Numerical Analysis of the Mixing Process of a Heterogeneously Viscous System with High Concentration Slurry Liquids in a Stirred Vessel

Meguru KAMINOYAMA¹, Kazuhiko NISHI¹, Ryuta MISUMI¹, Tomoyuki INOUE² and Hiroshi TAKEDA³

¹Faculty of Engineering, Yokohama National University, 79-5, Tokiwadai, Hodogaya-ku, Yokohama-shi, Kanagawa 240-8501, Japan
²Graduate School of Engineering, Yokohama National University, 79-5, Tokiwadai, Hodogaya-ku, Yokohama-shi, Kanagawa 240-8501, Japan
³Rflow Co., Ltd., 1-10-45, Takanashi Bldg., Takasago, Soka-shi, Saitama 340-0015, Japan

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We tried in this study to numerically analyze the mixing process of high concentration slurry liquids having a plastic rheological property, and in which unevenness of both viscosity and concentration disappear along with agitation despite the initial condition in which the viscosities are different between the upper and lower halves of a stirred vessel. We verified the reliability of the analyzed results by comparing the viscosity and concentration with measurements of mixing time and the power consumption required to attain mixing. We clarified the relation between the mixing time and the initial concentration (viscosity) difference of the two layers or the final concentration of the mixture, both experimentally and analytically. We ascertained that a mixing performance index, $N_M''$, can be well correlated with the Reynolds number, $Re_M$, and the viscosity correction term for each initial concentration difference of the two layers.

The numerical analysis enabled us to understand the correlation equations of the mixing process of a heterogeneously viscous system in which the initial viscosity varies through the vessel and to estimate the mixing performance in the vessel from a local point of view.

Introduction

Processes involving the mixing of highly concentrated slurry liquids in a stirred vessel are frequently used in several industrial fields, such as biochemistry, paints, ceramics, cosmetics, pharmacy, and wastewater treatment. Thus, the mixing of heterogeneously viscous fluids is not unfamiliar in these fields. However, few studies on the mixing properties of homogeneously viscous slurry liquids have been reported (Arai et al., 1990; Hirata and Aoshima, 1996; Nagafune and Hirata, 1999; Kamiwano et al., 2000), since the liquids are generally opaque. Estimation of the mixing property of a heterogeneously viscous system has been reported much less (Kaminoyama et al., 2004), since its non-Newtonian rheological property, substantially plastic rheology, varies intricately as the mixing proceeds. On the other hand, numerical analyses have been attempted for the fields of highly viscous non-Newtonian fluids, including slurry liquids (Kaminoyama et al., 1990a, 1990b, 1993, 1994a, 1994b, 1997, 1999; Tanguy et al., 1999). We try, in this study, a numerical analysis of the mixing process of heterogeneously viscous slurry liquids in a stirred vessel to investigate the effects of initial conditions of slurry properties and operating conditions on the process.

1. Procedure and Method for Analysis

1.1 The conception of the problem for the analysis

Figure 1 shows the conceptual diagram of the problem analyzed in this study. The figure indicates a vertical section of the right-hand side of a stirred vessel. The fluid in the vessel comprises two layers at the beginning: the upper layer, a less highly concentrated slurry, and the lower layer, a more highly concentrated slurry, for preparing a stable initial state for experimental measurement. We numerically analyze the mixing process of the two layers, which have different viscosities at the initial state, until the two layers
attain a homogeneous viscosity. We investigate the influence of the initial difference in viscosity of the two layers and the final homogeneous viscosity and the impeller rotational speed, \( n \), on the mixing time.

### 1.2 Physical properties of the slurry

The fluids analyzed are highly concentrated aqueous slurries of calcium carbonate, ranging from 35 to 70 wt% concentration. Table 1 shows the slurry density, \( \rho \), depending on the concentration. Figure 2 shows the rheological property of each concentrated slurry measured with a rheogoniometer (Nihon Rheology Kiki Co., Ltd.). The property is substantially plastic. We, however, approximately express the non-Newtonian viscosity, \( \eta \), with a two-parameter power law model, that is, no yield stress model, as follows:

\[
\eta = k \gamma^{m-1}
\]

where \( k \) is a function of the concentration, \( w \) [wt%], as shown in Eq. (2).

\[
k = 10^\beta
\]

\[
\beta = 0.00136w^2 - 0.08757w + 0.56575 \tag{2}
\]

where \( m = 0.506 \), independent of the concentration.

