DYNAMIC BEHAVIOR AND SOLID VOLUME FRACTION DISTRIBUTION IN A JETTING FLUIDIZED BED

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A reflection-type fiber optic probe was used to measure local solid volume fraction in a cylindrical jetting fluidized bed with a conical distributor using three kinds of particles different in sizes. By analyzing the local voidage fluctuations, the jet contours, and the time-averaged solid volume fraction profiles were studied. The jet extension half angle was found to be much lower than those calculated by Merry's correlation derived for beds with flat distributors. The solid volume fraction measured on the axis increased almost linearly with the distance from the nozzle. In the annulus, it was about one tenth lower than that at the minimum fluidization. The radial solid volume fraction profile in the jet was found to be parabolic.

Introduction

A gas jet injected vertically upwards in a fluidized bed may generate a variety of flow patterns. Possible patterns include a permanent cavity standing at the orifice (Rowe *et al.*, 1979), a jet plume consisting of a series of cavities divided by periodic "necks" or constrictions (Yang and Kearnins, 1974; Knowlton and Hirson, 1980; Kimura *et al.*, 1994), or chains of bubbles from the orifice (Rowe *et al.*, 1979). Which one of the patterns is formed depends on the fluidization mode of the bed above the jetting region, the configurations of the bed and the gas injector, and particle properties (Massimilla, 1985).

Many studies have so far been carried out on jet configurations, and various correlations for jet height have been proposed by different authors. However, discrepancies among these correlations exist probably because of the differences in the definitions of the jet height, experimental conditions, and the measuring methods (Massimilla, 1985). Rowe *et al.* (1979) defined the jet as a permanent void, but it has been found that the disturbance due to the gas jet extends farther than the height of the permanent void and the void configuration changes with time periodically. Knowlton and Hirsan (1980) used L_B (the deepest penetration of jet bubbles into the bed before losing their momentum), $L_{\max}$ (maximum penetration length of series cavities), and $L_{\min}$ (minimum penetration length) to describe the jet penetration. Some authors defined the jet height, H_j , as the distance between the orifice and the bubble bottom when an initial bubble detaches from the jet (Wen *et al.*, 1982; Tsukada and Horio, 1990; Kimura *et al.*, 1994). Various methods

as well as visual observations have also been used to measure jet configurations. Wen *et al.* (1982) measured the percentage of void time along the axis by a fiber probe and found that it gives a minimum at a certain height, which coincides with the jet height, H_j , defined as above. Raghunathan *et al.* (1989) determined jet heights based on momentum profiles measured by Pitot tubes. They defined the jet height as the height where the jet momentum flux decreases to that of the emulsion phase. Frame by frame analysis of high-speed movies of the jet shows this is the height at which the bubble detaches from the top of the jet. They also reported that contrary to isolated jet experiments, in which an increase in the nozzle velocity always results in a significant increase in the jet height, in the case of multiple jets, after the orifice velocity reaches a certain value, further increasing the orifice velocity decreases the jet height.

The jet boundary is another important parameter of the jet configuration. Many experimental data on it are available, mostly based on observation by photography. Momentum profiles measured by Pitot tubes have also been used to determine the jet boundary (Behie *et al.*, 1970; Behie *et al.*, 1971; Behie *et al.*, 1973; Raghunathan *et al.*, 1989). The extension half angle is usually used to describe the boundary. Based on various data over a wide range of operation conditions, Merry (1975) proposed a correlation to predict the extension half angle in the case of a flat distributor. Yang *et al.* (1986) surveyed solid circulation behavior in a large jetting fluidized bed with a force probe that could measure the impact direction of the particles. They identified three major solid flow regions, *i.e.* the central bubble street region in which solids move predominantly upward, the outer annulus region in which solids move predominantly downward, and the transient region between the

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above two regions. In the transient region, the solid flow is alternatively upward and downward, depending on the approach and departure of the bubbles. They also studied solid mixing by injecting tracer particles and found that the axial mixing of solids in the bubble street is very fast.

