A STAGewise ALGORITHM FOR MULTICOMPONENT MULTISTAGE BATCH DISTILLATION CALCULATION

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A stagewise algorithm is proposed for the solution of multicomponent multistage batch distillation problems. In order to make the algorithm more precise and flexible, a generalized implicit Euler equation is implemented for numerical integration of the process variables on each stage at succeeding time steps. An overall normalized \( \theta \) method is also employed to accelerate convergence for each time step. The relative performance of the proposed method compared with the Runge-Kutta method is shown using a numerical example; the CPU time can be decreased by up to 1/300. Comparison of the simulation results with the experiment suggests the adequacy of the proposed method.

Introduction

Recently, chemical industries have become more versatile, pursuing the production of a variety of high value added chemicals. Thus, batch distillation is attracting more attention as a possible separation process which provides high product purity as well as high yield.

Two approaches to model the change in the state inside the batch distillation column with time have been developed; the short-cut approach and the rigorous approach. Since the solution of batch distillation problems has an intrinsically integral nature, it requires a tremendous amount of calculation for design or optimization purposes. Short-cut approaches are employed in such situations to reduce computation time at the sacrifice of accuracy. They enable us to obtain approximate solutions within a limited time. Rigorous approaches may be used to check the consistency of the solution and to make detailed adjustment if necessary. The role of rigorous approaches becomes more significant whenever a more reliable solution is needed.

The primary concern in development of the rigorous simulation method for batch distillation is to include more practical models, though the increase of calculation load is to be minimized as well. This can be accomplished by employing an integration formula with high accuracy. Distefano(1968) presented a rigorous mathematical model and investigated the applicability of numerical integration methods. Guy(1983) extended Distefano’s model by including tray hydraulics. In practice, however, models to be considered will change in each application and sometimes customization may be required. Flexibility is another factor to be considered when developing algorithms.

In the following, we try to propose an algorithm with high accuracy for rigorous batch distillation calculations and discuss its numerical characteristics through examples.

1. Batch Distillation Model

**Figure 1** shows a typical batch distillation column with rectifying stages. The transient balance equations around the stages are the same as those in Distefano(1968) and Guy(1983), provided that the following primary assumptions are included; adiabatic column, negligible vapor holdup, and negligible stage hydraulics.

Total material balance:

\[
\frac{dU_j}{dt} = g_j^N
\]

(1)

\[
g_j^N = V_j - (L_j + D)
\]

(2)

\[
g_j^M = V_{j+1} + L_j - V_j - L_j
\]

(3)

\[
g_j^N = L_{N-j} - V_j
\]

(4)

Component material balance:

\[
\frac{d(U_jx_{j,i})}{dt} = g_j^N_i
\]

(5)

\[
g_j^N_i = V_jy_{j,i} - (L_j + D)x_{j,i}
\]

(6)

\[
g_j^M_i = V_{j+1}x_{j+1} + L_jx_j - V_jx_{j,i} - L_jx_{j,i}
\]

(7)

\[
g_j^N_i = L_{N-j} - V_jx_{j,i}
\]

(8)

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We do not discuss plate efficiencies further. The reader may choose a suitable plate efficiency model according to the required precision of calculation.

For enthalpies, $h$ and $H$, we assume a linear mole-fraction average of pure substances.

$$h = \sum_{i=1}^{N} x_i^h$$

$$H = \sum_{i=1}^{N} \gamma_i H_i$$

2. Numerical Integration

The authors have presented a successive relaxation algorithm for steady state multicomponent distillation calculation (Mori et al., 1987). In this attempt to reduce the number of iterative calculations required for convergence, they successfully applied a generalized implicit Euler equation, Eq.(20), in numerical integration.

$$\frac{v^{n+1} - v^n}{\Delta t} = f + \delta \left( \omega f^{n+1} + (1 - \omega) f^n \right)$$

The results also imply the possible application of this equation to the solution of batch distillation problems with higher accuracy.

2.1 Composition and temperature

In the same way as in Mori et al. (1987), we can rewrite Eq.(5) into a working form using Eq.(20) with a time increment $\delta$.

$$\sum U_j^i \chi_{j,i} = U_j \chi_{j,i} + \delta \left( \omega \chi_{j,i}^{n+1} + (1 - \omega) \chi_{j,i}^n \right)$$

where a superscript $v$ represents properties at time $t$ and $v+1$ at time $t + \delta$.

