AUTOMATIC TUNING OF PID CONTROLLER USING SECOND-ORDER PLUS TIME DELAY MODEL

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A new identification method using the second order plus time delay model for the automatic tuning of the PID controller and a simple and explicit tuning method are proposed. We present a relay feedback test combined with a P controller to identify the process using the second order plus time delay model. Thus, the difficulty to determine the initial proportional gain is overcome. Since the proposed identification method uses the second order plus time delay model to identify the process, it can incorporate more various processes such as underdamped and high order processes than the identification method using the first order plus time delay model. In addition, a simple and explicit tuning relation for the second order plus time delay model is proposed in this paper. This tuning relation shows almost the same performance as the optimal tuning parameters. The proposed method needs no numerical technique and shows good performances in both simulation and experimental study.

Introduction

A major controller which has been used in industry is the PID (Proportional-Integral-Derivative) type controller. It has been recognized as a simple and robust controller and it is familiar to the field operator. However, it is difficult and tedious to determine the tuning parameters of the PID controller. Therefore, several different methods have been proposed for tuning.

Åström and Hägglund identified the ultimate process information from a relay feedback test to tune the PID controller automatically and many applications in industry are reported. They approximated the output of the relay (two step square wave) by a sinusoidal signal using Fourier expansion and developed relations to estimate the ultimate information and then, the PID controller was tuned by Ziegler-Nichols-type tuning rule. Sung et al. used six step square wave and improved the accuracy of Åström and Hägglund's identification method by reducing high order harmonic terms. However, these methods inherit limitations of Ziegler-Nichols-type tuning rule. Li et al. obtained parametric models from two relay feedback tests. Lee and Sung obtained the first order plus time delay model from a relay feedback test combined with a P (Proportional) controller. However, Li et al.'s approach uses the true process time delay as a model time delay. It is not reasonable to choose the true time delay as the time delay of the model because the structure of the model can be severely different from that of the real plant. Lee and Sung's method shows good accuracy for low order plus time delay processes. However, even though the identified model is exact for the first order plus time delay process, the performance is poor for high order processes or underdamped processes because it uses the first order plus time delay model whose structure is overdamped and low order.

Yuwana and Seborg proposed a P control method to obtain the first order plus time delay model using a few transient data points. It is poor for the process which has a long time delay because the first order Padé approximation is used to manipulate the time delay term in the theoretical development step. It was improved by Jutan and Rodriguez, Lee, Chen and Sung et al.. Jutan and Rodriguez used the second order Padé approximation to obtain more exact process parameters. Lee and Chen used a pole matching technique and the fact that the cross-over frequency of the closed loop transfer function is the same as that of the open-loop transfer function. Sung et al. obtained an explicit solution of the first order plus time delay process controlled by the P controller and their simple identification method provides the exact process parameters for the first order plus time delay process. Since these methods
use the first order plus time delay model, control performances for some processes such as underdamped or high order processes can be poor. Lee et al. suggested a P control method to identify the second order plus time delay model. To estimate the parameters of the PID controller, a frequency domain method based on the methods of Edgar et al. and Harris and Mellichamp is applied, yielding a good controller setting. Their method shows a good control performance for the set point change process in comparison with previous identification methods to tune the PID controller automatically. However, in practice, the method is too complicated to be applied in industry because it uses a root finding technique and an optimization procedure to obtain the model parameters and the controller setting. In addition, the performance of the input disturbance rejection is not considered systematically and the initial proportional gain of the P controller should be determined to guarantee the underdamped closed loop characteristics. This difficulty is common in identification methods using a P controller.

There are many tuning rules developed for the first order plus time delay model such as ZN (Ziegler-Nichols), IMC (Internal Model Control), ITAE (the Integral of the Time weighted Absolute value of the Error), Cohen-Coon tuning methods (Ziegler and Nichols, Lopez et al., Cohen and Coon, Morari and Zafiiriou). The IMC tuning rule shows a superior performance to the other tuning rules for the set point tracking but an inferior performance in the input disturbance rejection because this tuning rule used the concept of the IMC control (pole-zero cancellation). Usually, the ITAE-load tuning rule shows a superior performance in the input disturbance rejection in comparison with Cohen-Coon, ZN (Miller et al.) and IMC tuning rule. To tune the PID controller for the second order plus time delay model, several methods using frequency response criteria or desired trajectory can be used (Ziegler and Nichols, Hougen, Edgar et al., Harris and Mellichamp, Sung et al.). However, there are few simple and explicit tuning methods to tune the PID controller for the second order plus time delay model.

