HEAT TRANSFER BY NATURAL CONVECTION IN RECTANGULAR CAVITIES WITH LOW GRASHOF NUMBER AND VARIOUS HEATER LOCATIONS

GUI-LIN ZHANG, TAMOTSU HANZAWA
AND NOBORU SAKAI

Department of Food Engineering, Tokyo University of Fisheries,
Tokyo 108

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The velocity profiles, temperature distributions, average Nusselt numbers near heating wall, and maximum velocities of a circulating flow were calculated numerically by fundamental equations under different operating conditions including changing the Grashof number (at low Gr range), the location and number of heating walls, and the temperature difference between heated and cooled walls.

The number of circulating flows is one or two owing to the configurations of heated and cooled walls under the present operating condition. Generally, the circulating rate was very slow and the temperature distributions were similar to those of conduction alone. In the case of a heated top wall, a domain of cold temperature extended widely as the circulating flow rate was increased, and it could be seen that heat transfer was suppressed.

To check these calculated results, the velocity profiles and temperature distributions in small enclosures were measured experimentally and observed by a tracer under the same operating conditions as those of theoretical analysis. The calculated results agreed closely with the experimental ones.

Introduction

Many experimental and theoretical studies have been taken to natural convective heat transfer in various enclosed cavities. However, these investigations are restricted approximately to a Grashof number range larger than 10 (Davis, 1968; Elder, 1965; Kirkpatrick and Bohn, 1986; Sernas and Lee, 1981; Turner and Flack; 1980, Wilkes and Churchill; 1966). If the size of the enclosure is small, the Grashof number is reduced and the natural convection becomes small. Thus, it is conjectured that heat transfer rates are greatly reduced by stagnant flow in the enclosure. In porous food, of which the materials are composed of a solid continuous matrix containing small vacant spaces of air, it is conjectured that the Grashof number in the vacant spaces and the heat transfer rates are affected very much by the heat convection, geometry and heating operating conditions. The absence of data for low ranges of Grashof number and the practical importance of this problem in the field of porous food motivated this work.

In this study, as basic research on heat transfer through porous food, fundamental equations were derived and the velocity profiles, temperature distributions, and the rate of heat transfer with natural convection were investigated numerically for a cubical

enclosure heated respectively from below, the side, or the top.

Furthermore, calculated results were compared with experimental results and observations with a tracer.

1. Fundamental Equations and Numerical Calculations

Heat transfer phenomenon with natural convection in a two-dimensional cubical enclosure is considered where one or two side walls are heated and the other wall is kept at constant or adiabatic temperature, as shown in Fig. 1. If it is assumed that the physical properties of the fluid are constant and independent of temperature, and that the buoyancy is directly proportional to the temperature difference, the dimensionless fundamental equations at steady state and boundary conditions are obtained as follows (Chu *et al.*, 1976; Davis, 1968; Hanzawa *et al.*; 1975, Ozoe *et al.*; 1974; Wilkes and Churchill; 1966):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \nabla^2 U \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \nabla^2 V - Gr \cdot T \quad (3)$$

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Fig. 1 Coordinate system for a small enclosure

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \nabla^2 T \quad (4)$$

(Boundary conditions)*

$$\left. \begin{aligned} X=0; & \quad U=V=0, \quad T=1 \\ X=1; & \quad U=V=0, \quad T=0 \\ Y=0; & \quad U=V=0, \quad T=0 \\ Y=H/D; & \quad U=V=0, \quad T=1 \end{aligned} \right\} \quad (5)$$

The dimensionless stream function ϕ and vorticity ξ were introduced to Eqs. (1)~(5) and each equation was written in a finite difference form by the "upwind method". They were then solved numerically by a relaxation method. The optimum values of the relaxative condition were found by trial and error for a given system of grid points. Representative values of grid size ($\Delta X, \Delta Y$) were 0.0107~0.0333, and those of iteration number were 800~2000 for good convergence.

Six configurations of the heating wall were tested with low Grashof number. The first was a configuration with a heated bottom and the other walls cooled. The second was a configuration with only one heated side wall. The third configuration was with a heated top. The fourth was mixed, with a heated bottom and single side wall. The fifth was mixed, with one side wall heated and a heated top. The last was a hot and a cold side wall with an adiabatic bottom and top.

The calculations were performed over the ranges, $H/D=1.0$, $Gr=0.492 \sim 1.06 \times 10^5$ and $Pr=0.71$.

2. Experimental Apparatus and Procedure

To check the calculated results, observation of the flow pattern by a tracer and measurement of the

* In this case, one side and the top walls are heated, and the other walls are kept at a constant cold temperature.

** Figure 2 shows the case of configuration with a heated bottom and cooled other walls.

Fig. 2 Experimental apparatus

temperature distributions in the enclosure were performed.

