CONSTANT PRESSURE FILTRATION OF POWER-LAW NON-NEWTONIAN FLUIDS

MOMPEI SHIRATO, TSUTOMU ARAGAKI, EIJI IRITANI,
MASAAKI WAKIMOTO, SATOSHI FUJIYOSHI,
AND SHÖJI NANDA

Department of Chemical Engineering, Nagoya University,
Nagoya 464

A filtration theory for the power-law non-Newtonian fluids is developed on the basis of the extended Rabinowitsch-Mooney equation, and several definitions of non-Newtonian filtration characteristics are defined. The equations presented in this paper may be considered as equations applicable both for non-Newtonian filtration and for conventional Newtonian filtration. Methods for evaluating the overall characteristics of non-Newtonian filtration are suggested by using the compression permeability data. In order to confirm the validity of the theory, constant pressure filtration experiments are carried out under various conditions of the flow behavior index N ranging from 0.404 to 0.504 and the filtration pressure p from 1000 to 3000 G/cm², and it is shown that the methods presented in this paper are valid. It is also shown that the average specific filtration resistance varies considerably with change in the N -value of the power-law, and the cakes formed from non-Newtonian filtration of pseudo-plastic fluid are denser than those from usual Newtonian filtration.

Introduction

In spite of the basic importance of non-Newtonian filtration in broad fields of petrochemical and food processing industries etc., very little has been studied in theories and experiments, while W. Kozicki *et al.*^{7,8)} have made valuable contributions to the filtration theory of power-law non-Newtonian fluids.

In this paper, the conventional filtration theory of Newtonian fluids at constant pressure is reexamined in view of the power-law for flow of non-Newtonian fluids. In order to provide a useful mathematical tool of simplified form to industrial filtration, a generalized theory which is applicable to both non-Newtonian and conventional Newtonian filtration is presented in this paper. It will also be demonstrated that the non-Newtonian filtration behaviors can be calculated on the basis of compression-permeability cell measurements and the estimated results are compared with constant pressure filtration experiments of pseudo-plastic non-Newtonian fluids.

1. Experimental Equipment and Procedures

The experimental filter, shown in Fig. 1, essentially consists of a plexiglass cylinder of 130 mm inside diameter, a brass upper plate with a connection for applying air pressure and a stainless-steel bottom

plate which supports a perforated plexiglass plate with a filter paper on it.

In order to conduct non-Newtonian filtration experiments, the selection of solid materials is important and it is essential that addition of non-Newtonian liquid to suspensions may not substantially affect particle flocculation, and that the viscous characteristics of the filtrate do not vary during filtration process¹⁷⁾. Several kinds of slurry materials (i.e., Gairome clay, Korean kaolin, calcium carbonate, fine silica sand, Filter-Cel, Standard-Supercel, Hyflo-Supercel and Radiolite) have been examined. The last two materials have proven to be appropriate in the above mentioned views and Radiolite (# 1100)* is used for the experiments attempted in this study.

The non-Newtonian fluids used in this study are

Fig. 1 Schematic diagram of experimental apparatus

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* Diatomaceous filter aid, Showa Kagaku Kogyo Co., Ltd.

aqueous solutions of sodium polyacrylate at concentrations of 0.2 to 0.3 weight percent. In order to dissolve sodium polyacrylate in deionized water, the aqueous solutions are agitated for about two days. Slurry is prepared by suspending Radiolite in a vacuum agitated vessel. Slurry concentration is 10.0% by weight through all runs.

Constant pressure filtration experiments are carried out by applying air pressures of 1.0 to 3.0 Kg/cm² and filtrate volume per unit area v vs. time θ relations are determined. The viscous flow characteristics of filtrate during filtration are measured by a cone-plate type rotational rheometer. The constancy of viscous characteristics of filtrate during filtration is confirmed by several measurements of filtrate obtained at different stages of a run.