In this paper, we propose an on-line process identification method for the automatic tuning of the PID controller, which is based on the second order plus time delay (SOPTD), with a simple and explicit tuning method for the second order plus time delay model.

1. Theory for On-Line Process Identification

For most processes, stable oscillation can be obtained from a relay feedback test. A big advantage of the relay feedback identification method is to provide the ultimate data without any stability problem for the open loop stable processes (Åström and Hägglund). As shown in Fig. 1, test signal is generated by the same relay feedback test combined with a P controller as Lee and Sung’s method. Let the relay feedback test and the P control test be phase 1 and phase 2, respectively. In phase 1, we can estimate the ultimate frequency just by measuring the period of relay feedback signal (Åström and Hägglund). The ultimate frequency obtained by the relay feedback test is almost the same as the true value in usual processes (Li et al.). We know that the ultimate frequency of the closed loop transfer function is the same as that of the open loop transfer function. With this fact, time delay can be obtained from the ultimate frequency measured in the phase 1 while the time constant and the damping factor of the closed loop SOPTD model are estimated from the transient data in phase 2. In addition, the time constant and the damping factor of the open loop SOPTD model can be obtained from the transient data in phase 2 with the approximation of the time delay term using the second order Taylor series expansion. Detailed equations to obtain the open loop SOPTD model are as follows.

As shown in Fig. 1, the phase 2 begins by exchanging the relay with the P controller when the process output reaches the valley value ($y_v$). From extensive simulation results, we can recognize that the error of the exchange time from the phase 1 to the
phase 2 can't almost affect the modeling performance. Even the biggest error (that is, a quarter of the period) results in negligible variation of the model parameters within 10 percent. The overshoot and the period of the closed loop oscillatory response are not almost affected by the initial state of the phase 2. Therefore the proposed method using the overshoot and the period shows good robustness to the error in determining the valley value. Here, the P controller has the following form.

\[ u(t) = d - k_c(y + y_v) \]  \hfill (1)

where, \(d\) and \(k_c\) denote the magnitude of the relay and the proportional gain of the P controller, respectively. Consider the following proposition.

**Proposition** Assume that the phase 2 begins at \(t = \text{time delay}(\theta)\), the process is the SOPTD and (1) is used as a controller. Then, the response of the closed-loop system after the time delay is the same as that of the assumed system with a step set point change \((y_v)\) at \(t = 0\) whose initial state is a steady state \((y = y_v\) and \(u = u_{bias}\) for all \(t \leq \theta)\) and satisfies the following equations. That is, we can assume that the initial state is a steady state and a step set point change is done at \(t = 0\) and the phase 2 begins at \(t = \text{time delay}(\theta)\).

\[ d = u_{bias} + k_c y_v \]  \hfill (2)

\[ y_m = \frac{k_m(d - k_c y_v)}{1 + k_c k_m} \]  \hfill (3)

where, \(u_{bias}\) and \(y_m\) are the controller output corresponding to \(y = y_v\) and the steady state value of the process output, respectively and \(t = \theta\), \(k_m\) are the time of the phase 2 and the static gain of the process, respectively. In the phase 2, it is assumed that \(G_y(s) = k_c\) and the feedback gain \(k_c\) is so large that the closed loop system can be an underdamped system (the reason will be mentioned later). We want to approximate the closed loop responses of the phase 2 using the second order plus time delay model with a step input. Using the above proposition, we can assume that the initial state of the phase 2 is steady state so that the following equation can be easily obtained as in Coughanowr and Koppel. Here, (4) can be easily obtained from well-known properties of the second order plus time delay process with a step input.

\[ \frac{y(s) + y_v(s)}{y_v(s) + y_v(s)} = \frac{k \exp(-\theta_m s)}{\tau_c^2 s^2 + 2 \tau_c \xi_c s + 1} \]  \hfill (4)

where

\[ \theta_m = \frac{\pi}{2} + \arctan \left( \frac{2 \tau_c \xi_c}{\tau_c^2} \right) \]

\[ \omega_n = \frac{2 \pi}{\sqrt{\tau_c^2 - 4 \tau_c \xi_c}} \]

\[ k_m = \text{static gain of the process} \]

\[ k_c = \text{proportional gain of the P controller} \]

\[ y_v(s) = \text{Laplace transform of the set point} \]

\[ y(s) = \text{Laplace transform of the process output controlled by the P controller} \]

