ON THE MECHANISM OF DRYING OF GRANULAR BEDS

Mass Transfer from Discontinuous Source*

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Introduction

In the course of drying of wet granular materials, a period of constant drying rate is observed in most cases. It is usually stated that the rate of evaporation from the beds of such fine granular materials as clay is equal to that from a free water surface during the period of constant rate. In the case of relatively coarse granular materials such as sand, however, it was shown by Cegléske\(^1\) that these rates are slightly different each other. Shishkov\(^2\) also reported that such an equality did not hold under a severe drying condition. Nevertheless, difference between the above two rates of evaporation is sufficiently small for granular materials of practical interest. In explaining the constant drying rate, it has been assumed that the ratio of wet surface area to the sectional area of the beds remains constant during the concerned period, even though the moisture concentration decreases and pores at the surface increase with the time elapse. In most cases it has also been assumed that the surface of granular material is completely covered with a water film during this period.

In the drying of granular, non-hygroscopic materials such as sand or glass spheres, liquid water within the beds is drawn to the evaporating plane by a capillary action. The menisci at the surface must then have smaller radii of curvature than those in the interior of the beds, and there may be a dried region above each surface of the non-hygroscopic solid particles as shown in Fig. 1. The water evaporating from the surface is necessarily replaced by the air which enters the beds through large pores at the surface, and then there must be vacant pores at the surface of the solids being dried.

In other words, the evaporation takes place from the discontinuous menisci at the surface of the beds during the constant drying rate period, the ratio of the area of the menisci to the drying surface decreases with decreasing surface moisture concentration.

Heat and mass transfer from such a discontinuous source also occur in the other unit operations such as

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Greek Letters

\(\xi\) = non-dimensional axial distance = \(z/R_w\)
\(\eta\) = non-dimensional radial distance = \(r/R_w\)
\(\varepsilon\) = \(k_s/C_p \rho G \zeta\)
\(\mu\) = viscosity of fluid [kg/m·h]
\(\nu\) = kinematic viscosity of fluid [m²/h]

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Literature cited

1) S. Hatta and S. Maeda: Kagaku Kikai, 12, 56 (1948)
2) D. Kunii and J.M. Smith: A. I. Ch. E. J., 6, 97 (1960)
6) S. Yagi and D. Kunii: Kagaku Kagaku, 18, 576 (1964); A. I. Ch. E. J., 3, 373 (1967)
transpiration cooling, solid extraction, gas adsorption on porous solids and so on. At present situation, however, knowledges on the transfer phenomena from such a discontinuous source is unsatisfactory.

It is the object of this paper to solve the problem of mass transfer from a discontinuous surface to the fluid flowing over the surface, and to explain the drying mechanism of granular materials during the constant drying rate period.

Problem of mass transfer from a continuous surface to a fluid is fairly well understood based on the boundary layer theory in hydrodynamics. In studying mass transfer from discontinuous source therefore, it may be helpful to extend the analytical procedure already used in success for the continuous surface. However, it is very difficult to solve the boundary layer problems in such a case, because the boundary conditions for the partial differential equations are highly complicated.

In this paper, analytical as well as approximate solutions of mass transfer from a discontinuous source will be given for some simplified models of the boundary layers over the surface.

**Stagnant Boundary Film**

Let us consider a case of mass transfer from a layer of static fluid immediately adjacent to the surface of granular beds. This stagnant boundary film, as illustrated in Fig. 2, is the simplest model of a boundary layer over the granular beds being dried. When the mass fluxes within the layer are approximated to be two-dimensional, the equation of vapor concentration distribution at a steadystate becomes

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} = 0$$

(1)

Boundary conditions for the layer enclosed by dotted lines in Fig. 2 are

- $C = C_a$ at $y = \delta$
- $C = C_s$ at $y = 0$, $0 < x < \phi D/2$
- $\frac{\partial C}{\partial y} = 0$ at $y = 0$, $\phi D/2 < x < D/2$

where $\delta$ is the thickness of the layer, $D$ is the unit length of the discontinuous sources, which is taken equal to the diameter of granular particles, $\phi$ is the fraction of wet area, $C_s$ and $C_a$ are vapor concentrations of the air at the water menisci and in the drying air flow, respectively.