By considering the relations in Eqs.(6)–(8), Eq.(21) can be rearranged to give $\chi_{j,i}^{n+1}$'s.

$$\chi_{j,i}^{n+1} = \frac{U_j V_j \mu_j + (1 - \omega) \chi_{j,i}^n + \delta \left( \omega \chi_{j,i}^{n+1} + (1 - \omega) \chi_{j,i}^n \right)}{U_j + \delta \omega \chi_{j,i}^{n+1}}$$

$$\chi_{j,i}^{n+1} = \frac{1}{U_j + \delta \omega \chi_{j,i}^{n+1}}$$

$$\chi_{i,j} = \frac{U_j V_j \mu_j + (1 - \omega) \chi_{j,i}^n + \delta \left( \omega \chi_{j,i}^{n+1} + (1 - \omega) \chi_{j,i}^n \right)}{U_j + \delta \omega \chi_{j,i}^{n+1}}$$

Combined with a stoichiometric constraint, Eq.(14), we can determine $\chi_{j,i}^{n+1}$ and $\chi_{j,i}^n$ provided that any variables and terms except $K_{j,i}$ in the left-hand side of Eqs.(25) and (26) are fixed. $K_{j,i}$ is considered to be a function of $\chi_{j,i}^n$ due to a vapor pressure and $\chi_{j,i}^n$ due to activity coefficients.

$$\sum_{i=1}^{N} U_j \chi_{j,i} + \delta \left( \omega \chi_{j,i}^{n+1} + (1 - \omega) \chi_{j,i}^n \right) = 1$$

$$\sum_{i=1}^{N} U_j \chi_{j,i} + \delta \left( \omega \chi_{j,i}^{n+1} + (1 - \omega) \chi_{j,i}^n \right) = 1$$

When ideal stages are assumed $E_{j,i}^{n+1} = 1$. Then $E_{j,i}^n = 1$ and $y_{j,i} = y_{j,i}^n$. 

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\[
\sum_{j=1}^{N} U_{N,j} + \delta \alpha \Delta \frac{V_{N,j}^{\text{in}}}{V_{N}^{\text{in}}} = 1
\]  

(26)

2.2 Flow rates

This part of the model is strongly related to the operation mode or controlling system of the distillation column. For a reasonable selection of process variables to be controlled, we can specify two variables among distillate D, reflux ratio R or reflux rate L, and reboiler heat duty Q_{R}. These variables that are to be specified may not necessarily be constant with time and may be functions of time. For instance, we describe two combinations of D and Q_{R}, and R and Q_{R} in the following.

For constant molar holdup assumption, Eqs.(2)-(4) become

\[
k_{j}^{M} = 0 \quad (j = 1, \ldots, N - 1)
\]  

(27)

and

\[
k_{N}^{M} = - D
\]  

(28)

When we specify the values of the distillate D and the reboiler heat duty Q_{R} as an operating condition, Eq.(28) holds at any instance. Besides, when we assign the reflux ratio R instead of D, D should vary with time. Applying Eq.(20) to Eq.(1) with the relation of Eq.(28) yields

\[
\dot{U}_{N} = - \delta \left\{ \alpha D + (1 - \alpha) \Delta \right\}
\]  

(29)

By taking a summation of Eq.(9) from j = 2 to N, we obtain

\[
\frac{d}{dt} \sum_{j=2}^{N} U_{j} h_{j} = \sum_{j=2}^{N} \dot{g}_{j}^{o}
\]  

(30)

where

\[
\dot{\sum}_{j=2}^{N} \dot{g}_{j}^{o} = Q_{R} - V_{j} h_{j} + L_{j} h_{j}
\]  

(31)

Again we can rewrite Eq.(30) as a difference equation using Eq.(20).

\[
\sum_{j=2}^{N} U_{j}^{\text{in}} h_{j}^{\text{in}} - \sum_{j=2}^{N} U_{j} h_{j} = \sum_{j=2}^{N} \dot{g}_{j}^{o} + \delta \left\{ \alpha \sum_{j=2}^{N} g_{j}^{o} \right\} + (1 - \alpha) \sum_{j=2}^{N} \dot{g}_{j}^{o}
\]  