As well as jet configurations, gas and particle velocity distributions, and solid volume fraction distributions are also of great concern, especially in some processes including fast or selective chemical reactions, such as coal gasification processes, and polycrystalline silicon production. It is known that the transverse profiles of gas and solid velocity are of the Schlichting type and the turbulent jet theory is a suitable tool for describing gas dispersion in the jet region (Behie *et al.*, 1970; Behie *et al.*, 1971; Behie *et al.*, 1973; Donsi *et al.*, 1980; Massimilla *et al.*, 1981; Massimilla, 1985; Raghunathan *et al.*, 1989). But data on the solid volume fraction distributions in the jetting region are rather scarce, especially for three-dimensional beds. Basov *et al.* (1969) used radioscopy to measure the density distribution along the jet in a fluidized bed, but only average density at a transverse section can be measured by this method. Gidaspow *et al.* (1983) measured the time average voidage profiles in two-dimensional beds with a circular nozzle and with a rectangular nozzle by use of a γ -ray densitometer. A region of maximum porosity was found at a finite length above the jet inlet in the case of the circular nozzle jet. This maximum moved up the bed with increasing jet velocity. For the rectangular nozzle jet, the profiles of concentration contours were close to elliptical near the jet inlet. The jet heights were also determined from the voidage profiles, but they concluded that it is difficult to compare their data with various correlations, because the actual length of jet penetration depends upon its definition adopted. Kuipers *et al.* (1992) also measured the voidage distribution in a two dimensional rectangular bed with a jet by employing the IR-LED light transmission technique. Their experimental results also showed the existence of elliptic concentration contours near the dilute jet region. The porosity fluctuations in the jet region were also reported as evidence of the inherent oscillatory behavior of the fluidized bed.

Recently fiber optic probes have been used to measure local voidage in fluidized beds (Horio *et al.*, 1989; Hong and Tomita, 1995) and in spouted beds (He *et al.*, 1994). The advantage of the fiber optic probe technique is that it can be used to measure the local voidage in three dimensional beds.

In the present work, a reflection-type optic fiber probe connected to a PC interfacing system was used to measure the instantaneous local solid fraction in a three dimensional cylindrical jetting fluidized bed with a conical distributor. The conical distributor was

Fig. 1 Schematic diagram of experimental setup

adopted because it is often used in practical processes such as coal gasifiers (U-gas process, Patel, 1980; Westinghouse process, Schwartz *et al.*, 1982; and ICC process, Huang *et al.*, 1993), and polycrystalline silicon production units, but very few data on its dynamic characteristics are available in the published literature. The objectives of this study are to investigate the boundary between jet and annulus, penetration length of the jet, and solid volume fraction profiles and to elucidate the effects of operation conditions on them. The results will be used in the development of mathematical simulation of coal gasification, polycrystalline silicon production and other related processes.

1. Experimental

1.1 Apparatus

The experimental apparatus are schematically shown in Fig. 1. The fluidized bed is composed of a transparent acrylic resin circular column with 80 mm I.D. and 500 mm height. A perforated distributor of 60° cone angle and 128 holes of 0.5 mm I.D., and equipped with a circular center nozzle of 6 mm I.D. was employed. Three types of closely sized alumina particles with a density of 1500 kg/m³ were used as the bed material. Particle characteristics and experiment conditions are shown in Table 1. In all cases, the background air from the distributor was maintained at minimum fluidization level and the static bed height was maintained at 170 mm. The fluidizing gas was air supplied from a compressor. The flow rates of the fluidizing air injected through the distributor and through the center nozzle were controlled by needle valves and measured by orifice flow meters independently. In order to prevent electrostatic effects in the bed, humidifiers (with water temperature of 0°C) were employed through which two parts of the working air were forced.

As an auxiliary investigation, a half-column bed was also employed for observation with a video tape recorder (VTR). It was also made of transparent acrylic resin and all parameters were exactly the same

Table 1 Experiment conditions, results

Run	Average particle diameter D_p (μm)	Shape factor ϕ	Minimum fluidization velocity U_{mf} (cm/s)	Solid volume fraction at U_{mf} $1 - \varepsilon_{mf}$	Nozzle air velocity u_0 (m/s)
A-1	275	0.63	2.92	0.455	18.8
A-2	275	0.63	2.92	0.455	10.0
B-1	463	0.95	10.5	0.560	18.8
B-2	463	0.95	10.5	0.560	37.6
C-1	650	0.97	19.4	0.569	18.8
C-2	650	0.97	19.4	0.569	37.6