By introducing the dimensionless variables

- $Y = y/D$, $X = x/D$, $\theta = (C - C_a)/(C_s - C_a)$

the Eqs. (1) and (2) become

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0$$

(3)

**B. C.**

- $\theta = 0$ at $Y = \delta/D$
- $\theta = 1$ at $Y = 0$, $0 < X \leq \phi/2$
- $\frac{\partial \theta}{\partial Y} = 0$ at $Y = 0$, $\phi/2 < X \leq 1/2$
- $\frac{\partial \theta}{\partial X} = 0$ at $X = 0$ and $1/2$

The mass transfer coefficient may be calculated from the following equation

$$k_s = \frac{D}{2} \int_0^{\phi/2} \frac{\partial Y}{\partial X} \frac{\partial C}{\partial y} \frac{dX}{D}$$

(5)

The coefficient for a free surface under the same condition is given by

$$k_{s}^f = -\frac{2 \delta}{D} \int_{\phi/2}^{1/2} \frac{(\partial Y)}{\partial X} \frac{\partial C}{\partial y} \frac{dX}{\delta}$$

(6)

where $\phi^*$ is the solution of the Eq.(3) under the condition of $\phi = 1.0$. The ratio of the apparent mass transfer coefficient of the discontinuous source to that of the free surface becomes

$$\frac{k_s}{k_{s}^f} = -\frac{2 \delta}{D} \int_{\phi/2}^{1/2} \frac{(\partial Y)}{\partial X} \frac{\partial C}{\partial y} \frac{dX}{\delta}$$

(7)

The Eqs.(3) and (4) are characterized by two dimensionless parameters $D/\delta$ and $\phi$. Accordingly the ratio of mass transfer coefficients, $k_s/k_{s}^f$, depends only on these two parameters. Its functional form is given by

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**Fig. 2** Model of stagnant boundary film over the granular beds
conformal mapping as

\[
\frac{k_y}{k_y^*} = \frac{K_1(k) K_1'(k)}{K_1'(l) K_1(k)}
\]

\[
\frac{\delta}{D} = \frac{K_1(k)}{2 K_1'(l)}
\]

\[
\phi = \frac{sn^{-1}(k/l', l')}{K_1'(l')}
\]

where \(K_1\)'s are the complete elliptic integral of the first kind, \(sn(a, b)\) is the Jacobian elliptic function, \(k\) and \(l\) are moduli of these functions. Primed and unprimed quantities are related through

\[
k' \equiv \sqrt{1-k^2}
\]

\[
l' \equiv \sqrt{1-l^2}
\]

\[
K'(k) \equiv K(k')
\]

\[
K'(l) \equiv K(l')
\]

A numerical calculation of \(k_y/k_y^*\) was made on the digital computer NEAC-2230 and the results are shown in Fig. 3. It is seen from this figure that, as \(\delta/D\) increases, \(k_y/k_y^*\) becomes independent of \(\phi\).

**Boundary Film with Fluid Flow**

**General equations**

As second model, a boundary film with a fluid flow within it will be considered under the following assumptions.

1. Steady state.
2. Fluid flows in the \(x\)-direction and velocity profile is fully developed.
3. Diffusion of the vapor in the direction of the flow is negligible.
4. Mass fluxes are approximated to be two-dimensional.

With a constant molecular diffusivity \(D\) for the vapor in the air, the mass transfer equation and boundary conditions then take the form

\[
\nu(y) \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial y^2} = 0
\]

B. C.

\[
C = C_0 \quad \text{at} \quad y = 0, \quad \theta = 0, \quad \phi = 0
\]

\[
C = C_s \quad \text{at} \quad x = 0
\]

where \(\nu(y)\) is the velocity profile of the fluid. The origin was selected at the leading edge of the wet surface. On introducing the dimensionless variables

\[
Y = \frac{y}{\delta}, \quad X = \frac{\partial}{\partial \nu} (\delta \nu), \quad \theta = \frac{C - C_s}{C_s - C_0}
\]

\[
V(Y) = \frac{\nu(y)}{\nu(\delta)}
\]