The straight lines in the figure were drawn according to Eq. (2).

### 1.3 Governing equations and calculation procedure

The governing equations are composed of an equation of continuity for a compressible fluid, Eq. (3), an equation of motion for a non-Newtonian fluid, Eq. (4), a transport equation of \( \alpha \) for the concentration analysis, Eq. (5), and an equation of the density of the slurry mixture, Eq. (6), as well as the viscosity constitutive equations mentioned above, Eqs. (1) and (2).

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{3}
\]

\[
\rho \frac{D \mathbf{v}}{Dt} = -\nabla p + \nabla \eta \nabla \cdot \mathbf{v} + \rho \mathbf{g} \tag{4}
\]

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{v}) = 0 \tag{5}
\]

\[
\rho = \rho_0 \alpha + \rho_* (1.0 - \alpha) \tag{6}
\]

where \( \alpha \) is the volumetric occupation ratio of the lower concentration slurry to the total volume of each mesh discretized. The equation of motion, Eq. (4), is numerically solved without any turbulence model under the restriction of Eq. (3) to obtain a velocity vector \( \mathbf{v} \). After that, the distribution of \( \alpha \) is calculated according to Eq. (5) using the \( \mathbf{v} \) obtained. The value of \( \alpha \) is converted to the concentration, \( w \). The distribution of \( \eta \) is calculated according to Eqs. (1) and (2) using \( w \) in order to substitute \( \eta \) in Eq. (4).
The same procedure is repeated until \( \rho \) sufficiently reaches the average value of the lower density, \( \rho_l \), and the higher density, \( \rho_h \), of the initial two layers.

1.4 Analysis conditions

Figure 3 shows the schematic diagram of the apparatus for the analysis. The diameter of the vessel, \( D \), is 0.1 m equipped with a 4-blade paddle impeller of 0.05 m diameter, \( d \). Table 2 indicates the analysis conditions, such as the final average concentration, \( C_\infty \), and initial concentrations, \( C_0 \), of the two slurry layers. Table 3 indicates operating conditions including the volume ratio of the two layers and the rotational speed of the impeller.

1.5 Analysis model

We analyzed the region, a quarter of the whole vessel shown in Figure 4, in the circumferential direction considering the periodic symmetry. The total number of meshes discretized at unequal intervals is 48,944 (38 \( \times \) 28 \( \times \) 46; the radial, circumferential, and axial directions, respectively). The time step was set at the period, that is, the time required for one rotation of the impeller at each rotational speed, \( n \). We have done all the calculations with the commercially available software, RFLOW (Rflow Co., Ltd.), by incorporating the viscosity constitutive equations as a user subroutine. The two-parameter power law model has an infinity problem at a zero strain rate. We set the threshold values to avoid the problem, that is, the upper limit of \( \eta = 10^{10} \) Pa·s and the lower limit of the strain rate \( \dot{\gamma} = 10^{-2} \) s\(^{-1}\). The approximate turn around time for one case study was 4 to 8 h with a Windows PC of between 2 and 3 GHz and a memory of 30
Figure 5 shows, as an example, for Case No. 4 in Table 2, the temporal change of the dimensionless difference in concentration, $\Delta C^*$, which was calculated from $\Delta C$ between the two layers in the vicinity of Points ③ indicated in Figure 3. When $\Delta C^*$ reaches 0, the complete mixing is attained. We, however, defined the mixing time, $t_M$, as the time when $\Delta C^*$ reaches 0.2, for the reduction of the calculation load and the clearness independent of sensitivity, as shown in Figure 5.