From the fact that the ultimate frequency of the closed loop transfer function is the same as that of the open loop transfer function, time delay can be obtained as follows.

\[ \lambda = \frac{\pi + \arctan \left( \frac{2 \tau_c \omega_n \xi_c}{1 - \tau_c^2 \omega_n^2} \right)}{\omega_n} \]

\[ \omega_n = \frac{2 \pi}{\tau_c} \]

The measurements, \(y_m, y_p, \Delta t, p_r\) are given in Fig. 1. Here, \((y_p - y_m)/(y_v + y_v)\) and \(2 \Delta t\) denote the overshoot and the period of the oscillatory response, respectively.

We choose a second order plus time delay transfer function as a process model (open loop transfer function):

\[ G_m(s) = \frac{k_m \exp(-\theta_m s)}{\tau_m s^2 + 2 \tau_m \xi_m s + 1} \]  \hfill (6)

Then we obtain the following closed loop transfer function.

\[ \frac{y(s) + y_v(s)}{y_v(s) + y_v(s)} = \frac{k_c k_m \exp(-\theta_m s)}{\tau_m s^2 + 2 \tau_m \xi_m s + 1 + k_c k_m \exp(-\theta_m s)} \]  \hfill (7)

From the comparison of (4) and (7), we obtain the process model (6) using the second order Taylor series expansion of the time delay term in the denominator of (7) as follows.

\[ k_m = \frac{y_m}{u} \]  \hfill (8)
\[ \theta_m = \theta_c \]  

\[ \tau_m = \sqrt{(1 + k_c k_m) \tau_c^2 - \frac{k_c k_m \theta_c^2}{2}} \]  

\[ \xi_m = \frac{2(1 + k_c k_m) \tau_c \xi_c + k_c k_m \theta_c}{2\tau_m} \]  

where

\[ u_m = \text{steady state value of the controller} \]

To determine the \( k_c \) value of the P controller in the phase 2, the following equation is needed (Åström and Hägglund).

\[ k_{cu} = \frac{4d}{\pi a} \]  

\[ a = \frac{y_s + y_p}{2} \]  

Here, more accurate ultimate frequency and gain can be obtained by using Sung et al.'s method. We know that the \( k_c \) value of the P controller in the phase 2 must be determined so large that the closed loop system is underdamped. Usually, when the \( k_c \) value is greater than 0.3 \( k_{cu} \), the closed-loop system is underdamped if the P controller is used to control the system. The \( k_{cu} \) value can be calculated using (12). However, as the \( k_c \) value increases, a longer settling time is required. Therefore, we recommend that the \( k_c \) value is below 0.5 \( k_{cu} \) and above 0.3 \( k_{cu} \) to guarantee an underdamped and stable closed loop system. We simulated the first order plus time delay process with the ratio of the time delay to the time constant between 0.1 to 5.0 to inspect the effects of the \( k_c \). From extensive simulation results, we can recognize that a higher order plus time delay model shows more oscillatory response than the first order plus time delay process. The \( k_c \) value between 0.5 \( k_{cu} \) and 0.3 \( k_{cu} \) provides an appropriate oscillation and convergence rate. The steady state value of the process output can be estimated from the following equations with the assumption of the linear time invariant system, that is,

\[ y_v = a, \ y_w = k_m u_m. \]

\[ k_c = \beta \frac{4d}{\pi a} \]  

\[ y_w = \frac{k_m d - k_c k_m y_v}{1 + k_c k_m} = \frac{k_m d (1 - \beta \frac{4}{\pi})}{1 + \beta \frac{4k_m d}{\pi a}} \]  

Therefore, a smaller \( k_c \) value results in a larger steady state value.

The process parameters, \( k_m, \theta_m, \tau_m, \xi_m \) calculated from (8), (9), (10) and (11) are then used to determine the controller setting through the following tuning rule.

2. Development of On-Line Tuning Method

Many simple tuning rules are proposed for the first order plus time delay model such as IMC (Internal Model Control), ITAE (Integral of the Time weighted Absolute Error), Z-N (Ziegler-Nichols) tuning rules (Ziegler and Nichols, Lopez et al., Cohen and Coon, Morari and Zafiriou). Several tuning methods using frequency response criteria or desired trajectory can be used to tune the PID controller for the second order plus time delay model (Hougen, Edgar et al., Harris and Mellichamp, Sung et al.). However, these methods require some numerical techniques or the user should define the desired trajectory. There are few simple and explicit tuning rules proposed for the second order plus time delay model without numerical techniques. Even the Z-N tuning rule must use a root finding technique and tuning results show poor performances.

