A TWO-FLUID TURBULENCE MODEL FOR GAS-SOLID TWO-PHASE FLOWS

YUNLIANG WANG and SATORU KOMORI
Department of Chemical Engineering, Kyushu University,
Fukuoka 812-81
MYUNG KYOON CHUNG
Department of Mechanical Engineering, Korea Advanced Institute of
Science and Technology, Yusong-gu, Taejon, South Korea

Key Words: Particles, Suspension Flow, Turbulence Modeling, Computation, Two-way Coupling

On the basis of a two-equation turbulence model of single-phase flows, two-fluid turbulence models, namely, the $k-e-k_p-e_p$ model and the $k-e-k_p$ model are developed to describe turbulent gas-solid two-phase flows. Governing equations for the turbulent kinetic energy and the kinetic energy dissipation rate of the particulate phase are derived from their momentum equations, and unknown terms at two-equation closure level are modeled. The developed models show that the turbulence intensity of the particulate phase is often larger than that of the gaseous phase in the gas-solid flow in a 90° bend, which is in accordance with the experimental results. The conventional $k-e-A_p$ model using a $k-e$ turbulence model for the carrier fluid and Tchen-Hinze's formula for the eddy viscosity of the particulate phase, however, shows that the particulate turbulence intensity is always smaller than that of the gaseous phase. Therefore, the two-phase turbulence models developed here are superior to the conventional turbulence model. The turbulence models are also applied to predict the effect of particles on gaseous phase turbulence in the gas-solid flow in a vertical pipe. The predicted results are favorably compared with the available experimental data.

Introduction

The phenomena of two-phase suspension flows appear widely in nature and industrial processes, such as the transport of sand, pneumatic conveyance, fluidized beds, and feeding processes of fine chemical powders into a chemical reactor. Hence, it is of great importance to understand the mechanism of gas-solid two-phase flows. Compared to turbulent single-phase flows, turbulent two-phase flows are quite complicated because of the interaction between two phases. Soo (1967), Boothroyd (1971), and Hinze (1972) reported their earlier studies on the mechanism of exchange and interaction between two phases.

Choi & Chung (1983) extended the mixing-length theory to close the relevant momentum equations for two-phase flows, and evaluated the velocity profiles and friction factor of the turbulent suspension flow in a pipe. Pourahmadi & Humphrey (1983) developed two-phase turbulence transport equations, and applied their model to predict erosion wear in a bend.

To study turbulent gas-solid suspension flows, Chung et al. (1986), Lee & Chung (1987), and Han et al. (1991) developed several models of scalar eddy viscosity under the assumption of turbulence in a state of local equilibrium between the production and dissipation of the turbulent kinetic energy for both phases. Similar works can be found in the references of Elghobashi & Abou-Arab (1983), Elghobashi et al. (1984), and Rizk & Elghobashi (1989).

In the above-mentioned numerical studies, however, the Hinze-Tchen's formula and revised versions of it were widely used to calculate the eddy viscosity of the particulate phase. The eddy viscosity of the particulate phase ($v_{up}$) was often assumed to be proportional to that of the primary fluid ($v_i$). The proportional coefficient is a function of the ratio of particle relaxation time scale ($\tau_p$) to primary fluid Lagrangian integral time scale ($\tau_e$) (cf. Choi & Chung, 1983; Pourahmadi & Humphrey, 1983). The simplest form is:

\[ \frac{v_{up}}{v_i} = \frac{k_p}{k} = \left(1 + \frac{\tau_p}{\tau_e}\right)^{-1} \]  

(1)

This model indicates that the local turbulent kinetic energy of the particulate phase ($k_p$) is always smaller than that of the gaseous phase ($k$). However, the available LDV data show that the turbulence intensity of the particulate phase is not always smaller but often larger than that of the gaseous phase, as shown by Tsuji & Morikawa (1982), Kliafas & Holt (1987), and Kulick et al. (1994).