Run	Ratios of nozzle gas velocity to U_{mf} U_n/U_{mf}	Jet height H_j (mm)	Jet extension half angle θ_1		Merry's correlation (Eq. (1)) θ ($^\circ$)	Avg. solid volume fraction in annulus $1 - \varepsilon_a$
A-1	3.7	105	5.5	15.5	18.1	0.410
A-2	2.0	95	5.3	15.2	18.1	0.417
B-1	1.0	113	5.0	12.5	20.7	0.523
B-2	2.0	127	5.6	10.8	20.7	0.516
C-1	0.55	78	4.9	12.9	22.7	0.515
C-2	1.1	145	5.5	11.8	22.7	0.519

* θ_1 : corresponding to $f=f_m/2$; θ : corresponding to $f=0$

as those of full-column bed.

A reflection-type optic fiber probe with an outside diameter of 1.5 mm, consisting of 230 fibers of 0.05 mm diameter was used for measurement. The green light emitted from a laser diode was carried by light-projecting fibers to the measurement point. The light reflected by the particles was carried back through the light-receiving fibers and amplified to an A/D converter (SD2120, made by Graphtec Corp.), being converted into digital data according to the intensity of the signal. The digital data were taken by a personal computer and then the instantaneous signals were transformed into time-average solid volume fraction, void frequency and time fraction of lean solid concentration period ($1-\varepsilon < 0.1$). For each measurement point, the data were taken three times and 4096 data for each time at the rate of 2000 data per second. So all the data shown in the paper, *i.e.*, the time-average solid volume fraction, void frequency, and time fraction of lean solid concentration period were obtained over 6 seconds.

In the measurement, the probe was inserted vertically from the top of the bed and the tip of the probe was adjusted to the measurement location by a precision three-dimensional positioning mechanism. The probe was bent at the tip part (standard product made by Takenaka Electronic Industrial Co. LTD.) so that the tip was directed horizontally. The head of the probe was 4 mm wide. The disturbance of the probe on the flow might be expected, so observations were conducted through the transparent flat wall of the half-column bed. But obvious differences in the particle flow patterns could not be observed with and without the probe. Therefore, the effect of the probe on the measured particle concentration is assumed to be insignificant.

1.2 Calibration of the fiber probe

Some previous researchers took the relationship between solid volume fraction and the output voltage of the fiber optic probe as linear (Matsuna *et al.*, 1985; He *et al.*, 1994). Other researchers (Kuipers *et al.*, 1992; Werther *et al.*, 1993), however, have pointed out that optical theory does not provide a sound theoretical basis to relate quantitatively the recorded light transmission or reflection to the solid volume fraction at present, so appropriate calibration is necessary.

The key technique for the calibration is to realize a homogeneous gas-solid fluidized bed while the calibration measurement is conducted. Also it should be possible to measure enough points for this solid volume fraction over a wide range to make a calibration curve. So far two techniques have been used. One is the liquid-solid fluidization technique and the other is the vibrofluidization technique.

The liquid-solid fluidization technique provides a homogeneous expanded bed of solid particles over a wide range of solid fractions. Some researchers used this method to make the calibration. However, this method employs a different fluidizing medium (*i.e.*, a liquid) rather than the actual one used in the experiments (*i.e.*, air) and the absorption and reflection characteristics of light in the air-solid system and liquid-solid system are not identical. The difference in optical behavior between these two systems has been noted by Kuipers *et al.*, (1992). So Kuipers *et al.* used the vibrofluidization method to modify the calibration results made by liquid-solid fluidization method.

Figure 2 shows the schematic diagram of the experimental set-up of the vibrofluidized bed for the calibration of the sensor used in the present research.

Fig. 2 Vibrofluidization assembly for calibration of optical fiber probe

The calibration procedure is as follows: solid particles of certain weight are put into the vibrating cell. Next, the sensor to be calibrated is inserted into the cell. Then the calibrating cell was oscillated vertically (up and down) with a frequency of 20 Hz. The data taking method and procedure are just the same as those of actual measuring mentioned above. The average solid volume fraction in the measuring cell is calculated by using the following equation:

$$1 - \varepsilon = \frac{W_s}{\rho_s V_c}$$

where W_s is the weight of solid and V_c is the volume of the cell.

The correlations between the solid volume fraction and the intensity of signal were derived by the least squares method.