We propose an explicit and simple tuning rule for the second order plus time delay model. We obtained the optimal tuning parameters with the ITAE (Integral of the Time weighted Absolute Error) as an object function. Next, we fitted the obtained optimal data. The proposed tuning rule shows good tuning results and need neither root finding technique nor desired trajectory defined by the user.

We obtained about 900 optimal tuning parameter sets using Simplex method with the following object function. Here, we used DEC3000 workstation to solve the optimization problem. Even though the alpha-chip of DEC3000 is the fastest in the world, at least, about 25 minutes are consumed to obtain one optimal tuning parameter set with reasonable initial values. Therefore, from the view point of the present computing power, it seems that on-line tuning strategy based on the on-line optimization is not realistic.

\[ k_c, \tau_s, \tau_d = \arg \left\{ \text{MIN} \int_0^\infty \{ t | y_s - y(t) | \} dt \right\} \]  

\[ \tau_m \frac{d^2 y(t)}{dt^2} + 2\tau_m \xi_m \frac{dy(t)}{dt} + y(t) = k_m u(t - \theta_m) \]  

\[ u(t) = k_c (y_s - y(t)) + \frac{k_c}{\tau_c} \int_0^\infty (y_s - y(t)) dt \]  

\[ + k_c \tau_d \frac{d(y_s - y(t))}{dt} + \text{disturbance} \]
Table 1 Proposed tuning rule for the second order plus time delay model

Set point change:

\[ k_{m_k} = -0.04 + \left[ 0.333 + 0.940 \left( \frac{\theta_m}{\tau_m} \right) \right] \xi_m, \xi_m \leq 0.9 \]

\[ k_{m_k} = -0.544 + 0.308 \left( \frac{\theta_m}{\tau_m} \right) + 1.408 \left( \frac{\theta_m}{\tau_m} \right) \xi_m, \xi_m > 0.9 \]

\[ \frac{\tau_m}{\tau_m} = \left\{ 2.055 + 0.072 \left( \frac{\theta_m}{\tau_m} \right) \xi_m, \xi_m \leq 1 \right\} \]

\[ \frac{\tau_m}{\tau_m} = \left\{ 1.768 + 0.329 \left( \frac{\theta_m}{\tau_m} \right) \xi_m, \xi_m > 1 \right\} \]

\[ \frac{\tau_m}{\tau_d} = \left\lfloor 1 - \exp \left( \frac{\left( \frac{\theta_m}{\tau_m} \right)^{1.060}}{0.870} \right) \left\lfloor 0.55 + 1.683 \left( \frac{\theta_m}{\tau_m} \right)^{1.090} \right\rfloor \right\rfloor \]

Disturbance rejection:

\[ k_{m_k} = -0.67 + 0.297 \left( \frac{\theta_m}{\tau_m} \right)^{2.001} \]

\[ + 2.189 \left( \frac{\theta_m}{\tau_m} \right)^{0.766} \xi_m, \xi_m < 0.9 \]

\[ k_{m_k} = -0.365 + 0.260 \left( \frac{\theta_m}{\tau_m} - 1.4 \right)^2 \]

\[ + 2.189 \left( \frac{\theta_m}{\tau_m} \right)^{0.766} \xi_m, \xi_m \geq 0.9 \]

\[ \frac{\tau_m}{\tau_m} = 2.212 \left( \frac{\theta_m}{\tau_m} \right)^{0.520} - 0.3, \frac{\theta_m}{\tau_m} < 0.4 \]

\[ \frac{\tau_m}{\tau_m} = -0.975 + 0.910 \left( \frac{\theta_m}{\tau_m} - 1.845 \right)^2 \]

\[ + \left\lfloor 1 - \exp \left( \frac{\xi_m}{0.15 + 0.33 \left( \frac{\theta_m}{\tau_m} \right)} \right) \right\rfloor \left\lfloor 5.25 - 0.88 \left( \frac{\theta_m}{\tau_m} - 2.8 \right)^2 \right\rfloor, \frac{\theta_m}{\tau_m} \geq 0.4 \]

\[ \frac{\tau_m}{\tau_d} = -1.9 + 1.576 \left( \frac{\theta_m}{\tau_m} \right)^{0.530} \]

\[ + \left\lfloor 1 - \exp \left( \frac{\xi_m}{-0.15 + 0.939 \left( \frac{\theta_m}{\tau_m} \right)^{-1.121}} \right) \right\rfloor \left\lfloor 1.45 + 0.969 \left( \frac{\theta_m}{\tau_m} \right)^{-1.121} \right\rfloor \]

