PREDICTION OF CLUSTER SIZE IN CIRCULATING FLUIDIZED BEDS

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A simple model is presented to predict cluster size and cluster voidage for a given set of suspension voidage and operating conditions of a circulating fluidized bed. In this model a cluster is supposed to be suspended by both gas flows inside and outside the cluster; and the gas around the cluster is supposed to form bubble like gas pockets that move upward by buoyancy force. Model predictions are successfully compared with experimental data in the literature.

Introduction

The suspension of particles in a circulating fluidized bed is known to be inhomogeneous, even in its upper dilute region. The cause of the inhomogeneity is the essential instability of homogeneous suspensions as demonstrated by Grace and Tuot (1979). The measurement of pierced cluster length by Horio *et al.* (1988) clearly shows that the inhomogeneity is rather uniform except for the annulus region over the wall. Furthermore, theoretical analysis by Ishii *et al.* (1989) shows that the core annulus structure cannot be obtained if suspensions in circulating fluidized beds are homogeneous.

Figure 1 is an example of the solid mass flux-gas velocity-suspension density relationship for a circulating fluidized bed riser (data source: Hirama *et al.* (1992)). The lower dense region data were adopted for the suspension densities in the diagram. Here it is worth noting that the data were taken with a test rig where solid mass flux and gas velocity were accurately controlled. Such a diagram for a granular system corresponds to the P-V-T diagram for molecular systems with the rough understanding that the suspension density qualitatively corresponds to the molar density V^{-1} , solid mass flux to the pressure, and gas velocity to the temperature. This understanding is reasonable and logical considering a number of facts. For instance in bubbling fluidized beds, the minimum fluidization velocity is the melting-point like index which discriminates solid-like states (fixed bed) and liquid-states (fluidized bed). Therefore, the pressure of the molecular system should correspond to the "granular pressure" in particle systems which should increase with increasing solid circulation.

Fig. 1 $G_s-u_0-\epsilon_{pd}$ phase diagram drawn from Hirama *et al.* (1992) data

From Fig. 1 it is clear that the suspension density in bubbling to turbulent fluidized beds is almost independent of solid mass flux, as if the liquid density is independent of the pressure in molecular systems. However, in fast fluidized beds the suspension density becomes quite dependent on solid mass flux, as if the density of supercritical fluids is dependent on the pressure. It is therefore quite rational to assume that the cluster formation is the essential nature of the suspensions in fast fluidized beds.

Wirth (1991) developed a model introducing the concept of rod like strands to explain the supercritical behavior of turbulent and fast fluidized suspensions, but he has not gone further into the detailed structure of suspensions. To obtain more quantitative information of the size and the shape of clusters, we have measured the meso-scale structure of fast fluidized suspensions by optical fiber techniques (Horio *et al.* (1988), Ishii *et al.* (1989, 1991), Horio *et al.* (1992a, b)) and laser sheet methods (Horio and Kuroki (1994), Kuroki and Horio (1994), Ito *et al.* (1995)). Recently, Tanaka *et al.* (1993) successfully demonstrated that Horio-Kuroki (1994) data can be reproduced, even by

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two dimensional simulation, with their discrete element code.

For understanding of the fundamental nature of the suspension and for practical calculations, it is still important to develop a theoretical but simple model to determine the particle density of the cluster phase and the mean cluster size. In the present paper such a model is proposed and examined by experimental data, where the behaviors of not only clusters but also bubbles or gas pockets between clusters are taken into account for the closure of the problem. This bubble concept is based on our laser sheet observation (Horio and Kuroki (1994)) of gas pocket behavior in dilute suspension flow.