Eqs.(10) and (11) become

\[
\frac{\partial Y}{\partial X} \frac{\partial^2 Y}{\partial Y^2} = 0
\]

\[
B.C.
\]

\[
\theta = 0 \quad \text{at} \quad Y = 1
\]

\[
\theta = 1 \quad \text{at} \quad Y = 0, \quad \phi = 1.0
\]

\[
\left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} = 0
\]

\[
\left. \frac{\partial \theta}{\partial Y} \right|_{Y=1} = 0
\]

\[
\left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} = 0
\]

\[
\left. \frac{\partial \theta}{\partial Y} \right|_{Y=1} = 0
\]

\[
\left. \phi \right|_{Y=0} = 0
\]

\[
\left. \phi \right|_{Y=1} = 0
\]

\[
\theta = 0 \quad \text{at} \quad X = 0
\]

\[
\theta = 1 \quad \text{at} \quad X = 0
\]

where the dimensionless parameter \(N\) was defined as

\[
N = \frac{\partial \nu}{\partial \nu} (\delta \nu)
\]

The averaged mass transfer coefficient is given by

\[
k_y = \frac{\nu(\delta)}{\nu(X)} \int_0^{\delta} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=1} dX
\]

The mass transfer coefficient at the free water surface \((\phi=1.0)\) under the same condition becomes

\[
k_y^* = \frac{\partial \nu(\delta)}{\partial \nu(X)} \int_0^{\delta} \left( \frac{\partial \theta^*}{\partial Y} \right)_{Y=0} \left( \frac{\partial \theta^*}{\partial Y} \right)_{Y=1} dX
\]

where \(\theta^*\) is the solution of Eq.(12) for \(\phi=1.0\). The ratio of the mass transfer coefficients becomes

\[
\frac{k_y}{k_y^*} = \frac{\int_0^{\delta} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=1} dX}{\int_0^{\delta} \left( \frac{\partial \theta^*}{\partial Y} \right)_{Y=0} \left( \frac{\partial \theta^*}{\partial Y} \right)_{Y=1} dX}
\]

Eqs.(12), (13) and (17) are characterized by the dimensionless parameter \(N\), the number of the discontinuous sources \(n\), the dimensionless velocity profile \(V(Y)\) and the fraction of wetted area \(\phi\). In general, the solution consists of two parts; a periodic solution for the fully developed potential distribution at large \(X\), and an entrance solution at small \(X\) which disappears at large \(X\). If the total length of the discontinuous surface \(X=nN\) is sufficiently large, the potential distribution \(\theta(X, Y)\) may be approximated by the periodic solution, and the effect of the number \(n\) on the ratio of mass transfer coefficient becomes negligible. On the other hand, if the concentration boundary layer is not fully developed, then the thickness of the film \(\delta\) has no effect on the entrance solution at small \(X\), and the ratio of the mass transfer coefficient also does not depend on the thickness \(\delta\), nor on the dimensionless number \(N\).

Thus at large \(X\) the mass transfer coefficient depends on \(N, \phi\) and \(V(Y)\), whereas at small \(X\) on \(n, \phi\) and \(V(Y)\). In the following sections Eqs.(12), (13) and (17) are solved for the piston- and linear- velocity profiles and the influences of the \(N, n\) and \(\phi\) on the ratio of the mass transfer coefficient are calculated for each case.
Boundary Film with Piston-flow Velocity Profile

For this model of boundary film with a piston flow velocity profile, the dimensionless velocity \( V(Y) \) becomes
\[
V(Y) = 1.0 \quad (18)
\]
Eqs. (12) and (13) can be solved by the method of separation of variables.