![Fig. 2 Tuning results for the second order plus time delay model with \( \xi = 0.4 \)](image)

![Fig. 3 Tuning results for the second order plus time delay model with \( \xi = 1.0 \)](image)

\[ y(t) = \theta(t) = 0.0, \ t \leq 0.0 \quad (19) \]

where \( y_t = 1.0 \) and disturbance = 0.0 for the set point change process and \( y_t = 0.0 \) and disturbance = 1.0 for
### Table 2  Tuning results of the proposed tuning method for the servo problem

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$k_c$</th>
<th>$\tau_i$</th>
<th>$\tau_d$</th>
<th>$\xi$</th>
<th>$k_c$</th>
<th>$\tau_i$</th>
<th>$\tau_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>3.743(3.820)</td>
<td>0.813(0.812)</td>
<td>1.199(1.256)</td>
<td>0.4</td>
<td>1.333(1.340)</td>
<td>0.819(0.799)</td>
<td>1.221(1.256)</td>
</tr>
<tr>
<td>0.6</td>
<td>5.635(5.750)</td>
<td>1.219(1.221)</td>
<td>0.807(0.833)</td>
<td>0.6</td>
<td>2.019(2.029)</td>
<td>1.228(1.203)</td>
<td>0.840(0.833)</td>
</tr>
<tr>
<td>0.8</td>
<td>7.537(7.682)</td>
<td>1.626(1.631)</td>
<td>0.611(0.623)</td>
<td>0.8</td>
<td>3.392(3.382)</td>
<td>2.047(2.037)</td>
<td>0.536(0.517)</td>
</tr>
<tr>
<td>1.0</td>
<td>9.417(9.609)</td>
<td>2.032(2.041)</td>
<td>0.494(0.497)</td>
<td>1.0</td>
<td>4.916(4.840)</td>
<td>2.865(2.870)</td>
<td>0.406(0.394)</td>
</tr>
<tr>
<td>1.2</td>
<td>10.96(11.54)</td>
<td>2.439(2.451)</td>
<td>0.416(0.413)</td>
<td>1.2</td>
<td>6.449(6.370)</td>
<td>3.684(3.691)</td>
<td>0.335(0.329)</td>
</tr>
</tbody>
</table>

( ): optimal tuning value

### Table 3  Tuning results of the proposed tuning method for the regulatory problem

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$k_c$</th>
<th>$\tau_i$</th>
<th>$\tau_d$</th>
<th>$\xi$</th>
<th>$k_c$</th>
<th>$\tau_i$</th>
<th>$\tau_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>34.21(33.93)</td>
<td>0.368(0.380)</td>
<td>0.253(0.256)</td>
<td>0.4</td>
<td>4.836(4.976)</td>
<td>0.883(0.884)</td>
<td>0.598(0.625)</td>
</tr>
<tr>
<td>0.6</td>
<td>36.76(36.47)</td>
<td>0.368(0.375)</td>
<td>0.238(0.240)</td>
<td>0.6</td>
<td>5.937(5.974)</td>
<td>0.883(0.888)</td>
<td>0.515(0.537)</td>
</tr>
<tr>
<td>0.8</td>
<td>39.32(38.96)</td>
<td>0.368(0.372)</td>
<td>0.225(0.226)</td>
<td>0.8</td>
<td>8.139(8.095)</td>
<td>0.833(0.887)</td>
<td>0.410(0.423)</td>
</tr>
<tr>
<td>1.0</td>
<td>41.87(41.46)</td>
<td>0.368(0.371)</td>
<td>0.214(0.214)</td>
<td>1.0</td>
<td>10.34(10.36)</td>
<td>0.883(0.881)</td>
<td>0.347(0.352)</td>
</tr>
<tr>
<td>1.2</td>
<td>44.42(44.20)</td>
<td>0.368(0.366)</td>
<td>0.203(0.203)</td>
<td>1.2</td>
<td>12.54(12.75)</td>
<td>0.883(0.872)</td>
<td>0.305(0.307)</td>
</tr>
</tbody>
</table>

( ): optimal tuning value
the regulatory process. Then we fitted those data as shown in Table 1. Here the data between $\theta_m/\tau_m=0.05$ and $\theta_m/\tau_m=2.0$ are used because the usual chemical processes have $\theta_m/\tau_m \leq 2.0$.

