TURBULENT FLOW PATTERN OF HYPERBOLOID STIRRING REACTOR

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3D periodical turbulent flow induced by a low shear hyperboloid stirrer in a fully baffled stirred tank reactor is experimentally studied at a constant Reynolds number, 5600, by using a diode fiber laser-Doppler anemometer. The ratio $D:T:H$ of the hyperboloid stirrer diameter, $D = 50$ mm, the inner tank diameter, T , and the filled fluid height, H , is 1:3:3. Three low stands of hyperboloid stirrer clearance, $D/10$, $D/7$ and $D/5$, are also targeted to evaluate influence on flow structure. The experimental results are applied to verification of CFD-code based on a standard $k-\epsilon$ model.

The flow field close to the hyperboloid stirrer is characterized by a series of vortex jets that are trailing periodically down within a radial extension of $0.5D$ to $0.7D$. The maximum measured tangential velocity was 96 % of the tip stirrer velocity. In this "near-of-flow area", the kinetic energy is extensively dissipated itself due to impingement of velocity components. The axially-symmetric separation zone in the spatial form of λ -character induces two large-scale vortices above and below the separation. A highly rated kinetic energy lies within the head stream of λ -separation. Variation of the clearance at a factor less than $D/5$ sensibly influences flow structure and kinetic energy distribution. It is considered that computing on a turbulent hyperboloid flow field based on a $k-\epsilon$ model at a relatively high Reynolds number may result in a discharge of kinetic energy induced within the "near-of area".

Introduction

The stirring process plays an important role in various industrial applications such as treatment of chemical, food and mining products, environmental technologies, and novel water treatment systems, as well as in an advanced biological engineering. The stirred tank reactor (STR) is one of the main components in a typical chemical facility. In a treatment process utilizing sorption, suspension and aeration, various types of stirrers (or impellers) are widely used to match the optimal flow configuration required to yield fine products. Rushton- and propeller-type stirrers are among the most common, since their characteristics have been well-documented by numerous studies, for instance by Bertrand *et al.* (1980), Gunkel and Weber (1975), Hofken *et al.* (1991), and Nouri and Whitelaw (1994).

However, in recent times, amidst a framework of advanced environmental standards, the problem of energy consumption becomes very important. To select a proper STR-configuration, one needs to consider this factor to reduce maintenance cost. To achieve higher energy conservation rates, a new stirrer system, dubbed a

hyperboloid stirrer, was recently developed by Höfken M. *et al.* (1991, 1994). A basic solution of the potential flow equations was employed in the construction of the stirrer unit in a way that fluid streams almost remain attached on the upper surface. The stirrer has already been applied for industrial purposes, mainly to biological water treatment.

It is common practice to investigate the integral quantities of stirred vessel flows and to present them in a form of diagrams of normalized quantities. Carrying out measurements for a stirrer operated in a round vessel with four baffles mounted at the tank wall, the relationships of Newton-Reynolds numbers and power-sorption numbers results are given in Fig. 1. As can be seen from the $Ne-Re$ relationship, operation of the STR at a high Reynolds number, $Re > 5600$, presents a constant and minimum value of the power number and, therefore, this operational range is most common in practice.

It is important to yield data for the layout of the stirrer, and to operate in a round vessel, but very little information can be deduced on stirrer improvements and mixing processes. Detailed investigations show that information on the local flow field is much more helpful to assess shortcomings of stirrers and to deduce from these the measures for flow improvements. To yield the local velocity information, LDA measurements can be performed as indicated in the present paper.

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Fig. 1 Newton-Reynolds and Power-Sorption numbers relationships for different stirrer types in ground vessel: 1-disk stirrer, Korr. Abwasser (1980), Wilke *et al.* (1991) 2-propeller, Wilke *et al.* (1991), 3-hyperboloid stirrer [present result]

The operation of the hyperboloid stirrer is sensible in a size ratio of its basic units, to $D:T:C$, and to a number and a height of ribs, h_{rib} . A common construction reveals the rates $T=3D$ and $C=[0.1-0.2]D$. Aiming at a specific stirrer application, the number of ribs is varied from 4 to 12, and their height is a ratio of $D/25$ to $D/15$.

Bulk flow driven in a hyperboloid stirrer has been experimentally studied by Nouri and Whitelaw (1994) at Re numbers from 24500 to 48000, and at uncommon high clearances of a ratio of more than $T/10$. The diameter of the stirrer was set at a ratio of $D=T/3$ to the diameter of the tank. The maximum value of velocity was obtained in the radial direction, and was found to be 24% of the circumference velocity of the stirrer. It was concluded that the flow field induced by a hyperboloid stirrer is less vigorous than those induced by Rushton turbines of similar radius that are associated with a significantly lower power number.