1.6 Experimental validation

For the validation of our numerical simulations, we obtained experimental data (Kaminoyama et al., 2004) with the same apparatus as depicted by Figure 3 and under the same conditions as indicated in Tables 2 and 3. The mixing time, $t_M$, was evaluated from the
Concentration data at Points ③ in Figure 3, measured with the electrical conductivity method (Nagata, 1975). The power, $P$, varies along with the mixing process since the viscosities of the upper and lower layers in the vessel are different. So, we evaluated the value as a cumulative power, $t_M P$, expressed by Eq. (7).

$$t_M P = \int_0^t P \, dt$$

The dynamic value of the power was measured by a torque meter (Shinto Kagaku Co., Ltd.).

2. Results and Discussion

Figure 6 shows the distributions of velocity vectors, concentration and viscosity for Cases 2, 5, 6, and 8 in Table 2 at the dimensionless time, $n \cdot t = 100$, as an example. The viscosity varies complicatedly, as it depends on the concentration, as well as the strain rate. There are higher value regions in the concentration and viscosity distributions in the vicinity of the bottom of the vessel, whereas the velocity value is very low. The low value of the viscosity around the impeller is attributable to the high strain rate thereupon.

We can find that the larger the initial concentration difference of the two layers, $\Delta C_0$, is, the larger the stagnant region becomes (Cases 5 and 6) and also that the higher the final average concentration, $C_\infty$, is, the larger is the stagnant region (Cases 2, 5, and 8). Cavern states are recognized in the vicinity of the vessel bottom, especially in Cases 6 and 8.

Figure 7 shows the relation between the mixing time, $t_M$, and the rotational speed, $n$, for each final viscosity. We can find that the larger $\Delta C_0$ leads to the longer the mixing time, $t_M$, regardless of the final average concentration of the layer mixture. The higher the final concentration of the mixture of the two layers is, the longer $t_M$ becomes. These phenomena in the vessel can be understood from the local point of view as is depicted in Figure 6. Roughly speaking, the effect of $\Delta C_0$ on $t_M$ decreases with an increase of the rotational speed, $n$. The results numerically analyzed here reproduce the experimental data fairly sufficiently, as shown in this figure, to validate the reliability of the calculation method of this study. The differences between the numerically analyzed results and the experimentally obtained data at $n = 10 \, \text{s}^{-1}$ may be attributable to the turbulent flow in the actual stirred vessel.

Further, we tried to derive a relation between the dimensionless mixing time, $N_M'$, (Hoogendoon and Den Hartog, 1967) with power consumption, $P$, the experimental value evaluated by Eq. (7) and the Reynolds number, $Re_M$, defined by Eqs. (8) and (9), respectively, based on the simulated results excepting the $P$ values.

$$N_M' = t_M^2 P / \eta_\infty D^3$$

$$Re_M = \rho D^2 / t_M \eta_\infty$$

Here we calculated the values of $\eta_\infty$ for the final average concentration, $C_\infty$, using the Metzner method (Metzner and Otto, 1957) with Metzner’s constant $= 11.0$ for the 4-blade paddle impeller. However, we failed to sufficiently correlate them, presumably because the viscosity in the vessel changed dynamically. So, we added a viscosity correction term ($\eta_l / \eta_h$) into the correlation equation, where we evaluated the values of $\eta_l$ and $\eta_h$ at the initial state.

We could improve the correlation using the viscosity correction term as shown in Figure 8. The results show that the relation between $N_M'$ and $Re_M$
cannot be correlated in a single way but correlated for each initial concentration difference of the two layers, \(\Delta C_o\), regardless of the values of \(C_o\). Finally we could correlated \(a\) and \(\gamma\) appeared in Eq. (10), with Eqs. (11) and (12), respectively, where \(Re_M\) is smaller than about 50.

\[
N_M' = aRe_M^{-7/2} (\eta_0/\eta_0')^{0.2} \tag{10}
\]

\[
a = 2.5 \times 10^4 \Delta C_0^{0.53} \tag{11}
\]

\[
\gamma = 0.86 \Delta C_o^{0.023} \tag{12}
\]

