 obtained from the experiment for $FB_{(RAN)}$. Here, Fig. 6a represents a series of projections taken from the origin O in the radial direction with the video camera. Figure 6b shows front views of the particle which were estimated by comparing the shapes of the particle observed from many directions with each projection shown in Fig. 6a. The value of Δr indicates the r -directional displacement during settlement. As can be seen from Fig. 6b, in the case of $FB_{(RAN)}$, the settling orientation in the liquid was approximately expressed as $PO_{(v_c, t_g)}$, in which the directions of the particle axes v and t correspond to those of centrifugal and gravitational forces, respectively (cf. Fig. 4).

The behavior of the particle through the guide pipe was observed when the particle was placed on the platform in random orientation. It was found that the settling

Fig. 6 Settling orientation of a representative particle, L6, in case of $FB_{(RAN)}$ at $n = 40 \text{ min}^{-1}$

orientation of the particle in the guide pipe became $PO_{(v_c, t_g)}$ as the particle moved downward along the wall of the guide pipe. This phenomenon would result from the interaction between the centrifugal and gravitational forces, and the wall effect for the particle in the guide pipe. This suggests that the initial settling orientation of particles in the container at the point S could be controlled automatically by using the feeding device with a guide pipe as used in FB.

In the cases of $FA_{(t_c, v_g)}$ and $FA_{(u_c, v_g)}$ for this particle, the settling orientations of the particle in the container were observed to be approximately the same as the initial orientation.

Figure 7 shows representative results obtained from the experiment in the rotating container at $n = 40 \text{ min}^{-1}$. The r -directional displacements, $\Delta r (= r_E - r_S)$, are plotted against the three-dimensional shape index $\psi_{(tuv)}$ for each particle.

Figures 7a and 7b represent the respective results of the experiment $FA_{(t_c, v_g)}$ and $FA_{(u_c, v_g)}$ in which the particles were fed to the initial settling point S with the device FA in the orientations $PO_{(t_c, v_g)}$ and $PO_{(u_c, v_g)}$, manually arranged on the starting platform as shown in Fig. 4. Figure 7a indicates that in the case of $FA_{(t_c, v_g)}$, Δr depends approximately on the particle shape $\psi_{(tuv)}$ regardless of the size d_s , and that Δr decreases with increasing $\psi_{(tuv)}$ until $\psi_{(tuv)} \approx 0.6$. Above 0.6, Δr becomes nearly constant. Fig. 7b suggests that, in the case of $FA_{(u_c, v_g)}$, there are no obvious correlations between Δr and $\psi_{(tuv)}$. These results suggest that the larger differences in r -directional settling characteristics among particles would arise in the settling orientation $PO_{(t_c, v_g)}$ rather than $PO_{(u_c, v_g)}$, as discussed in more detail later.

Figure 8 shows the relationships in the case of $FB_{(RAN)}$, where the particles were fed by the device FB

Fig. 7 Relations between r -directional displacement Δr and shape index $\psi_{(tuv)}$ for each particle at $n = 40 \text{ min}^{-1}$ in cases of $FA_{(tc, vg)}$ and $FA_{(uc, vg)}$

through the guide pipe to the initial settling point S in the container without arrangements for orientation by hand on the starting platform. In this case, it is obvious for each rotational speed n that Δr increases as $\psi_{(tuv)}$ is closer to unity, i.e., as the particle shape becomes more spherical, on the contrary to Fig. 7a ($FA_{(tc, vg)}$). In addition, Δr increased with rotational speed n for a given particle. The differences in the relation between Δr and $\psi_{(tuv)}$ for Figs. 7a and 8, would result from those in the corresponding orientation $PO_{(tc, vg)}$ and $PO_{(vc, tg)}$ settled in liquid, as investigated in the later discussion.

On the other hand, displacements of particles in the y -direction, Δy , were observed to be negligibly small in these experiments.

These results suggest that the effective separation of differently shaped particles would be performed by using differences in Δr among particles when the initial settling orientation could be controlled in the orientation of $PO_{(tc, vg)}$ or $PO_{(vc, tg)}$. The application of the feeding device with a guide pipe to this field would make separation possible.

3. Discussion

The settling characteristics in Figs. 7 and 8 were investigated on the basis of shape correction factor K as follows.

Here, it is assumed that in liquid in the container, the settling orientations of every particle under the feeding condition $FA_{(tc, vg)}$, $FA_{(uc, vg)}$, and $FB_{(RAN)}$, correspond to $PO_{(tc, vg)}$, $PO_{(uc, vg)}$, and $PO_{(vc, tg)}$, respectively. Under these conditions it would be supposed that the radial displace-

Fig. 8 Relations between Δr and $\psi_{(tuv)}$ in case of $FB_{(RAN)}$ at various rotational speeds n

ment of a particle in the liquid for each orientation is closely connected with the difference in mobility of the particle towards the centrifugal and gravitational directions.