The objective of the present study is, therefore, to estimate $v_{up}$ based on turbulence quantities $k$, $e$, $k_p$, and/or $\tau_p$, of which transport equations are derived in the following sections. Instead of using such a simple model (1), two new turbulence models, i.e., $k-e-k_p-e_p$ model and $k-e-k_p$ model are proposed. In the $k-e-k_p-e_p$ turbulence model, it is assumed that the turbulent fluctuation length scale of the particulate

* Received on October 14, 1996. Correspondence concerning this article should be addressed to Y. Wang.
phase \( \rho_p \) is proportional to \( k_p^{1.5} / \epsilon_p \). The eddy viscosity of the particulate phase is obtained by \( \nu_p = C_{\text{mp}} k_p^{0.5} / \epsilon_p \). In the \( k-e-A_p \) turbulence model, the particulate phase is assumed to have the same turbulent fluctuation length scale as that of the gaseous phase, i.e., \( \ell_p \sim \epsilon \). Hence the eddy viscosity of the particulate phase is given by \( \nu_p = C_{\text{mp}} \rho_p k_p^{0.5} / \epsilon_p \).

The present turbulence models are applied to predict the turbulent gas-solid suspension flows in a 90° bend and in a vertical pipe, and their predictions are compared with the LDV measurements. The present models are also compared with the conventional \( k-e-A_p \) model which is based on the single phase \( k-e \) turbulence model and the Hinze-Tchen's formula.

1. Governing Equations and Turbulence Models

1.1 Assumptions

In order to develop turbulence models for two-phase flows, the following assumptions are made:

1. Both the gaseous phase and the particulate phase are regarded as a continuum macroscopecally, but only the gaseous phase behaves as a continuum microscopically.

2. The mean flow is steady, incompressible and isothermal. The suspension flow is a dilute one, and the direct collisions between particles are negligible.

3. All the particles are spheres of the same diameter. The momentum exchange between the two phases is described by the viscous drag due to the velocity slip between two phases.

On the molecular level, collisions between particles and gas molecules are the main mechanism of momentum exchange between the two phases. The interaction between particles and the gas enhances the momentum transfer between the adjacent layers of the gaseous phase. When a particle is suspended in a shear layer, the particle rotates due to the velocity difference between its upper surface and its lower surface. The fluid parcel over the upper surface moves down and the parcel below the bottom surface goes up, each with its respective momentum. As a result, momentum transfer takes place between the adjacent layers. The augmentation of the laminar momentum transfer which is usually called the kinematic viscosity of the second fluid, has been investigated by Happel (1957), Frankel & Acrivos (1967), and many others.

Einstein's formula is widely used to compute the kinematic viscosity of the particulate phase:

\[ \nu_p \sim \alpha_v \nu \]  \hfill (2)

where \( \alpha_v \) is particle volumetric fraction.

For homogeneous shear flows where the particulate phase has the same velocity as that of the carrier fluid, the Einstein formula is a valid model. Unfortunately, in most of the actual cases, suspension flow situations are not consistent with the Einstein assumption. In this work, Choi & Chung's proposal (1983) is used, which assumes that the value of \( \nu_p / \nu \) is regarded as similar to that of the carrier fluid:

\[ \nu_p / \nu \sim \nu / \nu_i \]  \hfill (3)

Han et al. (1991) employed formula (3) to successfully analyze the dilute gas-solid two-phase flow in a pipe.

1.2 Governing equations

Using Reynolds decomposition for fluctuating variables and time averaging of the instantaneous equations, the time-averaged governing equations for both phases can be obtained as follows:

- **gaseous phase:**

\[ (\rho U_j)_i = 0 \]  \hfill (4)

\[ (\rho U_i U_j)_i = -P_i + [\mu(U_{i,j} + U_{j,i}) - \bar{u_i u_j}]_i + \left[ \rho_p (U_i - U_p) + \bar{p}_p (u_i - u_p) \right] / \tau_p \]  \hfill (5)

- **particulate phase:**

\[ (\rho_p U_i)_j = -\langle \rho_p \nu_p U_j \rangle_i \]  \hfill (6)

\[ \langle \rho_p U_i U_j \rangle_j = \rho_p \nu_p \langle U_{i,j} + U_{j,i} \rangle_j - \rho_p \mu \langle U_{i,j} \rangle_j \]

\[ + \alpha_c (\rho_p - \rho) g_i + \left[ \left( \rho_p \bar{p}_p \nu_p U_j + U_{i,j} \bar{p}_p U_i + \bar{p}_p U_{i,j} \right) \right]_j \]

\[ + \left[ \nu_p \rho_p \bar{p}_p (U_{i,j} + U_{j,i}) \right]_j + \left[ \bar{p}_p (U_i - U_p) \right] \]

\[ + \bar{p}_p (u_i - u_p) / \tau_p \]  \hfill (7)