(32)

From Eqs.(32) and (29) with the relationships, \( V_{2} = (1 + R) D \) and \( L_{1} = RD \), we can derive the following equation to evaluate the distillate rate \( \dot{D} \) at time \( t + \delta t \).

\[
\dot{D} = \frac{\delta \alpha (1 + R) h_{N}^{\Delta} h_{j}^{\Delta} h_{N}^{\Delta} h_{N}^{\Delta}}{\delta \alpha Q_{R} - \sum_{j=2}^{N} U_{j}^{\text{in}} h_{j}^{\text{in}} + (1 \alpha) \sum_{j=2}^{N} \dot{g}_{j}^{o} + (1 - \alpha) \sum_{j=2}^{N} \dot{g}_{j}^{o}}
\]  

(33)

With the known value of \( \dot{D} \), the bottom holdup \( U_{N} \) is obtained from Eq.(29). Then, the vapor and liquid flow rates on each stage at time \( t + \delta t \) are calculated as follows.

\[
\dot{V}_{N} = \frac{O_{R} - \dot{D} h_{N}^{\Delta}}{h_{N}^{\Delta} h_{N}^{\Delta}}
\]  

(34)

\[
\dot{L}_{N-1} = \dot{V}_{N} - \dot{D}
\]  

(35)

\[
\dot{V}_{j} = \frac{\left[ (h_{j+1}^{\Delta} - h_{j-1}^{\Delta}) \dot{V}_{j+1}^{\text{in}} - (h_{j}^{\Delta} - h_{j-1}^{\Delta}) \dot{L}_{j-1}^{\text{in}} \right]}{h_{j}^{\Delta} h_{j+1}^{\Delta}}
\]  

(36)

\[
\dot{L}_{j-1} = \dot{V}_{j} - \dot{D}
\]  

(37)

where

\[
\dot{g}_{j}^{o} = \frac{\left[ (U_{j}^{\text{in}} h_{j}^{\Delta} - U_{j} h_{j}^{\Delta} - \delta \alpha (1 - \alpha) \sum_{j=2}^{N} \dot{g}_{j}^{o}) \right]}{\delta \alpha}
\]  

(38)

2.3 Calculation Algorithm

Since the integration formulas described above are implicit, the calculation for succeeding each time increment must be iterative.

For a given column specifications and initial and operating conditions the calculation steps are summarized below.

\textbf{Step 1} Solve Eqs.(25) and (26) for \( \dot{x}_{j}^{o} \) and \( \dot{x}_{j}^{1} \) from the bottom (\( j = N \)) to the top stage (\( j = 2 \)) with the convergence criterion.

\[
\left| \sum_{j=1}^{N} \dot{x}_{j}^{1} - 1 \right| \leq \epsilon_{\theta}
\]  

(39)

Then, calculate \( \dot{x}_{j}^{o} \) with Eq.(22). When the temperature of the reflux is assigned to its bubble point, execute a bubble point calculation to get \( T_{j}^{L} \).

After each calculation through the column, vapor compositions \( \dot{Y}_{j}^{o} \) are calculated using Eqs.(13) and (16). If the plate efficiency is specified as other than unity, vapor temperature \( T_{j}^{v} \) is obtained from a dew-point calculation.

\textbf{Step 2} Evaluate vapor and liquid enthalpies \( h_{j}^{o}, h_{j}^{1} \) from the values of \( \dot{x}_{j}^{o}, \dot{x}_{j}^{1}, \dot{Y}_{j}^{o}, T_{j}^{v} \) obtained in Step 1.

\textbf{Step 3} Calculate \( \dot{D} \) from Eq.(33) if \( R \) and \( Q_{R} \) are specified. Calculate \( \dot{U}_{N} \) from Eq.(29), then \( \dot{V}_{N} \) and \( \dot{L}_{N-1} \) using Eqs.(34)–(38) from \( j = N \) to 2.