1. Theoretical Development

1.1 Assumptions

The model proposed here is based on the following three major assumptions:

1. Clusters, flowing upward with velocity v , are suspended by both gases flowing through and flowing around them (cf. Fig. 2(a)). Thus we write

$$-g(\rho_p - \rho_f)(1 - \varepsilon') = (1 - \varepsilon^*) \left(\frac{dp}{dz} \right)_{cl} + \varepsilon^* \left(\frac{dp}{dz} \right)^* \quad (1)$$

where ε^* is the volume fraction of cluster-free space. $(dp/dz)_{cl}$ and $(dp/dz)^*$ are the vertical pressure gradients in the cluster phase and in the lean phase, predicted by following Eqs. (2) and (3), respectively (see Appendix for derivation):

$$\left(\frac{dp}{dz} \right)_{cl} = - \frac{18\mu}{d_p^2} \frac{1 - \varepsilon_{cl}}{\varepsilon_{cl}^{2(n-1)}} (u_{cl} - v) \quad (2)$$

$$\left(\frac{dp}{dz} \right)^* = - \frac{\rho_f}{3d_{cl}^o} \frac{1 - \varepsilon^*}{\varepsilon^{*2(n-1)}} (u^* - v)^2 \quad (3)$$

2. The gas in the lean phase (i.e., outside of the cluster phase) tends to form bubbles which move up driven by buoyancy force (cf. Fig. 2(b)). Thus we write for bubble rise velocity

$$u_b = \gamma \sqrt{gD_b} + \delta_1 u_b \varepsilon_b \quad (4)$$

Taking into account the gas throughflow, the lean phase gas velocity u^* is related to u_b by

$$u^* = u_b + \delta_2 u_{cl} \varepsilon_{cl} \quad (5)$$

3. Based on geometrical considerations, the cluster diameter and the bubble diameter are related by

$$\frac{d_{cl}^o}{D_b} = \left(\frac{2}{3} \frac{l_{cl}}{d_{cl}} \right)^{1/3} \left(\frac{2}{\sqrt{3}} \frac{D^*}{D_b} - 1 \right) \quad (6)$$

Fig. 2 Cluster-bubble model for cluster size prediction

where d_{cl}^o is the diameter of the sphere having a volume equivalent to a cluster, l_{cl}/d_{cl} is the aspect ratio of a cluster and D^* is the center-to-center distance between neighboring bubbles.

To obtain D^* let us apply the closest packing model introducing a parameter $\varepsilon_{\max}^*$, the maximum packing density. Then we write

$$D^*/D_b = (\varepsilon_{\max}^*/\varepsilon^*)^{1/3} \quad (7)$$

Substituting Eqs. (4), (5) and (7) into Eq. (6) we obtain

$$d_{cl}^o = \left(\frac{3}{2} \frac{l_{cl}}{d_{cl}} \right)^{1/3} \left(\frac{2}{\sqrt{3}} \left(\frac{\varepsilon_{\max}^*}{\varepsilon^*} \right)^{1/3} - 1 \right) \frac{1}{\gamma^2 g} [(1 - \delta_1 \varepsilon^*)(u^* - \delta_2 u_{cl} \varepsilon_{cl})]^2 \quad (6')$$

Formulation of the problem

First of all, the governing equations can be derived as follows:

1) voidage relationship

$$1 - \varepsilon' = (1 - \varepsilon^*)(1 - \varepsilon_{cl}) \quad (8)$$

2) mass balance for gas

$$u_0' = u^* \varepsilon^* + (1 - \varepsilon^*) \varepsilon_{cl} u_{cl} \quad (9)$$

where u_0' is the local superficial gas velocity.

3) Mass balance for solid

$$G_s' = (1 - \varepsilon') \rho_p v' \quad (10)$$

where G_s' is the local mass flux.

4) Force balance for a cluster: Eq. (1)

5) Pressure balance in gas phase

$$dp/dz = (dp/dz)_{cl} = (dp/dz)^* \quad (11)$$

Fig. 3 Iteration loop to determine d_{cl}^o and ε_{cl}

Solution of the above equations for d_{cl}^o

Substituting Eqs. (11) and (3) into Eq. (1) and rearranging it for d_{cl}^o , we can write

$$d_{cl}^o = \frac{1}{3(1 - \varepsilon_{cl})\varepsilon^{*2(n-1)}} \frac{\rho_f}{g(\rho_p - \rho_f)} (u^* - v)^2 \quad (12)$$

Substituting Eqs. (11) and (2) into Eq. (1) and rearranging it for the gas velocity in the cluster phase u_{cl} , we can write

$$u_{cl} = [(1 - \varepsilon)\varepsilon_{cl}^{2(n-1)}/(1 - \varepsilon_{cl})]u_t + v^* \quad (13)$$