At the surface of the water meniscus,
\[
\theta = 1 - Y + \sum A_{n,k} \exp(-k^2\pi^2 N) \sin(k\pi Y) \quad \text{for } nN < X \leq (n + \phi)N
\]
under the boundary condition of the first kind
\[
\theta = 1 \quad \text{at } Y = 0
\]
On the dry surface,
\[
\theta = \sum B_{n,k} \exp\left[\frac{1}{2} \left( j - \frac{1}{2} \right) \pi^2 N \right] \cos\left( j - \frac{1}{2} \right) \pi Y \quad \text{for } (n + \phi)N < X \leq (n + 1)N
\]
under the second kind boundary condition
\[
\frac{\partial \theta}{\partial Y} = 0 \quad \text{at } Y = 0
\]
The initial condition
\[
\theta = 0 \quad \text{at } X = 0 \quad (21)
\]
determines the coefficients \( \{A_{n,k}\} \) as
\[
A_{n,k} = \frac{-1}{2k\pi} \exp\left( -k\pi^2 N \right) \quad (22)
\]
From the conditions of continuity of these two solutions at \( X = mN \) and \( X = (m + \phi)N \), one gets
\[
1 - Y + \sum A_{n,k} \exp(-k^2\pi^2 mN) \sin(k\pi Y)
= \sum B_{n,k} \exp\left[\frac{1}{2} \left( j - \frac{1}{2} \right) \pi^2 mN \right] \cos\left( j - \frac{1}{2} \right) \pi Y
\]
and
\[
1 - Y + \sum A_{n,k} \exp(-k^2\pi^2 mN) \sin(k\pi Y)
= \sum B_{n,k} \exp\left[\frac{1}{2} \left( j - \frac{1}{2} \right) \pi^2 mN \right] \cos\left( j - \frac{1}{2} \right) \pi Y
\]
From Eqs. (23) and (24)
\[
B_{n,k} = \exp\left[\frac{1}{2} \left( j - \frac{1}{2} \right) \pi^2 (n + \phi)N \right] \left[ \frac{-\frac{1}{2}}{\pi^2} + \sum_{k} \frac{A_{n,k}}{\pi} \exp(-k^2\pi^2 (n + \phi)N) \sum_{k} \left[ \frac{1}{k + j - \frac{1}{2}} + \frac{1}{k - j + \frac{1}{2}} \right] \times \right]
\]
\[
A_{n,k} = \exp(k^2\pi^2 (n + 1)N) \left[ \frac{-\frac{1}{2}}{\pi^2} + \sum_{k} \frac{B_{n,k}}{\pi} \times \right]
\]
\[
\exp\left[\frac{1}{2} \left( j - \frac{1}{2} \right) \pi^2 (n + 1)N \right] \times \sum_{k} \frac{1}{k + j - \frac{1}{2}} + \frac{1}{k - j + \frac{1}{2}} \right] \times \quad (25)
\]
\[
\frac{1}{k + j - \frac{1}{2}} + \frac{1}{k - j + \frac{1}{2}} \right] \times \quad (26)
\]
By starting with Eq. (22), the coefficients \( \{A_{n,k}\} \) and \( \{B_{n,k}\} \) may be successively determined from Eqs. (25) and (26). The coefficients thus found are substituted in Eqs. (19) and (20), and then the mass transfer coefficient can be evaluated from Eq. (15). The solution \( \theta^* \) for the free water surface (\( \phi = 1.0 \)) is given by Eq. (19) with only the initial coefficients \( \{A_{n,k}\} \). Thus the ratio of the mass transfer coefficients may be calculated from the following equation
\[
k_x^* = \frac{\sum_{m=1}^{n} A_{n,k} \exp(-k^2\pi^2 mN) \exp(-k^2\pi^2 nM) - 1.0)}{\sum_{k} A_{n,k} \exp(-k^2\pi^2 mN) - 1.0} \quad (27)
\]
These equations were solved using the digital computer HITAC-5020E. The results for small \( X \) are presented in Fig. 4 and those for large \( X \) in Fig. 5.

It can be seen from these figures that the mass transfer coefficient does not change in proportion to the fraction of the wetted area. The fraction of the wetted area has little effect on the mass transfer coefficient at large \( n \) in the entrance region, and at small \( N \) in the region of large \( X \).

The entrance solution for the free water surface (\( \phi = 1.0 \)) should coincide with the solution by the penetration theory:
Table 1 Comparison of the solutions by the penetration theory \((n=1)\)

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>(\frac{kg}{kg_0})</th>
<th>asymptotic solution (Eq. 27)</th>
<th>penetration theory ((\sqrt{\phi}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.9481</td>
<td>0.9487</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.8932</td>
<td>0.8944</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>0.8347</td>
<td>0.8367</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.7719</td>
<td>0.7745</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.7036</td>
<td>0.7071</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.6280</td>
<td>0.6325</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.5423</td>
<td>0.5477</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.4405</td>
<td>0.4472</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.3080</td>
<td>0.3162</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Comparison of the solutions by the penetration theory \((\phi=1, \alpha=0.01)\)