\textbf{Step 4} Check the transient component material balance around the distillation process defined by the following equation.

\[
\Phi = \sum_{j=1}^{N} \left( \dot{U}_{j} \dot{x}_{j}^{1} - \delta t \left[ \alpha D \dot{x}_{j}^{1} + (1 - \alpha) \dot{D} \dot{x}_{j}^{o} \right] \right)
\]  

(40)

convergence criteria:

\[
|\Phi_{\text{max}} - 1| < \epsilon_{\Phi}
\]  

(41)

When Eq.(41) is satisfied, forward the calculation to the next time step while holding the calculated values at time \( t + \delta t \) as the stored values at time \( t \). Otherwise, correct \( \dot{x}_{j}^{o} \) using the overall normalized \( \theta \) method as
Table 1 Specifications for numerical example

<table>
<thead>
<tr>
<th>Column:</th>
<th>12 stages (including total condenser and still)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System:</td>
<td>Ethanol (1) / 2-Propanol (2) / Water (3)</td>
</tr>
<tr>
<td>Feed</td>
<td>15 mol</td>
</tr>
<tr>
<td>Feed Composition:</td>
<td>0.100 (1) / 0.025 (2) / 0.875 (3)</td>
</tr>
<tr>
<td>Total pressure:</td>
<td>760 mmHg</td>
</tr>
<tr>
<td>Heat duty:</td>
<td>95.3 W</td>
</tr>
<tr>
<td>Reflux temperature:</td>
<td>35 °C</td>
</tr>
<tr>
<td>Distillate:</td>
<td>2.58 × 10^{-4} mol / s</td>
</tr>
<tr>
<td>Holdup:</td>
<td>0.5 mol / reflux drum</td>
</tr>
<tr>
<td>Murphree plate efficiency:</td>
<td>E_{mr} = 1</td>
</tr>
</tbody>
</table>

Convergence criteria: \( \varepsilon_p = 10^{-12}, \varepsilon_g = 10^{-6} \)

Fig. 2 Distillation curves of numerical example

follows (Mori et al., 1994) and return to Step 1.

\[ r = \frac{k_{ji}^1 - \theta_j \phi_{ji}^1}{\sum_j q_j k_{ji}^1} \]

where

\[ \theta_j = \frac{\sum_j q_j k_{ji}^1}{1} \]

and \( \phi_j \) is defined in Eq.(40).

3. Numerical Examples and Discussion

The algorithm described above is coded in Fortran 77 and the program is executed on a work station hp712/60 (Hewlett Packard Co.) with double precision. We distinguish the problems as Type I, D and \( Q_R \) specified, and Type II, R and \( Q_R \) specified. Type I problems may arise from industrial batch distillation columns and Type II problems from laboratory experimental columns. To start the simulation, the column is first brought up to the steady state at total reflux, then distillate withdrawal is started at time \( t = 0 \). Table 1 shows the problem specifications (Type I) for separating a ternary mixture. Constant liquid molar holdup is assumed for the condenser and on each tray. The column is operated at atmospheric pressure and pressure drop is neglected. The temperature of the reflux is kept constant throughout the operation. The physical properties of the pure components are taken from Daubert and Danner (1989) and the activity coefficients for VLE calculation are estimated using ASOG (Tochigi et al., 1990). In this condition the vapor rate at the bottom \( V_N \) is about 2.35 × 10^{-3} mol/s. The calculation will terminate when the fraction distilled becomes 33.3% (molar base).

3.1 Weighting factor \( \omega \) and integration accuracy

Type I problem is solved using the proposed algorithm with a different combination of a time increment \( \delta \) and a weighting factor \( \omega \) to examine the integration accuracy. To obtain the reference solution, the same problem is solved using the Runge-Kutta method with successively decreased time increment (\( \delta = 3.6 \times 10^{-1} \sim 3.6 \times 10^{-4} \) s). Since the distillation curves coincide within a tolerance of \( \pm 10^{-5} \) between the results obtained with \( \delta = 3.6 \times 10^{-3} \) and \( \delta = 3.6 \times 10^{-4} \), hereafter we employ the latter, shown in Fig. 2, as the reference solution. In Fig. 3 the deviation of the calculated results using the Runge-Kutta method with different time increments is presented as relative errors to the reference solution using the water composition in the distillate. Comparing Fig. 2 with Fig. 3 shows that the error becomes large where the change in composition is large. This region is also important in that
Table 2  CPU time required for whole calculations