Various experimental and computational studies on the flow field generated by a hyperboloid stirrer have been recently carried out by Hofken *et al.* (1991, 1994). These investigations concentrated on practical, reasonable low clearances down to $D/10$. In fact the velocity field and the kinetic energy distribution close to the stirrer tip is a periodical flow. However, more detailed information is needed to optimize the flow structure within a STR equipped with a hyperboloid stirrer according to its specific application.

The main objectives of the present study are as follows :

1. Detailed LDA-measurements of flow driven by a hyper-boloid stirrer at low clearances, aiming at a precise pattern of velocity and kinetic energy distributions within a ground vessel.
2. Evaluation of the influence of stirrer clearance on velocity and kinetic energy distributions in the immediate vicinity of the stirrer element.
3. Validation of a numerical simulation on velocity field and kinetic energy distributions that is based on a $k-\epsilon$ model.

The experimental setup, the diode fiber LDA-measurements (DFLDA), and the data processing routines used for present experiments are described in Section 2. The main results considered for velocity and turbulent kinetic energy are presented in Section 3. Discussions of experimental results and their comparison with the numerical are selected to Section 4.

1. Experimental Setup

The experimental setup used for the present investigation consists of three subsystems that are (i) a STR-test rig, (ii) a diode fiber laser-Doppler anemometer (DFLDA) including a miniaturized optical LDA-probe which moves by a 3D-traversing system, and (iii) a series of electronic units providing automated and controlled measurements.

The STR-test rig is sketched in **Fig. 2**. The hyperboloid stirrer has been settled inside a cylindrical Duran-glass tank with inner diameter of $T=152.4$ mm. The tank was filled with a mixture of silicone oils to match the refractive index $n=1.468$ of the vessel glass wall that illuminates a distortion effect of a tank rounded surface on a path of laser beams. The basic physical properties of the oil mixture at room temperature of 20°C are characterized by a refractive index $n=1.473$, density $\rho=1039$ kg/m³ and kinematic viscosity $\nu=3.83 \times 10^{-6}$ m²/s. The fluid height, H , was equal to T . Mechanical construction of the vessel frames allowed rotation of the stirrer tank with respect to the rectangular Duran-glass vessel. The surrounding rectangular vessel was also filled with the same oil mixture to match equality of the glass and fluid refractive indexes.

Four vertical thick baffles were symmetrically installed at an angle of 45 degree with respect to the measurement plane. In order to neglect collection of particles and stagnation of flow at the cross-section of the baffle and tank wall, a narrow vertical gap of $0.017T$ was set. The 3D translation of the DFLDA-head allowed measurement of axial, tangential and radial velocity components as shown in **Fig. 3**. The data of refractive index of fluid and glass were introduced into the control software in order to compute the actual translation in the direction of the x -axis. To match a high angular resolution, an encoder was mounted on the motor top and measurement data were monitored at 1000 bin/rev. The rotational speed was a

Fig. 2 Test hyperboloid stirrer rig and shade of hyperboloid stirrer z1, z2, z3, z4 are specified zones for actual measurements

constant, $N=2060$ rev/min, that provided measurements at $Re=5600$. Three stirrer clearances, $C=D/10$, $D/7$ and $D/5$, were employed for the present investigation.

The diode fiber laser Doppler anemometer (DFLDA) was designed and developed by Stiegmeier and Tropea (1992). In this study, it was used for accurate measurements of the axial, U_a and u_{a-rms} , radial, U_r and u_{r-rms} , and tangential, U_t and u_{t-rms} , mean velocity and turbulence components respectively. The main components of this instrument are sketched in Fig. 4. Split beams, obtained with a diode monomode laser at wave length of 832 nm and power of 100 mW, were shifted at a frequency of 1 MHz, and transmitted through optic fibers to the translating head. Two transmitting optical configurations with focal lengths of 120 mm and 200 mm, as well as an additional variable band-pass filtering model, have been used for previous test measurements. Actual measurements have been done with a transmitting lens focal length of 120 mm. A portion of the TiO_2 -scattering particles, $d_p=3 \mu m$, after suspending in an ultrasonic field, were pre-mixed with the vessel fluid in order to obtain an optimal data rate.