Figure 2 shows a schematic diagram of the experimental apparatus**. The cubical enclosures, with cross-sectional areas of $5.0 \times 5.0 \text{ cm}^2$ and $2.0 \times 2.0 \text{ cm}^2$ (for low Gr range), and 25 cm in length, were made of copper plate. The heating and cooling sections were respectively made of a hot and cold water jacket attached to the copper plate in order to obtain isothermal condition.

TiCl_4 smoke was used to visualize the gas flow. Illumination was provided through a vertical slit in the enclosure and the flow pattern was observed.

To measure the wall temperature in the enclosure, CC thermocouples were placed in the heated and cooled walls. To investigate correctly the cross-sectional temperature distribution in the enclosure, gas temperature was measured by use of a sheathed CC thermocouple probe of very small diameter (0.25 mm ϕ).

After the temperature in the enclosure reached steady state, a sheathed thermocouple probe was put into the system from an end of the enclosure and was traversed in the cross-sectional direction by moving a travelling microscope to measure the local temperature in the enclosure.

The experiments were performed over the ranges $t_H=303 \sim 338 \text{ K}$ and $t_L=293 \sim 333 \text{ K}$.

3. Photographic Observation of Streamlines

Figure 3 shows photographs of streamlines in this enclosure for one heated wall and the other cooled walls. Figure 3(a) shows the streamlines in the case with a heated bottom for $Gr=6.80 \times 10^3$, $H/D=1.0$ ($D=2.0 \text{ cm}$) and $\Delta t=5.0 \text{ K}$. From this figure, the streamlines are two circulating flows, and the fluid moves upward in the central part of the enclosure and downward near the both side walls.

Figure 3(b) shows the streamlines in the case with a heated side wall for $Gr=6.80 \times 10^3$, $H/D=1.0$ ($D=2.0 \text{ cm}$) and $\Delta t=5.0 \text{ K}$. From this figure, the

Fig. 3 Experimental Streamlines in cross-section of enclosure with heated bottom (a) or one heated side wall (b) and other walls cooled for $Pr=0.71$, $H/D=1.0$, $Gr=6.80 \times 10^3$

streamlines are only one circulating flow moving upward near the heated wall.

4. Calculation Results and Comparison of Calculated with Experimental Values

4.1 Streamline and temperature distribution with one heated wall

Figures 4~6 show some typical streamlines and temperature distributions obtained numerically in the vertical cross-section through the enclosure for the cases with a heated bottom (Fig. 4), one heated side wall (Fig. 5) and a heated top wall (Fig. 6), respectively, for $H/D=1.0$, $Pr=0.71$ and $Gr=4.92 \times 10^2$ ($t_H=338$ K, $t_L=333$ K). The horizontal axis of the figure represents the dimensionless cross-sectional distance from the surface of left side wall, X , and the vertical axis represents the dimensionless longitudinal distance from the bottom of the enclosure, Y .

From Figs. 4(a)~6(a), the streamlines were only one circulating flow for heated side wall, but were two circulations for heated bottom or top wall. The circulating flow reduces, respectively, the rotation frequency in the order of heated side wall, bottom and

Fig. 4 Streamline and temperature distribution in case of heated bottom

Fig. 5 Streamline and temperature distribution in case of heated left side wall

Fig. 6 Streamline and temperature distribution in case of heated top wall

Fig. 7 Comparison of calculated temperature distributions with experimental ones in cross-sectional direction for $D=2.0, 5.0$ cm and $H/D=1.0$

top, and the circulating rate (number of streamlines), with the heated side wall increasing approximately ten-fold as compared with that of the bottom or top. It seems that in these cases, each circulating rate is extremely slow. The reason is the very small size of the enclosure.

A similar fluid motion was observed for the flow pattern of a TiCl_4 tracer as well, as shown in Fig. 3.

For the heated top wall in the enclosure, there are few works concerning the problems of natural convective heat transfer (Kirkpatrick and Bohn, 1986). From Fig. 6, it is seen that the circulating flow for the heated top wall rises toward the downward along the left and right side walls, moves upward in the central region of the enclosure, and is extremely slow.

According to our previous paper, as the Grashof number is large ($Gr > 10^6$), the circulating rates with the heated bottom are much larger than those of the heated top (Hanzawa *et al.*, 1975; Kirkpatrick and Bohn, 1986). However, the present result suggests that the circulating rate becomes almost the same

*** Equation (4) could be solved numerically in the case of $U=V=0$.

between the heated bottom and top through a small enclosure ($Gr < 10^3$).