Suppose that only one of either centrifugal or gravitational forces is exerted on a particle in liquid in the above respective orientation. The larger the value of K for a particle in the directions of centrifugal or gravitational force ($K_{(c)}$ or $K_{(g)}$) becomes, the faster the particle will settle in each direction owing to less fluid resistance to motion. In the rotating container used in this paper, both forces simultaneously act on the particle. Therefore, a shape correction factor ratio $K_{(c)}/K_{(g)}$ was adopted as the representative value of the difference in mobility between the above two directions. The factors $K_{(c)}$ and $K_{(g)}$ were given by the values of K , which were obtained experimentally from the particle subjected to centrifugal and gravitational forces for a given orientation, respectively. The ratio $K_{(c)}/K_{(g)}$ was derived by applying Eq.(8) to each orientation as follows.

$$K_{(c)} / K_{(g)} = \psi_{(tuv)}^p \quad (9)$$

where, the exponents p obtained were

$$\begin{aligned} p = -0.23 & \quad \because K_{(c)}/K_{(g)} = K_{(t)}/K_{(v)} \\ & \quad [FA_{(tc, vg)}; PO_{(tc, vg)}] \\ p = -0.09 & \quad \because K_{(c)}/K_{(g)} = K_{(u)}/K_{(v)} \\ & \quad [FA_{(uc, vg)}; PO_{(uc, vg)}] \\ p = 0.23 & \quad \because K_{(c)}/K_{(g)} = K_{(v)}/K_{(t)} \\ & \quad [FB_{(RAN)}; PO_{(vc, tg)}] \end{aligned}$$

Figure 9 shows the relationships between $\ln \Delta r$ and $\ln (K_{(c)}/K_{(g)})$ for every particle under the above assumption for various experimental conditions. In this figure, the

Fig. 9 Logarithmic plots between Δr [mm] and $K_{(c)}/K_{(g)}$ [-] for each experimental condition

values of $K_{(c)}/K_{(g)}$ for each particle were estimated from Eq. (9) on the basis of $\psi_{(tuv)}$ measured. It indicates that the r -directional displacement Δr depends on $K_{(c)}/K_{(g)}$, and is approximated by the following expression.

$$\Delta r = a (K_{(c)} / K_{(g)})^b \quad (10)$$

where a and b are constants which depend on the rotational speed n and the feeding condition as listed in Fig. 9. By substituting Eq. (9) into Eq. (10), Δr is expressed as a function of $\psi_{(tuv)}$ for each feeding condition as follows.

$$\Delta r = a \psi_{(tuv)}^{pb} \quad (11)$$

where the values of p differ from each other for every feeding condition as shown in Eq. (9).

The relations between Δr and $\psi_{(tuv)}$ by Eq.(11) are superimposed on Figs. 7 and 8. The calculated results show that, in the case of the orientation $PO_{(tc,vg)}$ (in Fig. 7a) and $PO_{(vc,tg)}$ (in Fig. 8), the radial displacement Δr considerably increases with decreasing and increasing shape index $\psi_{(tuv)}$, respectively. In the case of $PO_{(uc,vg)}$ (in Fig.7b), however, Δr doesn't vary so much with $\psi_{(tuv)}$. The difference in the settling characteristics between the above three cases would be based on the gap in the effect of $\psi_{(tuv)}$ on $(K_{(c)}/K_{(g)})$ as recognized in Eq. (9). That is, the orientations $PO_{(tc,vg)}$ and $PO_{(vc,tg)}$ have an opposite sign of exponent p in Eq.(9), and have larger values of $|p|$ than $PO_{(uc,vg)}$. Furthermore, calculated values in Figs. 7 and 8 agree with experimental trends. Therefore, it was confirmed that the radial settling characteristics of various shaped particles, Δr , depended on the relations between $K_{(c)}/K_{(g)}$ and $\psi_{(tuv)}$, and that the assumption in Eq. (9) noted in the square brackets was reasonable.

Conclusion

The radial settling displacements of a particle, Δr , were measured in the stationary liquid subject to centrifugal and gravitational forces, simultaneously. When the initial settling orientations of each particle were set in a

given and appropriate direction, Δr depended approximately on the particle shape $\psi_{(tuv)}$, regardless of the size. The relation between Δr and $\psi_{(tuv)}$ could be explained in terms of the shape correction factor ratio $K_{(c)}/K_{(g)}$.

A simple feeding method for making the initial settling orientations of particles uniform was found. The shape separation of particles could be expected by applying such a kind of feeding method to this separation field.

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Nomenclature

a, b	= constants in Eqs. (10) and (11)	[m], [-]
D	= inside diameter of cylindrical container	[m]
d_s	= equivalent volume diameter of particle	[m]
$\bar{d}_s$	= average value of d_s about particles in a group	[m]
f_w	= correction factor for wall effect proposed by Faxen, given by Eq. (5)	[-]
g	= acceleration due to gravity	[m·s ⁻²]
K	= shape correction factor at constant volume defined by Eq.(7)	[-]
l	= maximum length of each projection of a particle	[m]
m	= mass of a particle	[kg]
n	= rotational speed of a rectangular container around origin O	[s ⁻¹]
O- r, y, z	= rectangular coordinate system fixed on a rotating container(see Fig. 3a)	[m]
p	= exponent in Eqs. (9) and (11)	[-]
Re	= Reynolds Number (= $d_s u_p \rho_f / \mu_f$)	[-]
s	= breadth of each projection of a particle	[m]
t, u, v	= rectangular coordinates located on a particle in direction of length, breadth, and thickness	[m]
u_i	= settling velocity of particle observed in finite medium	[m·s ⁻¹]
u_{ic}	= settling velocity of particle in infinite medium estimated by Eq.(4)	[m·s ⁻¹]
u_s	= settling velocity of sphere in infinite medium, given by Eq.(6)	[m·s ⁻¹]
Δr	= moving distance in r -direction (= $r_E - r_s$)	[m]
Δ_y	= moving distance in y -direction (= $y_E - y_s$)	[m]
μ_f	= viscosity of liquid	[Pa·s]
ρ_f	= density of liquid	[kg·m ⁻³]
ρ_p	= density of particle	[kg·m ⁻³]
ψ	= shape index defined by Eq. (2)	[-]
$\psi_{(tuv)}$	= three-dimensional shape index defined by Eq. (3)	[-]

<Subscripts>

$(t), (u), (v)$	= values measured in direction of length, breadth, and thickness of particle
S, E	= starting and end points of particle settling
$(c), (g)$	= values in direction of centrifugal and gravitational force

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