For the lean phase gas velocity u^* , we can use the following Eq. (9)':

$$u^* = [u_0' - (1 - \varepsilon^*)\varepsilon_{cl}u_{cl}]/\varepsilon^* \quad (9)'$$

For the cluster-free space fraction ε^* we can write

$$\varepsilon^* = 1 - (1 - \varepsilon^*)/(1 - \varepsilon_{cl}) \quad (8)'$$

The iteration loop to solve the above system for d_{cl} and ε_{cl} for a given set of u_0' , v' , ε' data is, therefore, designed as shown in Fig. 3.

2. Model Validation

2.1 Dense suspensions

A comprehensive set of hydrodynamic data is

a) radial distributions of $1 - \varepsilon'$ b) radial distributions of v'

Fig. 4 Comparison of experimental and predicted radial distributions of $1 - \varepsilon'$ and v' (Data in parentheses are thought to be inaccurate ones typical to cross correlation method)

available for dense suspensions in a small scale circulating fluidized bed (Horio *et al.* (1988), Horio *et al.* (1992a, b)). Table 1 summarizes the data for the present model validation. The voidage and velocity data at the center of the column were estimated from cross sectional average data based on the following expression, which is a modified version of the correlations by Patience and Chaouki (1995) where $\beta=4$ and $m=0.4$ have been used:

$$(\varepsilon^m - \varepsilon^*)/(\varepsilon^m - \varepsilon) = \beta(r/R_t)^6 \quad (\beta = 3, m = 0.7) \quad (14)$$

$$v^*/v^*(r=0) = 1 - (r/\alpha R_t)^2 \quad (r = 0 \sim \alpha_1 R_t) \quad (15-1)$$

$$= 1 - (\alpha_1/\alpha)^2 \quad (r = \alpha_1 R_t \sim R_t) \quad (15-2)$$

For ε' and v_0 at the center we obtain

$$\varepsilon^*(r=0) = \varepsilon^m \quad (16)$$

$$v^*(r=0) = \frac{(G_s/\rho_p)}{\left[\left\{ (1 - \varepsilon^m) + \frac{\beta}{4} (\varepsilon^m - \varepsilon) \right\} \left\{ 1 - \left(\frac{\alpha_1}{\alpha} \right)^2 \right\} \right] + (1 - \varepsilon^m) \left(\frac{\alpha_1^4}{2\alpha^2} \right) + (\varepsilon^m - \varepsilon) \left(\beta \frac{\alpha_1^{10}}{20\alpha^2} \right)} \quad (17)$$

As shown in Fig. 4, the radial distributions of both voidage (Eq. (16)) and solid velocity (Eq. (17)) agreed fairly well with the previous measurement of Horio *et al.* (1988) when $\alpha_1 = (4\alpha + 1)/5$. The gas was assumed to flow only in the core. When there was no data for α , the following Werther (1993) correlation was applied:

$$1 - \alpha = 1.1(u_0 D_t \rho_f / \mu)^{-0.22} (L/D_t)^{0.21} (1 - z/L)^{0.73} \quad (18)$$

Values thus estimated are also shown in Table 1. Furthermore, the following values are assumed for parameters: $\varepsilon_{max}^* = 0.7405$, $\delta_1 = 1$, $\delta_2 = 1$, $n = 5$, $l_{cl}/d_{cl} = 2$.