Here \( \tau_p \) is the particle relaxation time scale, and for \( Re_p < 1000 \), it is estimated by (Boothroyd, 1971):

\[ \tau_p = \rho_p d_p^2 / [18 \mu (1 + 0.15 Re_p^{0.687})] \]  \hfill (8)

The particle Reynolds number is defined as:

\[ Re_p = \sqrt{(U_j - U_p)^2} \]  \hfill (9)

In the above equations, the second-order correlation terms containing \( \rho_p \) are usually approximated by the gradient-type diffusion assumption:

\[ \bar{\rho_p u_i} = \nu_i / \sigma_{\rho p,i} \]  \hfill (10)

\[ \bar{\rho_p u_i} = \nu_p / \sigma_{\rho p,i} \]  \hfill (11)

where \( \sigma_{\rho p} \) is the turbulent Schmidt number, and its
value is 0.7 (Han et al., 1991). \( v_t \) and \( v_{\nu} \) are the eddy viscosities of the gaseous phase and the particulate phase, respectively. In this study, a more general form given by Simonin (1991) is employed to estimate these terms:

\[
-\tilde{\rho_p}u_{\nu} = \tau_u u_{\nu} u_{\nu, p, j}
\]

(12)

where \( \tau_u \) is the fluid Lagrangian integral time scale and is given by Pourahmadi & Humphery (1983):

\[
\tau_u = 0.41 k / \varepsilon
\]

(13)

Similarly, for \( \tilde{\rho_p} u_{\nu} \) we have

\[
-\tilde{\rho_p} u_{\nu} = -\tau_u u_{\nu} u_{\nu} \rho_{p, j}
\]

(14)

For the modeling of the Reynolds stress terms, the Boussinesq assumption is used:

\[
-\overline{u_i u_j} = v_i (U_{i, j} + U_{j, i}) - 2/3 \delta_{ij} k
\]

(15)

\[
-\overline{u_{\nu} u_{\nu}} = v_{\nu} (U_{\nu, j} + U_{j, \nu}) - 2/3 \delta_{ij} k_p
\]

(16)

where \( k = \overline{\nu_1^2} / 2 \) and \( k_p = \overline{\nu_1^2} / 2 \) are the turbulent kinetic energies of the gaseous phase and the particulate phase, respectively. The eddy viscosity of the gaseous phase \( v_i \) is given by

\[
v_i = C_{\mu} k^2 / \varepsilon
\]

(17)

Here \( v_{\nu} \) in Eq. (16) is related to both the turbulent kinetic energy \( (k_p) \) and the energy dissipation rate of the particulate phase \( (\varepsilon_p) \) by the following formula:

\[
v_{\nu} = C_{\mu \nu} k_p^2 / \varepsilon_p
\]

(18)

where \( C_{\mu \nu} = C_{\mu} = 0.09 \).

Chung et al. (1992) investigated the performances of various computational turbulence models for the third-order diffusive terms so as to predict the scalar dispersion problems. Finally they found that the simple gradient-type model has the best overall prediction performance among all the models. Therefore the third-order correlation terms \( \tilde{\rho_p} u_{\nu} u_{\nu} \) are calculated using the conventional simple gradient-type model (Lauder, 1976).

\[
-\tilde{\rho_p} u_{\nu} u_{\nu} = C_{\nu} k_p / \varepsilon_p [u_{\nu} u_{\nu} (\overline{\rho_p u_{\nu}}) + u_{\nu} u_{\nu} (\overline{\rho_p u_{\nu}}) + \overline{\rho_p u_{\nu}}] + \overline{\rho_p u_{\nu} u_{\nu}}
\]

(19)

where the value of \( C_{\nu} \) is 0.11.

The correlation terms of the type of \( \overline{\rho_p u_{\nu}} \) appearing in a form multiplied by the kinematic viscosity of the particulate phase, will be neglected due to their relatively small magnitudes (Elghobashi & Abou-Arab, 1983).