From Figs. 4(b)~6(b), it seems that there are large temperature gradients in the circumference of the heating section and the temperature changes slightly near the cooled wall. In Figs. 5(b) and 6(b), the dotted lines show the temperature distribution due to conduction alone***, while solid lines show the cases of $Gr=4.92 \times 10^2$. From these figures, the difference between the two kinds of lines is small and especially scarce in the case for the heated top wall. These facts show that if Gr is less than about 10^3 , overall heat transfer is affected predominantly by conductive heat transfer.

Figure 7 shows some typical cross-sectional temperature distributions with a heated bottom wall for $D=2.0$ cm and 5.0 cm, respectively, for $H/D=1.0$ and $Gr=5.17 \times 10^4$ and 5.84×10^5 with y as a parameter. The horizontal axis of these figures represents the cross-sectional distance, x , and the vertical axis represents the temperature, t . Each line and the marked points show the calculated values and experimental ones, respectively. In Fig. 7, the

(a) Streamlines (b) Isotherms
Fig. 8 Streamline and temperature distribution in case of heated left side and bottom wall

(a) Streamlines (b) Isotherms
Fig. 9 Streamline and temperature distribution in case of heated left side and top wall

temperature distributions represent the same kind as those shown in Fig. 4. From Fig. 7, the calculated values are in sufficiently good agreement with the experimental ones. Such good agreement between the calculated results and the experimental ones was also obtained in other heated walls. From these cases, this calculation method can be reliably used to investigate the heat transfer in small enclosures in the case of the present operating conditions.

4.2 Streamline and temperature distribution with plural heated wall

Figures 8 and 9 show some typical streamlines and temperature distributions through the small enclosure for the case with plural heated walls with $H/D=1.0$, $Pr=0.71$ and $Gr=4.92 \times 10^2$ ($t_H=338$ K, $t_L=333$ K). The former figure is the mixed configuration with one side wall and one bottom wall heated, and the latter is with heated one side and top wall.

Figures 8 and 9 show that the streamlines are one circulating flow for these combinations of two heated walls in which the heated side wall is involved. These

circulating rates are almost the same as those for one heated side wall.

5. Discussion

5.1 Temperature distribution with heated top wall

As described earlier, there are few works concerning natural convective heat-transfer for a heated top wall in an enclosure. It is assumed that the main reasons are an extreme decrease in the natural convective flow with the heated top wall and promotion of heat transfer is not expected. In this discussion, the heat characteristics on the heated top wall were considered by the effect of operating condition on the temperature distribution.

Figures 10 and 11 show the calculated streamlines and temperature distributions with the heated top wall for the case where $Gr=5.04 \times 10^3$ and 4.03×10^4 , respectively. These figures can be connected with those shown in Fig. 6. From these figures, it seems that as Gr increases, the isotherms

(a) Streamlines (b) Isotherms
Fig. 10 Streamline and temperature distribution in case of heated top with $Gr=5.04 \times 10^3$

(a) Streamlines (b) Isotherms
Fig. 11 Streamline and temperature distribution in case of heated top with $Gr=4.03 \times 10^4$

move upward and the temperature distributions becomes mostly flat with the same temperature as the cooled wall. Furthermore, from Figs. 10(a) and 11(a), as Gr increases, the circulating flow, which moves upward in the central part of the enclosure, increases. One should note that as Gr increases, a domain of cold temperature extends widely and the heat transfer is suppressed.

5.2 Grashof number and local velocity

To measure the fluid motion in a small enclosure, some typical Grashof numbers and maximum local velocities, $V_{\max}$, are shown in Figs. 12 and 13.

Figure 12 shows the effects of D on Gr and $V_{\max}$ in the case of $\Delta t=5.0$ K and one heated side wall. $V_{\max}$ might be obtained as follows:

$$V_{\max} = \sqrt{(u^2 + v^2)_{\max}}$$

From Fig. 12, as D decreases Gr and $V_{\max}$ suddenly becomes small. This tendency is especially

Fig. 12 Relationships of Gr and $V_{\max}$ with D for heated left side wall

notable for Gr . It seems that the effects of circulating flow on the heat transfer are negligible under about $D=1.0$ cm.

Fig. 13 Relationships of Gr and V_{max} with Δt for heated left side wall

Figure 13 shows the effects of temperature difference, Δt , on Gr and V_{max} , with one heated side wall for $D=3.0$ cm. From Fig. 13, it seems that the effects of Δt on Gr and V_{max} are the same as those of D in Fig. 12.

5.3 Nusselt number

The average Nusselt number, $\overline{Nu}$, from the heated side wall is obtained by the temperature gradients near the heated wall as follows:

$$\overline{Nu} = - \frac{1}{H/D} \int_0^{H/D} \left(\frac{\partial T}{\partial X} \right)_{X=0} dY$$

Figure 14 shows relationships between $\overline{Nu}$ and Gr with one heated and one cooled side wall, and the adiabatic top and bottom in the enclosure for the case with $H/D=1.0$. The abscissa of the figure represents Gr and the ordinate represents $\overline{Nu}$.