Table 1 Available experimental data and estimated value for model validation

Authors	u_0 m/s	$u'_{0,r=0}$ at the centerline, m/s	G_s kg/m ² s	$v'_{r=0}$ at the centerline, m/s	z m	L m	D_i m	α —
Horio <i>et al.</i> (1992)	0.7	1.47 ^{b)}	5.7	0.47 ^{e)}	0.98	2.39	0.05	0.69 ^{f)}
	0.8	1.63 ^{b)}	12.2	1.13 ^{e)}	0.98	2.39	0.05	0.70 ^{f)}
	1.1	2.12 ^{b)}	20.1	1.90 ^{e)}	0.98	2.39	0.05	0.72 ^{f)}
	1.3	2.44 ^{b)}	25.6	2.19 ^{e)}	0.98	2.39	0.05	0.73 ^{f)}
Horio <i>et al.</i> (1988)	1.17	2.53 ^{a)} , 3.04 ^{b)}	11.7	1.70 ^{e)} , 1.32 ^{d)} , -3.90 ^{e)}	0.36	2.79	0.05	0.68 ^{c)} , 0.62 ^{f)}
	1.17	2.13 ^{a)} , 2.39 ^{b)}	11.7	2.06 ^{e)} , 0.76 ^{d)} , 1.18	1.06	2.79	0.05	0.74 ^{c)} , 0.70 ^{f)}
	1.17	2.53 ^{a)} , 1.29 ^{b)}	11.7	2.30 ^{e)} , 3.24 ^{d)} , 1.07 ^{e)}	1.63	2.79	0.05	0.68 ^{c)} , 0.78 ^{f)}
Ito <i>et al.</i> (1995)	0.58	0.77 ^{b)}	0.045	0.14 ^{e)}	0.78	1.6	0.20	0.87 ^{f)}
	1.10	1.42 ^{b)}	0.087	0.30 ^{e)}	0.78	1.6	0.20	0.88 ^{f)}

Authors	ε cross section av., —	$\varepsilon_{r=0}$ at the centerline, —	$d_{cl,calcd.}$ m	$\varepsilon_{cl,calcd.}$ —	$d_{cl,obs.}$ m	$\varepsilon_{cl,obs.}$ —
Horio <i>et al.</i> (1992)	0.83 ^{g)}	0.88 ^{h)}	0.042	0.69	0.0143	0.83
	0.87 ^{g)}	0.91 ^{h)}	0.019	0.77	0.012	0.85
	0.90 ^{g)}	0.93 ^{h)}	0.011	0.83	0.0064	0.91
	0.90 ^{g)}	0.93 ^{h)}	0.014	0.83	0.0044	0.88
Horio <i>et al.</i> (1988)	0.80 ^{g)}	0.92 ^{c)} , 0.89 ^{h)}	0.037	0.79	0.0040	0.86
	0.88 ^{g)}	0.92 ^{c)} , 0.91 ^{h)}	0.011	0.83	0.0043	0.90
	0.936 ^{g)}	0.95 ^{c)} , 0.955 ^{h)}	0.007	0.86	0.0043	0.92
Ito <i>et al.</i> (1995)	0.9993 ^{g)}	0.9995 ^{h)}	0.0128	0.9982	0.020–0.040	—
	0.9994 ^{g)}	0.9996 ^{h)}	0.0552	0.9986	0.010–0.030	—

a) Estimated by $u'_0 = u_0/\alpha^2$ with exp. values of dimensionless core diameter, b) Estimated by $u'_0 = u_0/\alpha^2$ with calcd. values of dimensionless core diameter, c) Observed by optical fiber probe, d) Estimated by Eq. (17) with exp. values of dimensionless core diameter, e) Estimated by Eq. (17) with calcd. values of dimensionless core diameter, f) Estimated by Eq. (18), g) Determined from pressure gradient, h) Estimated by Eq. (16)

Fig. 5 An example of curves for Eq. (6)' and Eq. (12)

Figure 5 shows some examples of the d_{cl} vs. ε_{cl} curves obtained from Eq. (6)' with γ as a parameter and Eq. (12). From the intersection of the two curves we can determine the solution. Results are summarized in **Fig. 6**. In **Figure 6** the center values estimated from the above method were used when there were only cross section average data. When $\delta_1=0$ was adopted, the value of γ was greater than

Fig. 6 d_{cl}^0 and ε_{cl} as a function of γ

Fig. 7 Comparison of observed and predicted data for d_{cl}^o and ϵ_{cl}

unity, which seems unrealistic. This implies that bubbles should be treated as those in a swarm (*i.e.*, $\delta_1=1$). Predicted d_{cl} and ϵ_{cl} are compared with observed ones in Fig. 7. As can be seen in Fig. 7, the calculated results for $\gamma=0.9$ and $\delta_1=1$ are in reasonable agreement with the observed cluster voidage and diameter.