The accuracy of the fitting equation in Table 1 is very good. To compare the proposed tuning results with both the optimal and Ziegler-Nichols tuning results, we obtained controller settings by each method as shown in Tables 2 and 3 and compared simulation results as shown in Figs. 2 and 3. Figures 2 and 3 show the tuning performances in the set point tracking and the input disturbance rejection problems for $\xi_m=0.4$ and $\xi_m=1.0$, respectively. As shown in the simulation results, we realize that the control performance of the proposed tuning rule is almost the same as that of the optimal tuning. However, Ziegler-Nichols tuning method shows very poor performance especially for the underdamped process.

3. Simulation Study

To show the performances of the proposed identification method for the PID controller auto-
tuning, let us compare it with several previous identification methods for the following cases.

(i) Overdamped process with several poles:

$$G_p(s) = \frac{\exp(-0.15s)}{(s+1)(\frac{s}{2}+1)(\frac{s}{3}+1)}$$

(ii) Overdamped high order process:

$$G_p(s) = \frac{1}{(s+1)^5}$$

(iii) Underdamped process with time delay:

$$G_p(s) = \frac{\exp(-s)}{(9s^2+2.4s+1)(s+1)}$$

The identified model and data for the simulation are shown in Tables 4, 5, and 6 and simulation results are depicted in Figs. 4, 5, and 6. Here, to obtain the controller parameters of Lee's and Lee and Sung's methods, we used IMC tuning rule and ITAE tuning.
Fig. 4 Control results of the proposed method and several previous methods for process (i)

Fig. 5 Control results of the proposed method and several previous methods for process (ii)

Fig. 6 Control results of the proposed method and several previous methods for process (iii)

rule for the set point change and disturbance rejection processes, respectively. In process (i) and process (iii), \(k_c = 2.0\) and \(k_c = 1.0\) are used for Lee's identification work, respectively, while \(k_c = 1.5\) and \(k_c = 1.0\) are used for Lee's and Lee et al.'s method in process (ii), respectively. In all processes, the proposed method used \(k_c = 0.35 k_{io}\) for the phase 2. Since Lee and Sung's and Lee's methods use the first order plus time delay model, there are limitations to approximate all the frequency region of the process. Lee and Sung's model is estimated with a high frequency test signal (relay feedback signal) and Lee's method estimates the model parameters using a low frequency test signal (set point change with a P controller). Therefore, in Fig. 4, we recognize that Lee and Sung's method shows an aggressive control action as mentioned in their paper and Lee's method shows a better control performance for the set point change process than that for the input disturbance rejection process because the control action must be low frequency signal for the set point change. However, Lee and Sung's method shows a better control performance for the regulatory process than that for the set point change process because the controller must act in the high frequency region for the input disturbance rejection process. It is compared with the poor control performance of Lee's method. The proposed method is superior to Lee and Sung's method and Lee's method for both set point change and input disturbance rejection processes. In Fig. 5, Lee et al.'s method shows a good set point tracking behavior. However, their method needs a complicated calculation to estimate the model parameters and tune the PID controller and the capability of the input disturbance rejection is poor. In Fig. 6, Lee's method and Lee and Sung's method show a very poor
control performance because the underdamped process is approximated by the overdamped first order plus time delay model. The proposed method shows good control performances for both underdamped process and overdamped process in comparison with those of the previous works.

4. Experiment Study

We applied the proposed method to control the level of tanks. As shown in Fig. 7, the process output and input are the level of the lowest tank and the control signal to manipulate the value, respectively. We used the AD/DA converter using RS232C serial communication to acquire the process data from DP cell and send the control signal to the valve. The obtained model is as follows.

\[
G_m(s) = \frac{6.26 \exp(-(121.30 \text{ sec})s)}{(415.73 \text{ sec})^2 s^2 + 2 \times (415.73 \text{ sec}) \times 0.95s + 1}
\]

Experimental results are shown in Fig. 8. Two step set changes are done after the identification work is over. In the phase 1, since the process is nonlinear, the peak value is different from the valley value of the level and the effect of noises is severe in the PID control region because of the derivative action. Here, we chose 5 sec as the sampling time and \( k_c = 0.35 k_{cw} \) for the phase 2. From the results of the experiment, we can recognize that even though the process is nonlinear and noisy, the proposed on-line identification and tuning method shows such a good control performance that it promises to be applied in industry.