2. Theoretical Analysis

On the assumption of no slip velocity at the walls of flow conduits with arbitrary cross-sectional shapes, the average flow rate U_e of a time independent non-Newtonian fluid can be represented by the extended Rabinowitsch-Mooney Equation^{4,6,7,9},

$$f(\tau_w) = a\tau_w \frac{d\left(\frac{2U_e}{r_H}\right)}{d\tau_w} + b\left(\frac{2U_e}{r_H}\right) \quad (1)$$

Where a and b are geometric constants depending on the cross-sectional shapes of the conduits, τ_w the shear stress at the wall, U_e the average flow rate, r_H the hydraulic radius and $f(\tau_w)$ is the shear rate at the wall which depends on shear stress τ_w . We are concerned here with pseudoplastic fluids described by the power-law model

$$g_c\tau = K(\dot{\gamma})^N \quad (2)$$

Therefore, shear rate $\dot{\gamma}$ is given by the form

$$f(\tau) \equiv \dot{\gamma} = K^{-1/N} (g_c\tau)^{1/N} \quad (3)$$

Where K is the fluid consistency index and N is the flow behavior index. When Rabinowitsch-Mooney Equation (1) is applied to the filtrate flow in a filter cake, U_e must be the real flow rate along with the actual sinuous path pursued by an element of the filtrate and can be related to the velocity U_x along with the opposite direction of cake growth by introducing the so-called tortuosity concept^{1,2}.

$$U_e = TU_x \quad (4)$$

Where T is the tortuosity, i.e., the ratio of the length of actual flow path to the depth of a bed. Equation (1) can easily be integrated to yield

$$\frac{2U_e}{r_H} = \frac{1}{a} \tau_w^{-b/a} \int_0^{\tau_w} \tau^{b/a-1} f(\tau) d\tau \quad (5)$$

Substituting Eqs. (3) and (4) into Eq. (5) and integrating results in

$$U_x = \left\{ \frac{(1+\xi)N}{1+\xi N} \right\} \left(\frac{r_H}{TK_o} \right) \cdot \left(\frac{g_c\tau_w}{K} \right)^{1/N} \quad (6)$$

where

$$K_o \equiv 2(a+b) \quad (7)$$

$$\xi \equiv b/a \quad (8)$$

Kozicki has pointed out that ξ , the aspect factor is 3.0 for usual granular beds same as for circular conduits, and for such unusual situations as beds of abnormal porosities and beds comprised of sharp-edged particles, however, it may have a possibility of values different from 3.0⁶. The impermeability K_o is related to Kozeny's constant k by^{1,2}

$$k = T^2 K_o \quad (9)$$

The force balance between the shear stress τ_w on solids and the hydraulic pressure loss dp_L through a differential length of actual flow path dx' and the differential cake thickness dx gives

$$g_c\tau_w = r_H \cdot \frac{g_c\partial p_L}{\partial x'} = \frac{r_H}{T} \cdot \frac{g_c\partial p_L}{\partial x} \quad (10)$$

where

$$r_H \equiv \frac{\varepsilon}{S_o(1-\varepsilon)} \quad (11)$$

In above equations, p_L is the local hydraulic pressure, S_o the effective specific surface of cake solids and ε is the local porosity at the distance x from the cake bottom.

It should be noted that U_x in Eq. (6) is the relative velocity of liquid with respect to migrating solids rather than the true velocity of liquid with respect to a fixed coordinate, since solid particles in cake migrate towards the filter medium due to cake compaction, and one gets

$$U_x = \frac{q_x}{\varepsilon} - \frac{r_x}{1-\varepsilon} = \frac{1}{\varepsilon} (q_x - er_x) \quad (12)$$

Where $e \equiv \varepsilon/(1-\varepsilon)$ is the local voidage, q_x the apparent velocity of liquid, r_x the apparent migration rate of solids in cake and the subscript "x" implies the local values at the distance x from the medium.