Attach point of measurement, a measurement control volume (MCV) was formed which had main charac-

Fig. 3 Measurements of axial, tangential and radial velocity components

Fig. 4 Main units of DFLDA

Table 1 Basic optical setup

Angle of the beam intersection [degree]	7.62
Fringe spacing [μm]	6.2605
Diameter of MCV at $1/e^2$ intensity in air [μm]	37
Length of MCV at $1/e^2$ intensity in air [μm]	550
Frequency to velocity conversion [MHz/ms^{-1}]	0.160

teristics as presented in Table 1. Back scattered light was received by the same optical head and transmitted back to an APD. The flow field close to the stirrer element was measured as an angle resolved to take into account a periodic flow oscillation by assigning phase series. The spa-

tial resolution of the measuring flow grid was set at 1 mm. At each measurement point, 60,000 samples were assembled to minimize the biasing and statistical effects. In order to observe an integral flow pattern, the whole velocity field within a radial extension from 10 up to 74 mm and a vertical extension from 0 up to 144 mm was measured by using a time-average method. In this case, the spatial translation was scaled at 4 mm and accurate statistics were achieved through taking 10,000 samples for each measurement point.

The APD-output signal was fed into a counter processor, and via an interface board it was transferred to a PC. An Encoder counting the rotational phase within each revolution period was also connected to the interface board. The measured velocities corresponding to angular phases were stored in the form of binary files. These files were pre-processed in order to construct a matrix of the finite-element zones with respect to the measurement grids. Afterwards, the basic matrix was post-processed to bound the data with STR geometry and finally to build up numerous dynamic patterns of velocity components, velocity means and kinetic energy.

For detailed analysis of angular flow periodicity the different components were treated as a function of the rotational angle. Taking into account the geometry of the baffles setup at 45 degree with respect to measurement intersection plans and also the same angular periodicity of eight upper surface ribs, the flow periodicity corresponds to this fixed value. In terms of time scaling, this period is 3.64 ms.

An appropriate graphical presentation is necessary to facilitate access to the data obtained. For this purpose a post-processing software, TECPLOT, has been utilized. Optional software programs were written to introduce large amounts of data into the post-processor. The measured flow fields have been fully reconstructed due to program facilities organized.

2. Results

An integral vector flow field and normalized kinetic energy distribution for the case of 5 mm stirrer clearance is presented in Fig. 5. The figure reveals that numerous zones of different flow structure can be defined. Close to the tank wall, at radius $r=44-68$ mm and height $h=4-70$ mm, a large scale vortex is formed and fully developed. A secondary large scale vortex of less intensity is generated at radius $r=48-60$ mm and height $h=80-120$ mm. Both vortices are separated by a stagnation zone close to the wall at height $h=60-92$ mm.

In these vortex zones the means of specific turbulence kinetic energy, k/U_{tip}^2 , where tip velocity $U_{tip}=5.39$ m/s, is negligible and less than the order 10^{-4} . It is also important to outline that no disturbances are present close to the fluid top surface usually observed for flows induced by other types of stirrer unit, e.g. Rushton turbine. The area below the height $h=70$ mm and within the radius

Fig. 5 Integral hyperboloid flow field within intersection plan respecting to baffles setup at 45:

range $r=0-30$ mm is characterized by a gentle fully developed coaxial flow accelerating downwards. The most energy-concentrated zone is located in the vicinity of the stirrer element. This zone is referred to as "near-of area" and discussed in more detail below.

The "near-of area" can be divided into several mini-size zones which represent an individual flow configuration as clearly seen in Fig. 6. At the tip of stirrer an intensive jet stream separates the integral flow into two areas, upper vortex zone and down re-circulation zone as it is indicated by the vector fields depicted in the left columns of the present figure. The structure of this separation zone can be easily described in terms of lambda " λ ".

After ejection at $r=R$, the flow is separated. The major portion of the flow proceeds within a right arm of " λ " along a radial-axial path, $r=1.0R-1.4R$, until it reaches the tank bottom, where it is deflected again in the radial direction, $r>1.4R$, to form the lower large scale vortex described above. A fraction of the ejected flow proceeds within a left arm of " λ " forming down a re-circulation zone. In the case of 5 mm clearance, only one re-circulation zone can be observed, whereas larger clearances cause generation of a secondary re-circulation zone. This fact is confirmed by the distributions of the mean velocity, right columns in Fig. 8, which show a clear separation of the flow field beneath the stirrer element. Turbulence kinetic energy concentrates itself within the separation jet on the right arm of " λ ", as seen from the center columns in Fig. 6. Its mean reaches maximum values up to 5% of U_{tip}^2 . At a clearance of 10 mm, the highest values of turbulence kinetic energy occur in a spread area.