2.2 Lean suspensions

Ito *et al.* (1995) attempted three dimensional visualization of suspension structures based on the scanning laser sheet technique. In their experiment the cold test rig was the same as that of Ishii *et al.* (1989), Horii and Kuroki (1994), Ito *et al.* (1995), and Tsukada *et al.* (1997) with a riser of the diameter 0.2 m and height 1.6 m. Figure 8 shows the improved version of the images they presented. As can be seen in the figure, clusters of roughly several centimeters diameter still exist in very dilute suspensions. Since the exact values of suspension voidage have not yet been obtained due to the difficulty of calibration, $\epsilon=0.9993-0.9994$ was assumed. Results for the dilute suspension are shown in Fig. 9, where rather reasonable cluster sizes are obtained for $\gamma=0.3$ and $n=1$

Conclusion

Based on the assumption that a suspension in a circulating fluidized bed has a matrix structure composed of clusters and bubbles a mathematical model to predict the cluster diameter and the voidage in the cluster phase was developed. Predicted results from the model showed reasonable agreement with observed data previously reported. This model satisfies the scaling raw of circulating fluidized beds and is applicable to the calculation of large scale units.

a) $u_0=0.58$ m/s%0.53 u_{tr} , $G_s=0.045$ kg/m²s, turbulent regime
b) $u_0=1.10$ m/s%1.0 u_{tr} , $G_s=0.087$ kg/m²s, fast regime

Fig. 8 Three dimensional cluster images obtained by scanning laser sheet technique for dilute suspensions (clusters: white parts; column: $D_r=0.2$ m, particles: FCC, $d_p=61.3$ μ m, $\rho_p=1780$ kg/m³; scan velocity: 1.36 m/s; 1000 frames/sec)

Fig. 9 d_{cl}^o and ϵ_{cl} predicted for dilute suspension

Appendix A: Derivation of $(dp/dz)^*$

Assume that clusters are macroscopically suspended by gas flowing around them. For each cluster in the suspension having drag coefficient $C_{D,cl}$, the drag force N_{D^0} acting on it can be written as:

$$N_{D^0} = \frac{1}{2} \rho_f (u^* - v)^2 \frac{\pi}{4} d_{cl}^2 C_{D,cl} \quad (A-1)$$

Then, the drag force acting on the solid in a unit volume can be related to $(dp/dz)^*$, pressure gradient in lean phase, by:

$$\left(\frac{dp}{dz} \right)^* = \frac{1}{2} \rho_f (u^* - v)^2 \frac{\pi}{4} d_{cl}^2 C_{D,cl} \left(\frac{1 - \varepsilon^*}{\frac{\pi}{6} d_{cl}^3} \right) \quad (A-2)$$

$$= (1 - \varepsilon^*)(\rho_p - \rho_f)g \quad (A-2)'$$

If we extend the Richardson-Zaki (1952) correlation to the clusters suspended in the stream as done earlier by Yerushalmi *et al.* (1978), the slip condition of clustering suspensions can be expressed as:

$$u^* - v = u_{t,cl} \varepsilon^{*(n-1)} \quad (A-3)$$

where u^* is gas velocity in cluster-free space, v is velocity of clusters, ε^* is volume fraction of cluster-free space and the terminal velocity of a cluster, $u_{t,cl}$, is defined by Horio *et al.* (1992a) as follows:

$$\frac{1}{2} \rho_f u_{t,cl}^2 \frac{\pi}{4} d_{cl}^2 C_{D,cl} = \frac{\pi}{6} d_{cl}^3 (\rho_p - \rho_f)g \quad (A-4)$$

where $C_{D,cl}(=4/9)$ is the drag coefficient of a cluster defined based on the cross sectional area of a volume-equivalent spherical cluster. From the comparison of Eqs. (A-2) and (A-2)' with Eqs. (A-3) and (A-4), the drag coefficient of a cluster for homogeneously suspended clusters, $C_{D,cl}$, can be written as:

$$C_{D,cl} = \frac{C_{D,cl}^*}{\varepsilon^{*2(n-1)}} \quad (A-5)$$

Substituting Eq. (A-5) into Eq. (A-2) to estimate Eq. (3), we obtain:

$$\left(\frac{dp}{dz} \right)^* = - \frac{\rho_f}{3d_{cl}^2} \frac{1 - \varepsilon^*}{\varepsilon^{*2(n-1)}} (u^* - v)^2 \quad (A-6)$$