The dependency of \(\gamma\) on \(\Delta C_o\) is very small and the values are 0.92 ± 0.01. So roughly speaking, when \(N_M'(\eta_0/\eta_0')^{0.2}\) is designated by \(N_M''\), \((N_M'' Re_M)\) are almost constant and the intercept value a increases with an increase of \(\Delta C_o\). We can evaluate the mixing time of the mixing process of heterogeneously viscous slurry liquids, \(t_M\), based on Eq. (10) by setting the operating conditions such as \(n\), \(\Delta C_0\) and \(C_o\), and assuming or measuring the \(P\) value.

**Conclusions**

In this study, we tried a numerical analysis of the mixing process of heterogeneously viscous slurry liquids in a stirred vessel to investigate the effect of the initial conditions of slurry properties and operating conditions on the process. The followings were found as a result, which are in fairly good agreement with the results obtained experimentally:

1. The larger the initial concentration (viscosity) difference of the two layers is, the longer the mixing time is prolonged.

2. The effect of the initial concentration difference on the mixing time decreases with an increase of the rotational speed.

3. The higher the final concentration of the mixture of the two layers is, the longer the mixing time becomes.

4. The mixing performance index, \(N_M'\), can be well correlated with the Reynolds number, \(Re_M\), and the viscosity correction term for each initial concentration difference of the two layers.

\[
N_M' = 2.5 \times 10^4 \Delta C_0^{0.53} Re_M^{0.82} (\eta_0/\eta_0')^{-0.2} \tag{13}
\]

The numerical analysis is useful for understanding of the complicated mixing phenomena in a vessel from a local point of view.

**Nomenclature**

- \(a\) = parameter in Eq. (10) [—]
- \(b\) = impeller height [m]
- \(C\) = concentration [wt%]
- \(C_o\) = initial concentration [wt%]
- \(\Delta C\) = final average concentration [wt%]
- \(D\) = diameter of an agitated tank [m]
- \(d\) = impeller diameter [m]
- \(g\) = gravitational acceleration [m/s²]
- \(H\) = liquid depth [m]
- \(h\) = height of an impeller location [m]
- \(I\) = second invariant of rate of deformation tensor [s⁻²]
- \(k\) = fluid consistency index in the power-law rheological equation [Pa·sα]
- \(m\) = exponent in the power-law rheological equation [—]
- \(N_M'\) = mixing performance index based on power consumption defined by Hoogendoorn and Hartog, \(=t_M/\rho \eta_0 D^3\) [—]
- \(N_M''\) = \(N_M'(\eta_0/\eta_0')^{0.2}\) [—]
- \(n\) = rotational speed of an impeller [s⁻¹]
- \(P\) = power [W]
- \(P\) = pressure [Pa]
- \(Re_M\) = modified Reynolds number, \(=pD^2/\eta_0 R\) [—]
- \(t\) = time elapsed from beginning of agitation [s]
- \(t_M\) = time required to attain mixing [s]
- \(t_0P\) = cumulative power calculated by Eq. (7) [J]
- \(v\) = velocity vector [m/s]
- \(w\) = weight concentration of slurry [wt%]
- \(\alpha\) = volumetric occupation ratio of lower concentration slurry to the total volume of each mesh discretized [—]
- \(\beta\) = parameter in Eq. (2) [—]
- \(\gamma\) = parameter in Eq. (10) [—]
- \(\gamma\) = strain rate, \(= \left[(1/2)\dot{\gamma}\right]^{1/2}\) [s⁻¹]
- \(\Delta C\) = difference in concentration [wt%]
- \(\Delta C_o\) = initial difference in concentration [wt%]
- \(\Delta C'\) = final difference in concentration [wt%]
- \(\Delta C''\) = dimensionless difference in concentration, \(= (\Delta C_\infty - \Delta C_o)/\Delta C_\infty\) [—]
- \(\eta\) = viscosity of non-Newtonian fluid [Pa·s]
- \(\rho\) = density of slurry [kg/m³]
Literature Cited