The hyperboloid flow is axially symmetric, thus the separation λ -area has an umbrella shape in a 3D-space. An intrinsic bulk flow includes primary and, in cases of higher clearances, also secondary down re-circulation

fig. 6 Plan view of vicinity flow field elements for different stirrer clearances: 5 mm (top series), 7.5 mm (center series) and 10 mm (bottom series)

Fig. 7 Angular periodicity of normalized turbulent kinetic energy of stirrer flow under clearance $C=5$ mm at $h=5$ mm

structure. The reason for the separation is due to a specially designed stirrer shape. Downward flow on the upper hyperboloid surface lies in a field of negative axial acceleration, while the bottom shape of the stirrer is planar. Because of this, the flow at the stirrer tip circumference is impinged and transformed into a λ -separation jet.

The hyperboloid flow has an obvious periodicity within the fixed period of 45 degree. The spatial distribution of velocity and kinetic energy depends on angular phase. The strongest periodicity appears near the stirrer tip within the rib height, $h_r=2$ mm, beginning from the tank bottom and ending at the top of ribs. **Figure 7** presents a radial “near-of” extension of normalized turbu-

lence kinetic energy at a clearance of 5 mm.

The figure indicates that at a height level corresponding to the stirrer bottom, $h=C$, the flow becomes strongly dependent on the angular phase within the “near-of” area. Intensity of kinetic energy is significantly damped along the radial extension. From experiments, it was observed that going up to the level where transport ribs induced the flow, the radial component of velocity dominates the flow configuration developed, especially in the case of a high clearance.

Increased clearance is indicated by increased means of kinetic energy in the “near-of area”. Impingement zones start to develop at a certain ribs position, *i.e.* angular phase is 0 degree. A radial thickness of the impingement zone increases and trails because of the stirrer rotation. It is clearly seen that the angular phase of trailed impingement trends is over geometrical periodicity, 45 degree. The flow in this area is characterized by a vast transformation of velocity and turbulence components.

3. Discussion

Flow induced by the hyperboloid stirrer is strongly periodical within a well-defined area close to the stirrer tip, and is referred as “near-of” area. The flow structure, at Reynolds number over 5000, is also significantly dependent on stirrer clearance. Details of typical velocity and turbulence components oscillation for the case of $h=C=5$ mm are presented in **Fig. 8**. Velocity and turbulence fluxes oscillations at three radial positions, $r=28$ and 31 mm, near of the stirrer tip are presented. The inten-

Fig. 8 Angular periodicity of velocity components (U) and their fluxes (rms) at radial positions of 28 and 31 mm

sity of oscillations is damped when the radius is over a critical radius $r_{cr} \geq 31$ -35 mm.

To compare the flow characteristics for different clearances $C=5.0$ and 10.0 mm the radial distributions of velocity components, (A)-axial, (T)-tangential and (R)-radial, as well as specific kinetic energy near of the stirrer, $r=25$ -35 mm ($r/R=1.0$ -1.4), are plotted in **Fig. 9**. In both cases of given clearances C at the planar height position $h=C$, the axial velocities are negative, excluding small "near-of" area $r=25$ -26 mm. They lightly phase with the stirrer rotation. Radial distribution of tangential velocity shows an angular phasing, and for clearance of $C=5$ mm the velocity mean matches 96% of tip velocity, U_{tip} . The radial velocity component for present clearances is periodical and the highest radial velocity mean is cited of $C=10$ mm. In general, the velocity radial distribution is the same when $r \geq r_{cr}$.

The periodicity is also clearly seen in the turbulence kinetic energy distribution, **Fig. 9**. At the start phase, $\alpha=0$ degree, the highest energy, $k_n = k/U_{tip}^2 = 0.042$, is observing for the case of $C=10$ mm at the stirrer tip $r=R$. For this clearance, a number of energy peaks are presented in the radial extension, while for small clearance only one energy peak occurs. At an angular phase of $\alpha=15$ degree (time scale correspondents to $1.2 \mu s$) the highest energy peak moves to a clearance of $C=5$ mm, and is valued $k_n = k/U_{tip}^2 = 0.046$ at $r=29$ mm.