### 1.3 Turbulence model equations for \( k \) and \( \varepsilon \)

The exact equations of \( k \) and \( \varepsilon \) for the gaseous phase can be found in Elghobashi & Abou-Arab's work (1983). In the present study, the \( k \)-equation is derived by multiplying the \( u_i \) equation by \( u_i \) and then taking the time-average. The \( \varepsilon \)-equation is derived by differentiating the \( u_i \) equation with respect to \( x_i \), multiplying by \( u_i u_i x_i \), and finally taking the time-average. Similarly, the governing equations of \( k_p \) and \( \varepsilon_p \) are derived. The fourth-order correlation terms in the governing equations are neglected. We used Jones & Launder's method (1972) to model the 'single-phase' correlation terms that are not directly related to the interphase interaction. Coefficients appearing in the governing equations of the particulate phase are given by the same values as those of the gaseous phase for lack of available experimental data.

The equations of turbulent kinetic energy are:

- **gaseous phase**:

\[
\left( \rho u_k \right)_j = \left[ (\mu + \mu_t / \sigma_k) k, j \right] + G_k - \rho e
\]

\[
+ \left[ \rho_p (-2k + u_{\nu} u_{\nu}) - \left( U_i - U_{\nu} \right) \overline{p_p u_{\nu}} \right] / \tau_p
\]

(20)

- **particulate phase**:

\[
\left( \rho_p U_p k_p \right)_j = \left[ \rho_p (v_{\nu} + v_{\nu} / \sigma_{k_p}) k_p, j \right] + G_{k_p} - \rho_{p \varepsilon_p}
\]

\[
+ (1 - \rho / \rho_t) g \overline{p_p u_{\nu} u_{\nu}} + \left[ \rho_p (-2k_p + u_{\nu} u_{\nu}) \right] / \tau_p
\]

\[
+ \left( U_i - U_{\nu} \right) \overline{p_p u_{\nu} u_{\nu}} - \overline{p_p u_{\nu} u_{\nu}} + \overline{p_p u_{\nu} (u_i - u_{\nu})} / \tau_p
\]

(21)

where \( \sigma_k = 1.0 \). \( G_i \left( \rho \overline{u_i u_i} u_j \right) \) is the turbulence production term of the gaseous phase, and \( G_{k_p} \left( \rho_p \overline{u_i u_i} u_{\nu} \right) \) is the turbulence production term of the particulate phase.

The correlation terms in the turbulent kinetic energy equations are composed of the following types: \( \overline{p_p u_{\nu}} \), \( \overline{p_p u_{\nu} u_{\nu}} \), \( \overline{p_p u_{\nu} u_{\nu}} \), \( \overline{p_p u_{\nu} u_{\nu}} \), and \( \overline{p_p u_{\nu} u_{\nu}} \). The terms \( \overline{p_p u_{\nu}} \) and \( \overline{p_p u_{\nu}} \) are calculated by Eqs. (12) and (14), respectively. For the modeling of the second-order correlation terms \( u_{\nu} u_{\nu} \), the following assumptions are made.

If \( \tau_p / \tau_u = 0 \), the particles have adequate time to respond to the turbulent fluctuation of the carrier fluid. Namely \( u_i \) and \( u_{\nu} \) are completely correlated:

\[
\overline{u_{\nu} u_{\nu}} = \overline{u_{\nu} \nu} = 2k
\]

(22)

If \( \tau_p / \tau_u = 0 \), the particles do not have sufficient time to respond to the turbulent fluctuation of the
carrier fluid. Therefore $u_i$ and $u_{pi}$ are assumed to be not correlated:

$$
u_{pi} = 0$$  \hspace{1cm} (23)

However, the value of $\tau_p/\tau_r$ does not generally have these extremes. The term of $\nu_{pi}$ is estimated by an intermediate value within the range of $0-2k$ by the following model:

$$\nu_{pi} = 2k \cdot e^{d_k \cdot \tau_p/\tau_r}$$  \hspace{1cm} (24)

where $d_k$ is the damping factor, and in this paper $d_k=1$.