From Fig. 14, $\overline{Nu}$ decreases with decreasing Gr and $\overline{Nu}$ is very small under low Gr . When Gr is about 2×10^3 , $\overline{Nu}$ rapidly increases with increasing Gr . It seems that the main mechanism of heat transfer in this system changes from conduction to convection at this point of Gr .

Figure 14 also shows some previous results for $\overline{Nu}$ on a vertical heated surface in the system similar to

that used by the authors (Arnold *et al.*, 1976; Chu *et al.*, 1976; Elder, 1965; Ozoe *et al.*, 1974; Wilkes and Churchill, 1966). From this figure, although there are few reports of $\overline{Nu}$ in the case of low Gr ($Gr < 10^3$), it seems that the comparison is generally acceptable in the range of operating conditions to be compared.

Conclusion

Velocity and temperature distributions were investigated using six different configurations of differentially-heated and cooled vertical and horizontal surfaces in a cubical enclosure as basic research on heat transfer through a porous material. The fundamental equations derived under certain assumptions were solved numerically to analyze heat transfer in the enclosure under low Gr range. To check these calculated results, the temperature distribution was measured experimentally under the same operating conditions as those of the theoretical analysis. The following results were obtained in the case of $H/D=1.0$, $Pr=0.71$ and $Gr=0.5 \sim 10^5$ ($D=0.1 \sim 6.0$ cm, $\Delta t=0.1 \sim 30$ K).

(1) The number of circulating flow is one or two owing to the configurations of heated and cooled wall in the case of $H/D=1.0$, $D=1.0$ cm and $Gr=0.492 \sim 1.06 \times 10^5$. Generally, the circulating rate is very slow, and it is extremely slow with a heated top wall.

(2) The temperature changes gently from heated wall to cooled wall, and it can be seen that the temperature distributions are similar to those of conduction alone at less than about $Gr=10^3$.

(3) In the case of a heated top wall, a domain of cold temperature widely extends as the circulating flow rate is increased, and the heat transfer is suppressed.

(4) Calculated streamlines and temperature distributions agree closely with visualized streamlines and measured temperature distributions.

(5) There was reasonable agreement between the average Nusselt numbers calculated from the temperature gradients over the heated wall and those

Fig. 14 Average Nu compared with some previous results

cited in the literature in the range of operating conditions compared.

Nomenclatures

C_p	= heat capacity at constant pressure	[J/g·K]
D	= width of enclosure	[cm]
Gr	= Grashof number ($=g\beta D^3 (t_H - t_L)/\nu^2$)	[—]
g	= gravitational acceleration	[cm/s ²]
H	= height of enclosure	[cm]
h	= heat transfer coefficient	[W/cm ² ·K]
Nu	= Nusselt number ($=h \cdot D/\lambda$)	[—]
P	= dimensionless pressure ($=p \cdot D^2/\rho \cdot \nu^2$)	[—]
Pr	= Prandtl number ($=C_p \cdot \mu/\lambda$)	[—]
p	= pressure of fluid	[g/cm ²]
T	= dimensionless temperature ($=(t - t_L)/(t_H - t_L)$)	[—]
t	= temperature	[K]
U	= dimensionless velocity in X -direction ($=u \cdot D/\nu$)	[—]
u	= velocity in x -direction	[cm/s]
V	= dimensionless velocity in Y -direction ($=v \cdot D/\nu$)	[—]
$V_{\max}$	= maximum velocity of fluid in enclosure ($=(u^2 + v^2)_{\max}^{1/2}$)	[cm/s]
ν	= velocity in y -direction	[cm/s]
X	= dimensionless horizontal coordinate ($=x/D$)	[—]
x	= horizontal coordinate	[cm]
Y	= dimensionless vertical coordinate ($=y/D$)	[—]
y	= vertical coordinate	[cm]
β	= volumetric coefficient of expansion	[K ⁻¹]
ξ	= dimensionless vorticity	[—]
λ	= thermal conductivity of fluid	[W/cm·K]
μ	= viscosity of fluid	[g/cm·s]
ν	= kinematic viscosity of fluid ($=\mu/\rho$)	[cm ² /s]
ρ	= density of fluid	[g/cm ³]
ϕ	= dimensionless stream function	[—]
∇^2	= nabla operator ($=\partial^2/\partial X^2 + \partial^2/\partial Y^2$)	[—]

<Subscript>

H	= heating
L	= cooling

<Superscripts>

— = average

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