2. Results and Discussions

According to the phenomena observed by VTR in the half-column bed, the formation of a stream of voids (hereafter referred to as "bubbles"; the double quotation marks are used to distinguish the term from that of the usual meaning of bubble) from the nozzle mouth was found to be periodical. The stream of "bubbles" can be identified as the leading "bubble" and two or more following "bubbles" (the number of following "bubbles" increased with the increase of nozzle velocity u_0 , and up to 4 under the present experimental conditions). Figure 3 shows the formation pattern of a stream of "bubbles" and their

Fig. 3 Formation of "bubble" stream in jet

Fig. 4 Signal taken in jet

coalescence phenomena observed by VTR. The constriction between "bubbles" is accompanied with particle entrainment into the jet region. In a period it can be divided into the formation stage of a stream of "bubbles" and the coalescence stage of "bubbles." At the formation stage (from Fig. 3(a) to (c)), the number of "bubbles" in the stream increases with time, and with up-going of "bubbles," their sizes also increase. Although every "bubble" can be identified individually, the boundary line between two "bubbles" is not clear for the nose of the lower "bubble" seems to overlap the wake of the upper "bubble." During the coalescence stage (from Fig. 3(d) to (f)), one or more following "bubbles" catch up the leading "bubble" and they coalesce to form a bigger matured bubble and it detaches from the jet. The formation stage takes a relatively long time, while the coalescence of "bubbles" is completed within a shorter time. As a result of the propagation of these bubbles through the bed, the jet region is formed and the local solid volume fraction oscillates with time in the jet region (as shown in Fig. 4). The time-average solid volume fraction was obtained by treating the instantaneous signals.

Fig. 5 Solid volume fraction ($1-\varepsilon$) profiles in jetting fluidized bed under different experiment conditions

Test measurement indicated the symmetry jet center line coincided with the axis of the bed, therefore, measurements were carried out for only half of the bed cross section.

2.1 Profiles of solid volume fraction

Figure 5 shows the profiles of time-average solid volume fraction under six experiment conditions. The curves were obtained by simple linear interpolation of the measurements made at 139 spatial locations. From the inlet of the nozzle to a certain height, there was a region in the central part where the solid concentration was very low ($1-\varepsilon < 0.1$). Around this dilute region, elliptic iso-concentration contours exist. From Fig. 5, when the nozzle velocity increases, the contours of iso-concentration of solid extend significantly in the axial direction, whereas in the radial direction, the low solid concentration area does not seem to extend obviously. As to the effect of particle diameter, the low solid concentration area for the smaller particles seems larger when the nozzle gas velocity (u_0) is kept the same (A-1 vs. B-1 vs. C-1 in

Fig. 5). On the contrary, when keeping U_n/U_{mf} constant, the low solid concentration area extends more in the axial direction for larger particles (B-1 vs. C-2, A-2 vs. B-2 in Fig. 5). Regardless of nozzle velocity, the contours of iso-concentration of solid for particle A (with the smallest particle size) extend much more in the radial direction than those for the two bigger particles (particles B and C). However, the extension levels of low solid concentration area seem to be almost the same for B and C particles.

Similar elliptic iso-concentration contours have also been reported by Gidaspow *et al.* (1983) and Kuipers *et al.* (1992) from their experimental results obtained in two-dimensional beds with a flat distributor. But the very lean phase area was much smaller or cannot be seen in their curves. The lowest iso-concentration contours described are $1-\varepsilon = 0.32$ and $1-\varepsilon = 0.15$ in Gidaspow *et al.*'s and Kuipers *et al.*'s papers, respectively, while the very lean phase ($1-\varepsilon < 0.1$) occupies quite a large area in Fig. 5. The different ratios of superficial nozzle gas velocity to minimum fluidization velocity U_n/U_{mf} (0.17-0.54 in Gidaspow's; 0.53-1.05 in Kuipers; 0.55-3.7 in the present study) might be the cause of the differences. Another cause might be the difference between the two-dimensional bed they used and the three-dimensional bed in the present work.