Appendix B: derivation of $(dp/dz)_{cl}$

Particles in the cluster phase are microscopically suspended by the interstitial flow. The Richardson-Zaki (1952) expression for this situation can be written as:

$$u_{cl} - v = u_{t,cl} \varepsilon_{cl}^{(n-1)} \quad (B-1)$$

Then, the drag coefficient for a particle, $C_{D,p}$, in the cluster, which is supposed to be in the viscous dominant range, can be written as follows:

$$C_{D,p} = \frac{C_{D,p}^*}{\varepsilon_{cl}^{2(n-1)}} = \frac{24}{Re} \left(\frac{1}{\varepsilon_{cl}^{2(n-1)}} \right) = \frac{24\mu}{d_p(u_{cl} - v)\rho_f} \left(\frac{1}{\varepsilon_{cl}^{2(n-1)}} \right) \quad (B-2)$$

The pressure gradient $(dp/dz)_{cl}$ can be related to the drag force of particles in a unit volume of the cluster phase as:

$$\left(\frac{dp}{dz} \right)_{cl} = \frac{1}{2} \rho_f (u_{cl} - v)^2 \frac{\pi}{4} d_p^2 C_{D,p} \left(\frac{1 - \varepsilon_{cl}}{\frac{\pi}{6} d_p^3} \right) \quad (B-3)$$

Substituting Eq. (B-2) into Eq. (B-3) to estimate Eq. (2), we obtain:

$$\left(\frac{dp}{dz} \right)_{cl} = - \frac{18\mu}{d_p^2} \frac{1 - \varepsilon_{cl}}{\varepsilon_{cl}^{2(n-1)}} (u_{cl} - v) \quad (B-4)$$

Nomenclature

$C_{D,cl}$	= drag coefficient of cluster	[—]
$C_{D,p}$	= drag coefficient of particle	[—]
D^*	= center-to-center distance between neighboring bubbles	[m]
D_b	= bubble diameter	[m]
D_t	= column diameter	[m]
d_{cl}	= diameter of cluster cross section	[m]
d_{cl}^0	= equivalent spherical cluster diameter	[m]
G_s	= $v(1 - \varepsilon)\rho_p$, solid mass flux	[kg/(m ² s)]
g	= gravity acceleration	[m/s ²]
l_{cl}	= cluster length	[m]
n	= parameter in Richardson-Zaki correlation	[—]
P	= pressure	[Pa]
Re	= particle Reynolds number, $\rho_f u d_p / \mu$	[—]
R_t	= column radius	[m]
u_0	= superficial gas velocity	[m/s]
u^*	= gas velocity in cluster-free space	[m/s]
u_c	= onset velocity of bubbling-to-turbulent transition	[m/s]
u_{cl}	= gas velocity in cluster phase	[m/s]
u_k	= higher transition velocity for bubbling-to-turbulent transition	[m/s]
u_t	= particle terminal velocity	[m/s]
$u_{t,cl}$	= terminal velocity of cluster	[m/s]
u_{tr}	= transport velocity	[m/s]
v	= velocity of clusters	[m/s]
z	= height above distributor	[m]
α	= dimensionless core diameter	[—]
γ	= bubble velocity coefficient	[—]
δ_1	= parameter in Eq. (4)	[—]
δ_2	= parameter in Eq. (5)	[—]
ε	= voidage of suspension	[—]
ε_{cl}	= voidage in cluster phase	[—]
ε_{pd}	= particle volume fraction in lower dense region	[—]
ε^*	= volume fraction of cluster-free space	[—]
ε_{max}^*	= maximum possible value of ε^*	[—]
μ	= gas viscosity	[Pa s]
ρ_f	= fluid density	[kg/m ³]
ρ_p	= particle density	[kg/m ³]

<Superscript>

*	= cluster-free space, or bubble phase
,	= local value, single isolated
C	= core region
cl	= cluster phase

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