Conclusion

We proposed a new on-line process identification strategy for the automatic tuning of the PID controller and the ITAE tuning rule for the second order plus time delay model. In the simulation study, the proposed method provides better performances than the previous several methods, especially for the underdamped process. The proposed identification strategy and on-line tuning rule do not need any numerical technique and are applied to control the level of the pilot-scaled tank. Therefore, the proposed automatic tuning technique promises to be implemented easily in industry. Since it is based on the second order plus time delay model, the underdamped process can be identified and the overdamped process can be approximated more exactly than the previous techniques based on the first order plus time delay model.

Appendix : Proof of Proposition

Since the process is the second order plus time delay, the future responses \( y(t), t \geq \theta \) are determined by the past controller output for the time delay \( \theta \) \( u(t), 0,0 \leq t \leq \theta \), the process output and its first derivative value at \( t = \text{time delay} (\theta) \). In addition, when the relay state is changed, the peak value of the process in the phase 1 appears after the longer time than the time delay because the relative degree is 2 so that the time delay of the process is shorter than \( \theta_p/4 \).
Therefore, in Fig. 1, the past controller output for the time delay is \(d\) and the value and the first derivative value of the process output at \(t\) = time delay(\(\theta\)) are \(y\) and 0. Therefore, if (1) is chosen as a controller to have the same initial controller output as the relay output \((u(t) = d, 0 \leq t \leq \theta)\), the closed-loop responses of the system after the time delay is the same as that of the assumed system with a step set change of which initial state is in the steady state and the P controller has the following form.

\[
\begin{align*}
  u(t) &= u_{bias} + k_c(y - (y + y_0)) \\
  &= d - k_c(y + y_0)
\end{align*}
\]  

(A1)  

(A2)

with the following initial state \((t = 0)\).

\[
\begin{align*}
  y_0 &= 0.0 \\
  y &= -y_r \\
  u &= u_{bias}
\end{align*}
\]

(A3)

Therefore,

\[
  d = u_{bias} + k_c y_0
\]

In steady state, the following equation is obtained.

\[
\begin{align*}
  u_s &= d - k_c(y_s + y_0) \\
  y_s &= k_m u_s
\end{align*}
\]

(A4)  

(A5)

Therefore, the steady state value of the process output is

\[
  y_s = \frac{k_m (d - k_c y_0)}{1 + k_c k_m}
\]

(A6)

\[\text{Nomenclature}\]

\[
\begin{align*}
  a &= \text{peak value in phase 1} \\
  y(s) &= \text{Laplace transform of process output} \\
  y_{s\alpha}, y_{s\beta} &= \text{steady state value, the first peak of the process output in phase 2} \\
  y_p &= \text{peak value of the process output in phase 1} \\
  y_v &= \text{valley value of the process output in phase 1} \\
  d &= \text{magnitude of relay} \\
  G_c(s), G_p(s) &= \text{controller and process transfer function} \\
  k &= \text{closed-loop gain} \\
  k_{t\alpha}, k_c, k_m &= \text{ultimate, proportional, and static gain.} \\
  p_r &= \text{period of relay} \\
  y_s(s) &= \text{Laplace transform of set point} \\
  i &= \text{time} \\
  u(t), u_s &= \text{controller output and steady state controller output} \\
  y(t), y_s &= \text{process output and set point} \\
  \beta &= \text{constant to determine the } k_c \text{ value} \\
  \Delta t &= \text{half period in phase 2} \\
  \theta, \theta_c &= \text{time delays of open-loop and closed-loop model, respectively} \\
  \xi, \xi_c &= \text{damping factors of open-loop and closed-loop model, respectively} \\
  \tau_d, \tau_i &= \text{derivative and integral time of the PID controller}
\end{align*}
\]

\[\text{Time constants of closed-loop and open-loop model, respectively}\]

\[\text{Closed-loop}\]

\[\text{Model}\]

\[\text{Process}\]

\[\text{Set point}\]

\[\text{Literature Cited}\]


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