2.2 Jet penetration along the axis

Figure 6(a) shows axial solid volume fraction profiles under various conditions. From Fig. 6(a) the solid volume fraction on the axis increases almost linearly with the distance from the exit of nozzle in the jet. An increase in the nozzle air velocity significantly decreases the solid concentration at the axis. As mentioned above, the solid volume fraction in the jet is oscillatory. Each oscillation corresponds to the passage of a "bubble" at the measuring location. The void frequency, f , is defined as the number of valleys below the solid volume fraction of 0.1, and the time fraction of lean solid concentration period T_{lc} is defined as the time portion below the solid volume fraction of 0.1. The void frequency and time fraction of lean solid concentration period on the axis are shown in Fig. 6(b) and (c).

Jet heights have been reported by a number of researchers and their definitions are diverse. In order to determine the jet height, $h_{(a)}$ (the height where solid volume fraction along the axis reaches a certain value), $h_{(b)}$ (the height where the "bubble" frequency decreases to almost unchanged level) and $h_{(c)}$ (the height where time fraction of lean solid concentration period decreases to a constant level) were checked. As shown in Fig. 7 the comparison among them shows they are essentially equivalent. Therefore, the jet height is defined as the height where the "bubble" frequency decreases to an almost unchanged level. The determined jet heights under different conditions

Fig. 6 Axial dynamic parameters versus height with changes of particle size and nozzle velocity

Fig. 7 Comparison of jet heights determined by different methods

Fig. 8 Effects of particle diameter and nozzle gas velocity on jet height

are listed in Table 1. As expected, the jet height increased with increasing nozzle velocity, while the influence of particle diameter seems complex. Figure 8 shows the jet height (H_j) versus $U_n - U_{mf}$ for different particle sizes. Generally speaking, the larger the particle size, the higher the jet height under the present experimental conditions.

2.3 Jet boundary

According to the observation by VTR, the jet width was found to change periodically. Inspection of a stream of “bubbles” passing through a certain

height indicated the radial dimension of the first “bubble” to be much larger than those of the following “bubbles” as a result of greater resistance encountered by the first bubble in the axial direction than that encountered by the following “bubbles.” So a stable jet boundary does not exist, and the temporal boundary oscillates greatly. This characteristic is reflected in the radial variation of void frequency. Here we try to make the determination in terms of radial distribution of void frequency. Figure 9 shows the plots for void frequencies $f=0$, $f=1 \text{ s}^{-1}$, $f=f_m/4$, and $f=f_m/2$ (f_m is the void frequency on the axis, as shown in Fig. 6(b)). Figure 6(b) shows that axial void frequency, f , decreases with height but is larger than 2 s^{-1} in all cases, so both $f=0$ and $f=1 \text{ s}^{-1}$ were selected for plots in Fig. 9. The plots for $f=0$ represent the critical points from where the “bubbles” disappear. Unlike Patrose’s (1984) conclusion from a shallow (spouted) bed with flat distributor, which says the jet expanded only in the lower part very close to the nozzle mouth ($x < 30 \text{ mm}$) but not in the upper part, the present study showed that the jet expanded from the nozzle mouth to the top of the jet. Although the jet expands more in the nozzle mouth than in the upper part, the jet boundary could still be approximated as a cone. The half angle of the cone for $f=0$ corresponds to the jet extension half angle defined by Merry (1975).

Table 1 shows the data of the extension half angle θ for $f=0$ and θ_1 for $f=f_m/2$. It can be seen that the values of θ for the smallest size of particle (particle A, $275 \mu\text{m}$) seem larger than those for the two larger particles (B and C). However, all the values of θ_1 are very close to each other, and they are much smaller than those of θ . The difference between θ and θ_1 reflects great oscillation of the jet boundary. This is similar to Yang *et al.*’s finding (Yang *et al.*, 1986) that a transient region exists between the core jet region and the annulus region. It is, however, difficult to determine the inner boundary dividing the core jet region and the transient region.

Fig. 9 Jet boundary

From the cases of flat distributors Merry (1975) concluded a correlation for the extension half angle as follows:

$$\cot \theta = 10.4 \left(\frac{\rho_s D_p}{\rho_g D_n} \right)^{-0.3} \quad (\text{Merry (1975)}) \quad (1)$$

The calculated values of θ by Eq. (1) under different conditions are shown in Table 1. Compared to the values calculated by Merry's correlation, the present experimental data are much lower, especially for larger particles (particles B and C) which are about half of the values calculated by Merry's correlation. This coincides with the finding of Kimura *et al.* (1994) that the jet was narrowed and elongated when a conical distributor was employed compared to when a flat distributor was employed.