<table>
<thead>
<tr>
<th>(n)</th>
<th>mass transfer rate: (-\int_0^{x_0} \frac{\partial \theta}{\partial Y} dX)</th>
<th>error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>asymptotic solution</td>
<td>penetration theory</td>
</tr>
<tr>
<td>1</td>
<td>0.1115</td>
<td>0.1123</td>
</tr>
<tr>
<td>2</td>
<td>0.1582</td>
<td>0.1596</td>
</tr>
<tr>
<td>3</td>
<td>0.1941</td>
<td>0.1954</td>
</tr>
<tr>
<td>5</td>
<td>0.2510</td>
<td>0.2523</td>
</tr>
<tr>
<td>10</td>
<td>0.3555</td>
<td>0.3568</td>
</tr>
<tr>
<td>15</td>
<td>0.4367</td>
<td>0.4370</td>
</tr>
<tr>
<td>20</td>
<td>0.5038</td>
<td>0.5046</td>
</tr>
<tr>
<td>25</td>
<td>0.5648</td>
<td>0.5642</td>
</tr>
<tr>
<td>30</td>
<td>0.6215</td>
<td>0.6180</td>
</tr>
</tbody>
</table>

The ratio of the mass transfer coefficients for \(n=1.0\) in the entrance region is given as:

\[
k_0^s = \frac{2 \sqrt{D \pi N}}{\bar{u}} (38)
\]

The results are shown in Fig. 6.
The model of the boundary film with linear velocity profile may be suitable in this case. For a small Schmidt number, on the other hand, the velocity within the boundary film can not be approximated by the linear profile, but the model of piston flow velocity profile may be used as a limiting case. When the behaviour of the fluid flow over the solid surface is not well understood, the model of the stagnant boundary film may be used for a rough estimate of the mass transfer coefficient. This stagnant film is also considered as a model for the diffusion from discontinuous surface in the lateral direction to the fluid flow.

From the results of the calculation for these models, it is shown that the relation of mass transfer coefficient to the fractional area of the discontinuous source is affected by the dimensionless parameters $N$, $n$ and $d/D$. Under the usual drying conditions, the particle diameter $D$ is sufficiently small compared with the thickness of the boundary film $\delta (D<<\delta)$, and the parameter $N$ is small and the parameters $n$ and $d/D$ are large. In such cases, it was shown that the decrease of fractional area of wetted surface has little effect on the mass transfer coefficient, and the mass transfer coefficient at the surface of the granular beds can be approximated by that of the free water surface (except in the case of small $\phi$). This result is considered as one of the reasons why the drying rate remains constant during the initial drying period of granular beds.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{x, z}$</td>
<td>coefficient</td>
<td>$-$</td>
</tr>
<tr>
<td>$B_{x, z}$</td>
<td>coefficient</td>
<td>$-$</td>
</tr>
<tr>
<td>$C$</td>
<td>concentration of vapor</td>
<td>$[kg/m^3]$</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of granular particle</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$D$</td>
<td>diffusion coefficient of vapor</td>
<td>$[m^2/hr]$</td>
</tr>
<tr>
<td>$H$, $h$</td>
<td>constants defined by Eq. (39)</td>
<td>$-$</td>
</tr>
<tr>
<td>$j$, $k$</td>
<td>number</td>
<td>$-$</td>
</tr>
<tr>
<td>$k$, $k'$</td>
<td>moduli of the elliptic functions</td>
<td>$-$</td>
</tr>
<tr>
<td>$l$, $l'$</td>
<td>moduli of the elliptic functions</td>
<td>$-$</td>
</tr>
<tr>
<td>$m$, $n$</td>
<td>number of the discontinuous surfaces</td>
<td>$-$</td>
</tr>
<tr>
<td>$N$</td>
<td>dimensionless parameter</td>
<td>$-$</td>
</tr>
<tr>
<td>$V$</td>
<td>dimensionless velocity</td>
<td>$-$</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity of the fluid</td>
<td>$[m/hr]$</td>
</tr>
<tr>
<td>$X$, $Y$</td>
<td>dimensionless length</td>
<td>$-$</td>
</tr>
<tr>
<td>$x$, $y$</td>
<td>length</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$a$, $\beta$</td>
<td>eigenvalues</td>
<td>$-$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>thickness of the boundary film</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>dimensionless concentration of vapor</td>
<td>$-$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>fractional area of wetted surface</td>
<td>$-$</td>
</tr>
</tbody>
</table>

**Literature cited**