SIZE EFFECT ON SPECTRUM OF TURBULENT ENERGY IN A STIRRED VESSEL

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Introduction

A major problem in industrial processes is successful scale-up of equipment under turbulent conditions. One of basic assumptions is that scale-up holding geometric similarity is completely successful if the turbulent flow mechanism is maintained. When the energy spectrum function (*ESF*) is considered to be a

representative expression of the turbulent flow mechanism, *ESF* becomes the main criterion for scale-up. Although it is already clear that for a circular pipe, *ESF* is changed when the pipe become at least three times the size of the original one (Ogawa, 1986), there is no practical confirmation in a stirred vessel. The purposes of this paper are to experimentally elucidate the relationship between *ESF* and size of stirred vessel and to clarify the effect of kinematic energy on *ESF* for Newtonian liquids.

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Fig. 1 Experimental stirred vessel

1. ESF Form

We present the following simple formula of one-dimensional *ESF* for wide wavenumber ranges based on an information entropy conception (Ogawa *et al.*, 1985).

$$\frac{EK_b}{u^2} = \frac{1}{\sum_{j=1}^m 2^{j-1}} \sum_{j=1}^m \{6^{j-1} \exp\left(-3^{j-1} \frac{k}{K_b}\right)\} \quad (1)$$

This expression was derived under two assumptions; the turbulent flow field consists of m eddy-groups (a basic eddy-group and its sequential subharmonic $m-1$ eddy-groups) and each eddy-groups adopts the *ESF* curve which associates the maximum amount of information entropy under the condition of existence of the average wavenumber. It is confirmed that the formula appears to be a satisfactory approximation of the measured distribution of *ESF* in a flow; pipe flow, jet flow, downstream of a grid (Ogawa *et al.*, 1985).

2. Experiments

2.1 Experimental apparatus and method

The cylindrical stirred vessels used were three sizes (60, 180, 540 mm inner diameter) with geometric similarity. They are shown in Fig.1. A standard disk turbine with six flat blades was used as the impeller, with four baffles made of methacrylate plastic. The test liquids used were ion exchanged water and 10wt% glycerin aqueous solution. Each of test liquid contained 3×10^{-3} mol/l of $K_4Fe(CN)_6$ and $K_3Fe(CN)_6$, and 5×10^{-1} mol/l of KCl for using the electrode reaction velocimeter. The platinum sphere electrode (Ogawa *et al.*, 1973) for measuring the radial liquid velocity fluctuations was set at a constant point; $D_T/4$ from the impeller axis and $D_T/3$ from the bottom in the central vertical section between baffles. The point is located at

the impeller discharge flow region which represents the turbulent flow field in the stirred vessel. The counter electrode, made of thin platinum sheet, was attached to the bottom of the vessel. The kinematic viscosity of each test liquid was measured by using a new type of capillary viscometer (Ogawa *et al.*, 1991) at every experimental step. The experiments were carried out under conditions of constant Reynolds number ($Re = ND_T^2/\nu = 10^4$) and constant impeller tip velocity ($ND_T = 0.715$ m/s).

The measured velocity fluctuations were treated by a computer after passing through an analog-digital converter under the sampling frequency of 100Hz.

2.2 Experimental results

2.2.1 Relationship between *ESF* distributions and diameter of stirred vessel

As shown in Figs. 2 and 3, each measured *ESF* distributions can be expressed satisfactorily by a calculated *ESF* curve based on Eq.(1). The number of eddy-groups increases one by one in order of diameter of stirred vessel regardless of liquids used. That is to say, in the stirred vessel of $D_T = 60$ mm, the turbulent flow field consists of only the basic eddy-group (the number of eddy groups is $m=1$). In the stirred vessel of $D_T = 180$ mm, a new subharmonic eddy group appears (the number of eddy groups is $m=2$). In the stirred vessel of $D_T = 540$ mm, the second subharmonic eddy-group appears (the number of eddy groups is $m=3$).

This results prove that a new subharmonic eddy-group appears when the size of the stirred vessel is at least triples, and that scale-up of a stirred vessel holding geometric similarity is accomplished completely when the scale-up ratio is less than three. Of course, as predicted by Ogawa and Kuroda (1995), there is a possibility to accomplish scale-up successfully even if the scale-up ratio is more than three. When the power per unit volume is maintained, all the *ESF* curves overlap together in the higher wavenumber ranges. This fact indicates that scale-up is accomplished sufficiently by maintaining power per unit volume, if the higher wavenumber ranges play a significant role in the phenomena.

2.2.2 Relationship between kinematic viscosity and average wavenumber of basic eddy-group

Each fluid takes a constant value of the average wavenumber of basic eddy-group. The average wavenumber of the basic eddy-group can be calculated from Eq.(1) and the measured *ESF* curve. As shown in Fig.4, the average wavenumber of the basic eddy-group decreases according to the increase of kinematic viscosity. The relationship can be written by the following relational equation.

$$K_b = 0.266\nu^{-0.5} \quad (2)$$

Fig. 2 Relationship between *ESF* and size of stirred vessel for water ($Re = 10,000$)

Fig. 3 Relationship between *ESF* and size of stirred vessel for glycerin aq. sol ($Re = 10,000$)

Fig. 4 Relationship between average wavenumber of basic eddy-group and kinematic viscosity

The fact that the average wavenumber is proportional to the -0.5 power of kinematic viscosity is of much interest, because the same results can be derived from isotropic turbulence theory (Hinze, 1959); the square of micro scale is in proportion to the kinematic viscosity when the dissipation energy per unit mass per unit turbulent energy is constant.

Conclusion

It is elucidated that the *ESF* changes from one curve to another when the diameter of the stirred vessel is increased by at least a factor of three by the appearance of new subharmonic eddy-groups. Therefore, the scale-up of a stirred vessel holding geometric similarity is accomplished completely when the scale-up ratio is less than three.

The average wavenumber of basic eddy-group is in proportion to the -0.5 power of kinematic viscosity.

Nomenclature

E	= energy spectrum function	$[m^3/s^2]$
K_b	= average wavenumber of basic eddy-group	$[1/m]$
k	= wavenumber	$[1/m]$
m	= number of eddy-groups	$[-]$
N	= impeller revolutionary speed	$[1/s]$
Re	= Reynolds number	$[-]$
u^2	= mean-square turbulent velocity	$[m^2/s^2]$
ν	= kinematic viscosity	$[m^2/s]$

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