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>R-K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8 \times 10^2</td>
<td>38s</td>
<td>38</td>
<td>41</td>
<td>44</td>
<td>46</td>
<td>49</td>
<td>n.c.</td>
</tr>
<tr>
<td>7.2 \times 10^2</td>
<td>54s</td>
<td>58</td>
<td>60</td>
<td>64</td>
<td>67</td>
<td>69</td>
<td>n.c.</td>
</tr>
<tr>
<td>3.6 \times 10^2</td>
<td>68s</td>
<td>74</td>
<td>80</td>
<td>92</td>
<td>90</td>
<td>95</td>
<td>n.c.</td>
</tr>
<tr>
<td>1.8 \times 10^3</td>
<td>91s</td>
<td>100</td>
<td>105</td>
<td>112</td>
<td>117</td>
<td>124</td>
<td>n.c.</td>
</tr>
<tr>
<td>7.2 \times 10^3</td>
<td>147s</td>
<td>150</td>
<td>160</td>
<td>167</td>
<td>179</td>
<td>176</td>
<td>n.c.</td>
</tr>
<tr>
<td>3.6 \times 10^3</td>
<td>224s</td>
<td>219</td>
<td>232</td>
<td>239</td>
<td>250</td>
<td>261</td>
<td>n.c.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.8 \times 10^4</td>
<td>n.c.</td>
<td>331s</td>
<td>345s</td>
<td>360s</td>
<td>369s</td>
<td>383s</td>
</tr>
<tr>
<td>7.2 \times 10^4</td>
<td>n.c.</td>
<td>589s</td>
<td>613s</td>
<td>617s</td>
<td>632s</td>
<td>646s</td>
<td>1138</td>
</tr>
<tr>
<td>3.6 \times 10^4</td>
<td>n.c.</td>
<td>932s</td>
<td>953s</td>
<td>1010s</td>
<td>1034s</td>
<td>1052s</td>
<td>2141</td>
</tr>
<tr>
<td>1.8 \times 10^5</td>
<td>n.c.</td>
<td>1457s</td>
<td>1488s</td>
<td>1512s</td>
<td>1536s</td>
<td>1560s</td>
<td>4209</td>
</tr>
<tr>
<td>7.2 \times 10^5</td>
<td>n.c.</td>
<td>3106s</td>
<td>3085s</td>
<td>3114s</td>
<td>3139s</td>
<td>3204s</td>
<td>9928</td>
</tr>
<tr>
<td>3.6 \times 10^5</td>
<td>n.c.</td>
<td>5348</td>
<td>5275</td>
<td>5479</td>
<td>5569</td>
<td>5715</td>
<td>18604*</td>
</tr>
</tbody>
</table>

1: CPU time [s]
\( n.c. \): not converged
\(*\): satisfies a condition, maximum error < 3.6 \times 10^{-4} (see Fig. 4)

Table 3  Experiment and calculation conditions

<table>
<thead>
<tr>
<th>Column</th>
<th>12 stages (including total condenser and still)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>Ethanol (1) / 2-Propanol (2) / Water (3)</td>
</tr>
<tr>
<td>Feed</td>
<td>20 mol</td>
</tr>
<tr>
<td>Feed Composition</td>
<td>0.25 (1) / 0.05 (2) / 0.70 (3)</td>
</tr>
<tr>
<td>Total pressure</td>
<td>760 mmHg</td>
</tr>
<tr>
<td>Heat duty</td>
<td>127.7 W (experiment)</td>
</tr>
<tr>
<td>133.5 W (calculation)</td>
<td></td>
</tr>
<tr>
<td>Reflux temperature</td>
<td>20 °C</td>
</tr>
<tr>
<td>Reflux ratio</td>
<td>6.9 (time controlled)</td>
</tr>
<tr>
<td>Holdup</td>
<td>0.04 mol / stage</td>
</tr>
</tbody>
</table>
| Murphree plate efficiency | \( E_{\text{M}} = 0.75 \)

Fig. 5  Acceleration effect of the normalized \( \theta \) method

Fig. 6  Comparison of experimental and calculated distillate compositions

Runge - Kutta method (\( \delta = 3.6 \times 10^{-2} \)).