2.4 Radial distribution of solid volume fraction in the jet

The solid volume fraction in the jet was relatively low. Generally, it increased from the lowest value at the center to almost as high as that in the annulus at the jet boundary determined by $f=0$. Using a linear regression method, the solid volume fraction profiles in the jet has been concluded to be the following correlation:

Fig. 10 Radial solid concentration profile

Fig. 11 Solid volume fraction in the annulus

$$\frac{(1-\varepsilon) - (1-\varepsilon_m)}{(1-\varepsilon_a) - (1-\varepsilon_m)} = \left(\frac{r}{b} \right)^2 \quad (2)$$

That is to say, the solid volume fraction profiles in the jet are parabolic. This correlation is compared to the experimental data in Fig. 10. The correlation coefficient and the standard deviation for Eq. (2) are 0.930 and 0.082, respectively.

2.5 Solid volume fraction in the annulus

The solid volume fraction in the annulus ($1-\varepsilon_a$) seems not to be constant. When the nozzle air velocity was higher, the solid volume fraction in the annulus was a little bit lower, and it decreased with distance from the exit of the nozzle (see Fig. 11). As shown in Table 1, the solid volume fraction in the annulus ($1-\varepsilon_a$) was on average about one-tenth lower than that at minimum fluidization ($1-\varepsilon_{mf}$). This demonstrates that the common assumption that the voidage in the annulus is equal to the voidage at minimum fluidization (Donsi *et al.*, 1980; Massimilla *et al.*, 1981) is not fully accurate.

Conclusion

- 1) A reflection-type optical fiber probe technique has been developed for measuring local solid volume fraction in three-dimensional jetting fluidized beds. Employing a PC interfacing system makes the measurement of local solid volume fraction oscillation possible.
- 2) The time-average iso-concentration contours are nearly elliptic in shape, similar to the experimental results of Gidaspow *et al.* (1983) and Kuipers *et al.* (1992) obtained in two-dimensional beds with a flat distributor. From the inlet of the nozzle to a certain height, there is a region in the central part where the solid concentration is very low ($1-\varepsilon < 0.1$). This region extends significantly along the axis, but changes little in the radial direction when the nozzle velocity is increased.
- 3) The values of jet extension half angle measured in the three dimensional jetting fluidized bed with conical distributor are much lower than those calculated by Merry's correlation for the cases of flat distributors being employed.
- 4) The solid volume fraction on the axis increases almost linearly with distance from the exit of the nozzle in the jet. When larger particles are used, the jet height seems higher.
- 5) The radial solid volume fraction distribution in the jet is parabolic. The solid volume fraction in the annulus ($1-\varepsilon_a$) is found not to be constant and a little bit lower than that at minimum fluidization ($1-\varepsilon_{mf}$).

Nomenclature

b	= width from axis to boundary of the jet	[m]
D_n	= nozzle diameter	[m]
D_p	= particle diameter	[mm]
D_t	= column diameter	[m]
f	= void frequency	[s ⁻¹]
f_m	= void frequency on axis	[s ⁻¹]
$h_{(a)}$	= height where solid volume fraction along axis reaches certain value	[m]
$h_{(b)}$	= height where void frequency decreases to almost unchanged level	[m]
$h_{(c)}$	= height where time fraction of lean solid concentration period decreases to constant level	[m]
H_j	= jet height	[m]
r	= radius	[m]
T_{lc}	= time fraction of low concentration of solid	[%]
U_a	= superficial gas velocity from distributor	[m/s]
U_{mf}	= minimum fluidized velocity	[m/s]
U_n	= superficial gas velocity from nozzle	[m/s]
u_o	= nozzle gas velocity	[m/s]
x	= distance from nozzle exit	[m]
ε	= voidage	[-]
ε_a	= voidage in annulus	[-]
ε_m	= voidage on axis	[-]
ε_{mf}	= voidage under minimum fluidization	[-]
ϕ	= shape factor of particle	[-]
θ	= extension half angle of jet boundary	[°]
θ_1	= extension half angle corresponding to $f=f_m/2$	[°]

ρ_s = solid density of particle [kg/m³]

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