To evaluate the capability of a modeling on turbulent hyperboloid flow, the CFD-code FASTEST-3D-P was developed by the Institute of Fluid Mechanics, University of Erlangen. The numerical solution procedure is based on a collocated block-structured finite volume discretization, a full approximation scheme, a pressure-correction procedure for smoothing and coupling of variables, and a grid partitioning technique for parallelization.

The 3D unsteady flow is computed on the base of a standard $k-\epsilon$ turbulence model. The time-dependent geometry has been modeled by using stationary and rotating parts of the block-structured grid.

For the case of $C=h=5$ mm, the data obtained from the present measurements, and those modeled by $k-\epsilon$ tur-

Fig. 9 Radial distribution at level of clearance ($h=C=5$ and 10 mm) of velocity components (U_a -axial, U_r -radial and U_t -tangential) and normalized kinetic energy (k/U_{tip}^2) at three angular phases 0, 15 and 30 degree

bulence models are compared in **Fig. 10**. Three angular phases $\alpha=0^\circ$, 15° , 30° are referred to for radial distributions of axial U_a , tangential U_t and radial U_r velocity components. The trend of velocity distribution presented by the experimental data corresponds to the numerical data. Most discharge, with respect to normalized kinetic energy, k/U_{tip}^2 , occurs in the "near of" area, $r=1.0R$ - $1.1R$, because of the $k-\epsilon$ turbulence model used at high Re number. Experiments show that the energy value of the turbulent hyperboloid flow field is unisotropic, and this fact has to be taken into account to improve the numerical turbulent model.

Conclusion

Based on the present results it is possible to consider that for practical utilization of the hyperboloid stirrer, the following conclusions are most important :

1. An energy conservative operation of the hyperboloid stirrer is dealt with fully developed baffled turbulent flow, *i.e.* Reynolds numbers of $Re \geq 5000$.
2. The low clearances, $C \leq D/10$, allow to induce within the STR a spatial λ -separated flow field of concentrat-

Fig. 10 Radial distribution of velocity components A -axial, T -tangential and R -radial and normalized kinetic energy, k/U_{tip}^2 , at three angular phases: 0, 15, and 30 degree. (line-numerical, star-experimental)

ed turbulence kinetic energy within the right arm of the λ -flow configuration. A varying of the clearances permits matching of an optimal ratio of energy induced into the flow, and to control a spatial distribution of the flow and turbulence kinetic energy. This is important where suspension/aeration or/and sorption treatment processes have to be employed.

- The effect of periodicity has to be taken into account in a "near-of area" that lies in the range of radius $r/R=1.0-1.4$. In the treatment of substances with a sensitive molecular structure and a time response as low as a few ms, the induced impingement of velocity and energy may cause a phase transfer of these substances.

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Nomenclature

a	= ratio of interfacial area to liquid volume	[m ² /m ³]
C	= stirrer clearance at the bottom of the vessel	[m]
d_p	= particle diameter	[m]
D	= stirrer diameter	[m]
H	= liquid high in vessel	[m]
h	= axial coordinate	[m]
h_r	= height of hyperboloid upper surface ribs	[m]
k_n	= specific turbulence kinetic energy = k/U_{tip}^2	[dimensionless]
$k_L a$	= mass transfer coefficient	[1/s]
n	= refractive index	[dimensionless]
N	= stirrer rotational speed	[rev/s]
Ne	= Newton (or power) number = $P/\rho N^3 D^5$	[dimensionless]
Q	= air supply volumetric flow rate	[m ³ /s]
P	= power consumed = ωT	[dimensionless]
r	= radial co-ordinate	[m]
rms	= root of mean square of velocity flux	[m/s]
R	= stirrer radius	[m]
Re	= Reynolds number = ND^2/ν	[dimensionless]
T	= tank diameter	[m]

u_a	= axial rms	[m/s]
U_a	= axial mean velocity	[m/s]
u_r	= radial rms	[m/s]
U_r	= radial mean velocity	[m/s]
u_t	= tangential rms	[m/s]
U_t	= tangential mean velocity	[m/s]
U_{tip}	= stirrer tip velocity	[m/s]
V	= liquid volume in aeration tank	[m ³]
Y	= sorption number = $k_L a V / q$	[dimensionless]
α	= angular phase	[rad]
ν	= fluid kinematic viscosity	[m ² /s]
ρ	= fluid density	[kg/m ³]
ω	= stirrer angular velocity	[rad/s]
*MCV	= measurement control volume formed at the intersection of two laser beams	

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