The effect of the overall normalized \( \theta \) method (Step 4) on reducing the number of iterations for each time step is shown in Fig. 5. This effect is significant at larger time increments compared with a simple direct iteration.

3.3  Comparison with experiment

The experimental and calculation conditions (Type II problem) are shown in Table 3. The 10-stage Oldershaw column used has a 32 mm inner diameter and a 28 mm tray spacing. From preliminary experiments the molar liquid holdup on each plate and the heat duty at reboiler are determined. As for the plate efficiency, although there are many arguments, we assign \( E_{\text{M}} = 0.75 \) for all plates and components as a rough estimation. The experimental procedure consists of repeating the following three steps: (i) total reflux operation and vapor rate measurement, (ii) constant reflux operation, and (iii) distillate sampling for analysis. This operation mode is also considered in the simulation.

Figure 6 shows a comparison of the experimental data and the calculated results. There is a slight difference due to the assumption of no heat loss, constant heat duty, constant molar holdup, and constant plate efficiencies. The overall agreement, however, seems to be fairly good.

In order to adapt the proposed algorithm to a variety of situations, the readers may choose other problem specifications and select more realistic models, such as variable
liquid holdup and tray hydraulics. This will be further discussed in detail elsewhere.

Conclusion

We proposed a calculation algorithm for simulating batch distillation process based on a successive stagewise calculation. The results of the numerical examples show a higher accuracy and a lesser CPU time compared with the Runge – Kutta method. In one particular case, with the weighting factor $\omega = 0.5$, the proposed method can take a time increment up to $10^3$ greater in value than that of the Runge – Kutta method to yield equivalent accuracy and also the CPU time may decrease by 1/300. A comparison of the simulation with experimental results suggests the adequacy of the proposed method.

Since the proposed algorithm is based on a stagewise calculation, it can be applied easily to a wide variety of situations and configuration of batch distillation processes.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>total number of components</td>
<td>[-]</td>
</tr>
<tr>
<td>$D$</td>
<td>molar distillate rate</td>
<td>[mol s$^{-1}$]</td>
</tr>
<tr>
<td>$EMV$</td>
<td>Murphree vapor-phase stage efficiency</td>
<td>[-]</td>
</tr>
<tr>
<td>$E^v$</td>
<td>vaporization stage efficiency</td>
<td>[-]</td>
</tr>
<tr>
<td>$f$</td>
<td>function</td>
<td>[-]</td>
</tr>
</tbody>
</table>
| $g^M$  | total material balance                                                    | [mol s$^{-1}$]|}
| $g^w$  | component material balance                                                | [mol s$^{-1}$]|}
| $g^h$  | heat balance                                                              | [J mol$^{-1}$]|
| $h$    | vapor molar enthalpy                                                      | [J mol$^{-1}$]|
| $K$    | vapor-liquid equilibrium ratio                                            | [-]            |
| $L$    | molar liquid flow rate                                                    | [mol s$^{-1}$]|
| $N$    | total number of plates in distillation column                            | [-]            |
| $Q_C$  | heat load at condenser                                                    | [W]            |
| $Q_R$  | heat load at reboiler                                                    | [W]            |
| $R$    | reflux ratio ($= L/D$)                                                    | [-]            |
| $t$    | operation time                                                            | [s]            |
| $U$    | liquid molar holdup                                                       | [mol]          |

$V$ = molar vapor flow rate [mol s$^{-1}$]

$x$ = liquid mole fraction [-]

$y$ = vapor mole fraction [-]

$\delta$ = time increment for numerical integration [s]

$\varepsilon$ = convergence criterion [-]

$\theta$ = correction factor [-]

$\lambda$ = heat of vaporization [J mol$^{-1}$]

$v$ = indication of operation time [-]

$\phi$ = objective function [-]

$\omega$ = weighting factor [-]

$\text{ca}$ = calculated

$\text{co}$ = corrected

$i$ = component

$j$ = stage

$'$ = derivative

$^*$ = pure component

# = equilibrium state

Literature cited