The third-order correlation $\rho_{pi}^{n} u_{pi}$ can be calculated directly by Eq. (19). The terms, such as $\rho_{pi}^{n} u_i$ and $\rho_{pi}^{n} u_{pi}$ are modeled in a similar manner to Eq. (19):

$$-\rho_{pi}^{n} u_i = 2C_j C_l / \nu_{pi} (\rho_{pi}^{n} u_i)$$  \hspace{1cm} (25)

$$\rho_{pi}^{n} u_{pi} = 2C_j / \nu_{pi} (\rho_{pi}^{n} u_{pi}) + \nu_{pi} u_{pi} (\rho_{pi}^{n} u_i)$$  \hspace{1cm} (26)

The equations of turbulent energy dissipation rate are:

- **Gaseous phase:**
  
  $$(\rho U_{\phi})_j = [(\mu + \mu_i/\sigma_i)\text{Re}_j]_j + C_i G_{\phi}/k$$
  $$- C_2 \rho e^{2} / k + 2(\rho_\phi (\rho - \nu u_i) / \tau_p)$$
  $$- 2v (U_i - U_p) \rho_{pi}^{n} u_{pi} / \tau_p - 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$- 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$- 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$- 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$- 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$

- **Particulate phase:**
  
  $$(\rho_\phi U_{\phi})_j = [(\rho_\phi + \rho_\phi/\sigma_\phi)\text{Re}_j]_j + C_i G_{\phi} / k$$
  $$- C_2 \rho e^{2} / k + 2(\rho_\phi (\rho - \nu u_i) / \tau_p)$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$
  $$+ 2v (U_i - U_p) \rho_{pi}^{n} u_{pi}$$

where $C_i = C_j = 1.44$, $C_p = C_k = 1.92$, $\sigma_p = 1.3$.

In the dissipation rate equations, the terms $\rho_{pi}^{n} u_i$, $\rho_{pi}^{n} u_{pi}$, $\rho_{pi}^{n} u_{pi}$, $\rho_{pi}^{n} u_{pi}$, and $\rho_{pi}^{n} u_{pi}$ are negligible for high Reynolds number flows (Lauder, 1976). The terms $u_{i,pi}$, $u_{pi}$, and $u_{pi}$ are modeled in a similar way to Eq. (24).

$$u_{i,pi} = e \cdot \nu \cdot e^{-d_k \cdot \tau_p / \tau_r}$$  \hspace{1cm} (29)

$$u_{pi} = u_{pi} / e \cdot \nu \cdot e^{-d_k \cdot \tau_p / \tau_r}$$  \hspace{1cm} (30)

Here $d_k$ is another damping factor, and in this study $d_k=1$.

The third-order terms containing $u_i u_{pi}$ or $u_i u_{pi}$ are neglected because of their smaller magnitudes (Elghobashi & Abou-Arab, 1983).

### 2. Numerical Procedure and Boundary Conditions

For a two-dimensional flow, there are two continuity equations, four momentum equations, two turbulent kinetic energy equations and two turbulent energy dissipation rate equations.

This set of equations can be written in a general form of convection-diffusion equation for an arbitrary variable $\Phi$.

$$\frac{\partial (\rho \Phi)}{\partial x} + \frac{\partial (\rho \Phi)}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \Phi}{\partial x} \right)$$

$$+ \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \Phi}{\partial y} \right) + S$$  \hspace{1cm} (31)

Here $\Gamma$ is the effective diffusion coefficient, and $S$ is the source term. Body-fitted coordinates are used for the solution of Eq. (31). Instead of the staggered grid, the non-staggered grid arrangement is employed for the computation. A specific scheme developed by Rhie & Chow (1983) is used to suppress the pressure oscillation of the gaseous phase, which is similar to the SIMPLE algorithm.

The entrance velocities of both phases are given. The entrance gas pressure and the bulk density of the particulate phase are given too. The entrance turbulent kinetic energy and the dissipation rate of the turbulent energy of both phases are assumed by

$$k_{in} = 0.01(U_g^2 + V_g^2)/2$$  \hspace{1cm} (32)

$$\varepsilon_{in} = C_k \kappa_{in}^3 / (0.03 L)$$  \hspace{1cm} (33)

$$(k_p)_{in} = 0.01 [(U_p^2)_{in} + (V_p^2)_{in}] / 2$$  \hspace{1cm} (34)

$$\varepsilon_{in} = C_k \kappa_{in}^3 / (0.03 L)$$  \hspace{1cm} (35)

where $L$ is the width of the flow channel.

On the exit boundary a fully developed flow condition is set by

$$\frac{\partial \Phi}{\partial x} \bigg|_{exit} = 0$$  \hspace{1cm} (36)
A no-slip condition is used for the gaseous phase on the wall surfaces, i.e., \( U_{wall} = 0 \), \( V_{wall} = 0 \). Besides, the wall function method is used to bridge the near-wall region. The particles are assumed to slip over the wall surface and the particle slip velocity on the wall surface is given below (Han et al., 1991).

\[
U_p^1|_{wall} = U^1|_{y = d_p/2}
\]  
(37)

\[
U_p^2|_{wall} = 0
\]  
(38)

where \( r \) and \( n \) are the tangential and normal directions of the wall surface, respectively.

As the wall is impermeable, we have

\[
\frac{\partial \rho_p}{\partial y}|_{wall} = 0
\]  
(39)

3. Numerical Results and Discussion

With the appearance of the laser-Doppler velocimeter, more and more measurements on gas-solid two-phase flows have been performed. However, reliable experiments with sufficient measurements of the needed variables are still limited. For validation of the present turbulence models, the turbulent gas-solid two-phase flows in a 90° bend and in a vertical pipe are predicted.

3.1 The gas-solid flow in a 90° bend

The geometry of the 90° bend in the experiment of Kliafas and Holt (1987) is shown in Fig. 1. The normal width of the bend \( b \) is 100 mm. The bend mean radius of curvature \( R \) is 170 mm. Two kinds of glass spheres 50 \( \mu \)m and 100 \( \mu \)m in diameter were used as the dispersed particles. The material density of the particles was 2990 kg/m³. The measurements were carried out for two different Reynolds numbers, 2.2\times10^5 (low-speed flow) and 3.47\times10^5 (high-speed flow), and the corresponding loading ratios of particles were 1.5\times10^{-4} and 9.5\times10^{-5}, respectively. In the low-speed flow, the measurements were made up to the \( \theta = 45^\circ \) section of the bend for 50 \( \mu \)m particles, whereas for 100 \( \mu \)m particles, they were made up to the \( \theta = 30^\circ \) section.

For the flow with 50 \( \mu \)m particles, the measured and computed profiles of the mean streamwise velocity at four stations are plotted in Figs. 2 - 5. Here \( y \) is the normal distance from the concave surface of the bend and \( U_0 \) the bulk gas velocity. \( U \) and \( U_p \) are the mean streamwise velocities of gas and particles, respectively. For the most part, 50 \( \mu \)m particles were found to lag behind the gas. The particle velocity profiles are fairly flat at all measuring stations, which differ from those of the gaseous phase. The gas velocity profiles cross with those of the particles near the concave surface. This shows that the gas was affected by the unfavorable pressure gradient, while
the particles were not affected by it.

The particle mean streamwise velocities predicted by the $k-e-k_p-e_p$ model agree best with the measurements at the sections $\theta=0^\circ$, $15^\circ$ and $30^\circ$, compared with other models. At the station $\theta=45^\circ$, however, the predicted mean streamwise velocity profiles of both phases differ from the measurements near the concave surface. This may be due to the effect of the strong curvature of the bend.

The predicted mean streamwise velocity profiles of the gaseous phase are the same among three different turbulence models. This is due to the fact that the gas-solid suspension flow is a dilute one, and the total effect of particles on the gaseous phase is small.

Figures 6 – 9 show the turbulence intensity profiles for the flow with 50 $\mu$m particles. In the figures, $U_{RMS}$ and $U_{pRMS}$ are the corresponding turbulence intensities of two phases in the streamwise direction. The turbulence intensities of the particles are found to be higher than those of the gas. The streamwise turbulence intensity profiles are the same among three models, as are the streamwise mean velocity profiles. At the stations $\theta=15^\circ$, $30^\circ$ and $45^\circ$, the turbulence intensities of the gas are smaller than the particle turbulence intensities except for the region very close to the concave surface. The turbulence intensity of the particulate phase predicted by the $k-e-A_p$ model is always smaller than that of the gaseous phase, which is not in accordance with the measurements. The turbulence intensity of the particulate phase is over-estimated by the $k-e-k_p$ model. The predicted turbulence intensity of the particulate phase by the $k-e-K_p-e_p$ model is closer to the experimental result than those by the $k-e-k_p$ and $k-e-A_p$ models. The $k-e-k_p-e_p$ model also shows that the turbulence intensity of the particulate phase can be larger than that of the gaseous phase. These comparisons suggest that both $k-e-k_p-e_p$ and $k-e-k_p$ models are superior to the $k-e-A_p$ model in the prediction of the particulate phase turbulence intensity, and the $k-e-k_p-e_p$ model is the best among them.
In addition, numerical calculations were performed for flows with 100 μm particles. The profiles of mean and fluctuating quantities for both phases were similar to those with 50 μm particles. The calculated results are not shown in this paper.

3.2 The gas-solid flow in a vertical pipe

In order to study gas-solid two-phase flow in a vertical pipe, several LDV measurements have been performed. Maeda et al. (1980) measured the profiles of mean velocity and turbulence intensity for both phases. Lee and Durst (1982) measured the mean velocity profiles with four kinds of particles with different diameters. Tsuji et al. (1984) investigated experimentally the influence of particles on the turbulence intensity of the gaseous phase.

In the vertical gas-solid suspension pipe flow of Tsuji et al. (1984), the employed particles were polystyrene, and the material density was 1020 kg/m³. The diameters of the employed particles ranged from 200 μm to 3 mm. To study the behavior of two-phase flows by using the present two-fluid models, only the smaller size particles (200 μm) were selected for the comparative analysis according to assumption (1) made in section 1. The pipe diameter D was 30 mm.

Figures 10 and 11 show the mean velocity profiles of the gaseous phase and particulate phase at the loading ratio of β=2.1 and 1.0, respectively. The bulk Reynolds number of the gas flow is 3.3×10⁴. In the figures, R is the radius of the pipe, r the distance from the central line of the pipe and U; the gas velocity on the centerline.

It can be seen that the particle velocity in the core region of the pipe is about 90% of the gas velocity. The particle velocity is smaller than the gas velocity over the cross-section at r/R ≥ 0.8, and the particle velocity can be larger than the gas velocity in the near-wall region. Over the cross-section of the pipe, the particle velocity profiles are flatter than the gas velocity profiles. This is due to the fact that the particles slip on the wall surface, whereas the gas satisfies the no-slip condition at the wall.

The figures also show that for three turbulent models, the predicted results are not so different. As mentioned above, however, for the two-phase flow in a 90° bend, the predicted particle velocity profiles were quite different among three models. In vertical pipe flow, the particles follow the streamlines of the gaseous phase when moving upward. However, for the flows in a 90° bend, the streamlines of particles are quite different from those of the gaseous phase due to the effect of the centrifugal force. The present models properly account for the effect of the centrifugal force, while for vertical pipe flow the advantage was not fully reflected.

Figures 12 and 13 show the effect of particle loading ratios on the mean velocity and turbulence intensity of gas, respectively. The bulk Reynolds number of the gas flow is 2.3×10⁴. The predictions are obtained by the \(k−ε\)-\(k_p−ε_p\) turbulence model. Figure 12 shows that when more particles are present, the gas velocity profiles become steeper near the wall.
region of the pipe. With the increase of the loading ratio, such effects may be enhanced due to the velocity slip between the two phases. The predictions are in close accordance with the measurements. From Fig. 13 it is found that the turbulence intensity of the gas is suppressed in the gas-solid flow in a pipe, and with the increase of the loading ratio, the extent of the suppression is enhanced.

Recent numerical and experimental studies have concentrated on the mechanism of particle-turbulence interaction. Generally, the presence of particles will modify the turbulence of the carrier fluid: coarse particles enhance the turbulence while fine particles suppress it. By analyzing various types of particle-laden flows, Gore and Crowe (1989) attempted to generalize the experimental results on particle-turbulence interaction, and showed that whether the given particles suppress or enhance turbulence intensity depends on the ratio of particle diameter/turbulence length scale \(d_p/l\). They found a demarcation of 0.1, below which the particles decrease turbulence intensity and above which they increase it. In the core region of the vertical pipe flow of Tsuji et al.'s experiment, the ratio \(d_p/l\) is 0.065, which is less than the value 0.1 recommended by Gore and Crowe (1989). Therefore, Gore and Crowe’s criterion is suitable for the flow with 200 μm particles shown in Fig. 13. Although the predicted value is slightly larger than the measurement over the cross-section of the pipe, the predicted trend of the gas turbulence intensity is consistent with the measurements.

**Conclusion**

Two-fluid turbulence models, the \(k-e-k_p-e_p\) model and the \(k-e-k_p\) model, were presented to analyze gas-solid suspension flows. The proposed models include the turbulent kinetic energy equations and turbulent energy dissipation rate equations of two phases, and they can fully account for the two-way coupling effects between two phases. The models were applied to two gas-solid suspension flows in a 90° bend and in a vertical pipe.

For turbulent gas-solid suspension flow in a 90° bend, the predicted mean velocity profiles and turbulence intensity profiles of two phases by the \(k-e-k_p-e_p\) turbulence model agree well with the experimental data. The turbulence intensity of the particulate phase predicted by a conventional \(k-e-A_p\) model is always smaller than that of the gaseous phase, which is not consistent with the measurements. The \(k-e-k_p-e_p\) model and/or the \(k-e-k_p\) model can predict that the particle turbulence intensity is often larger than that of the gaseous phase, which is in good agreement with the measurements. Therefore, the present \(k-e-k_p-e_p\) and \(k-e-k_p\) models are better than the conventional \(k-e-A_p\) model. From both comparisons of the turbulence intensities and mean velocities with the measurements, the \(k-e-k_p-e_p\) model is concluded to be the best among the three models.

For two-phase flow in a vertical pipe, the \(k-e-k_p-e_p\) turbulence model predicts that in the presence of 200 μm particles, higher particle loading suppresses the gas turbulence intensity more and makes the gas mean velocity profile flatter across the pipe. These predictions are in agreement with the measurements. However, the proposed two-fluid turbulence models are not suitable for the flows with very large size particles with \(d_p/l>0.1\). A study on the effect of very large particles on turbulence is a future subject. Besides, the coefficients appearing in the governing equations should be optimized in further application of the proposed models to other kinds of two-phase suspension flows.

**Nomenclature**

- \(b\) = width of bend [m]
- \(C_{es}, C_1, C_2\) = constants in \(k-e\) model equations [-]
- \(C_{es}, C_{cs}, C_{ps}, C_{pe}\) = constants in \(k_p-e_p\) model equations [-]
- \(d_s, d_e\) = damping factors [-]
- \(d_p\) = particle diameter [m]
- \(D\) = pipe diameter [m]
- \(g\) = gravity components [m.s\(^{-2}\)]
- \(k, k_p\) = turbulent kinetic energies [m\(^2\).s\(^{-2}\)]
- \(l, l_p\) = turbulent length scales [m]
- \(L\) = system characteristic length [m]
- \(r\) = distance from central line of pipe [m]
- \(R\) = radius [m]
- \(Re_p\) = particle Reynolds number = \(UL/\nu\) [-]
- \(Re_s, Re_p\) = particle Reynolds number in Eq. (9) [-]
- \(U_s, U_p\) = velocity fluctuating components [m.s\(^{-1}\)]
- \(x, y\) = Cartesian coordinates [m]
- \(\alpha_s\) = particle volumetric fraction [-]
- \(\beta\) = particle loading ratio [-]
- \(e, e_p\) = turbulent energy dissipation rates [m\(^2\).s\(^{-3}\)]
- \(\nu, \nu_p\) = kinematic viscosities [m\(^2\).s\(^{-1}\)]
- \(\nu_p, \nu_e\) = eddy viscosities [m\(^2\).s\(^{-1}\)]
- \(\rho\) = gas density [kg.m\(^{-3}\)]
- \(\rho_s\) = particle material density [kg.m\(^{-3}\)]
- \(\rho_p\) = bulk density of particle phase [kg.m\(^{-3}\)]
- \(\sigma_e, \sigma_s\) = effective Prandtl number [-]
- \(\sigma_p\) = turbulent Schmidt number [-]
- \(\tau_e\) = Lagrangian integral time scale in Eq. (13) [s]
- \(\tau_p\) = particle relaxation time scale in Eq. (9) [s]

**Subscript**

- \(i\) = partial differential with \(x_i\)
- \(c\) = pipe center
- \(in\) = inlet
- \(i\) = laminar
- \(p\) = particle
- \(t\) = turbulent

**Superscript**

- \(f\) = fluctuating quantity
- \(\cdot\) = time-averaged quantity
- \(RMS\) = root mean square of fluctuating quantity
Literature Cited