Substituting Eqs. (10), (11) and (12) into Eq. (6), and rearranging with the aids of Eq. (9) and the relation $dw_x = \rho_s(1-\varepsilon)dx$, in which ρ_s is the true density of solids, yields the basic flow equation for filtration of the power-law non-Newtonian fluids in the form

$$(q_x - er_x)^N = \frac{g_c\partial p_L}{K\gamma_x\partial w_x} = - \frac{g_c\partial p_s}{K\gamma_x\partial w_x} \quad (13)$$

where

$$\gamma_x \equiv \left\{ \frac{1+\xi N}{(1+\xi)N} \right\}^N \left\{ \frac{T\varepsilon^2}{kS_o(1-\varepsilon)} \right\}^{1-N} \cdot \alpha_x \quad (14)$$

$$\alpha_x \equiv \frac{T^2 K_o S_o^3 (1-\varepsilon)}{\rho_s \varepsilon^3} = \frac{k S_o^3 (1-\varepsilon)}{\rho_s \varepsilon^3} \quad (15)$$

γ_x is the local specific filtration resistance for the

Fig. 2 Compression test results

Fig. 3 Permeability test results

Fig. 4 Schematic diagram of cake structure

power-law non-Newtonian fluid and depends on both the flow behavior characteristics of fluids and the cake structure, α_x the local specific filtration resistance which has been conventionally defined for the Newtonian fluid filtration^{3,14}, w_x the mass of dry solids per unit area in the distance x from the filter medium and p_s is the local solid compressive pressure which can be related to the local hydraulic pressure by

$$dp_L + dp_s = 0 \quad (16)$$

or on integration

$$p_L + p_s = p \quad (17)$$

where p is the applied filtration pressure.

Since ϵ and α_x are functions of p_s alone as obtained from compression permeability cell measurements, S_0 can be calculated by using Eq. (15). Provided the values of ξ , T , and N are given, γ_x can be calculated as a function of p_s by using compression permeability cell data. Numerical calculations attempted in this paper are executed on the assumption of the value $\sqrt{2}$ of T recommended by Carman^{1,2} and the value 3 of ξ through a filter cake asserted by Kozicki⁵.

Figures 2 and 3 show the results of compression and permeability tests^{3,10,11} of Radiolite-Water suspension. It is postulated that the C-P cell data on the water suspension may serve as the basic tool for this

study, since the C-P cell measurements on non-Newtonian fluids are quite time-consuming and it appears that the existence of polymer does not substantially affect the degree of flocculation in the case of Radiolite suspended in sodium polyacrylate solutions. Furthermore, for executing the rigorous estimations of non-Newtonian filtrations under various conditions, it should be noted that C-P cell data on one non-Newtonian fluid can be applicable only to the filtration of the same one of the non-Newtonian fluid.

As the solid compressive pressure p_s is zero at the cake surface where $w_x = w$, and is a maximum value ($p - p_m$) at the cake bottom where $w_x = 0$ as shown in Fig. 4, integration of Eq. (13) over the entire cake yields

$$\begin{aligned} \int_0^1 \left(\frac{q_x}{q_1} - e^{-\frac{r_x}{q_1}} \right)^N d \frac{w_x}{w} &= - \frac{g_c}{K w q_1^N} \int_{p-p_m}^0 \frac{1}{\gamma_x} dp_s \\ &= - \frac{g_c}{K w q_1^N} \int_0^{p-p_m} \frac{1}{\gamma_x} dp_s \end{aligned}$$

and rearranging yields the analytical relation for filtration velocity in the form:

$$\begin{aligned} q_1^N &\equiv \left(\frac{dv}{d\theta} \right)^N \\ &= \frac{g_c (p - p_m)}{K \left[\left\{ \int_0^1 \left(\frac{q_x}{q_1} - e^{-\frac{r_x}{q_1}} \right)^N d \left(\frac{w_x}{w} \right) \right\} \cdot \left\{ \frac{p - p_m}{\int_0^{p-p_m} \frac{1}{\gamma_x} dp_s} \right\} \right] w} \\ &= \frac{g_c (p - p_m)}{K \gamma w} \quad (18) \end{aligned}$$

Where q_1 is the filtration velocity, v the filtrate volume per unit area, θ the filtration time, w the total mass of solids in cake per unit area, p_m the pressure drop through the medium and γ is the average specific filtration resistance for power-law non-Newtonian fluids, and is defined by

$$\gamma \equiv J \cdot \gamma_K = \int_0^1 \left(\frac{q_x}{q_1} - e^{-\frac{r_x}{q_1}} \right)^N d \frac{w_x}{w} \cdot \frac{p - p_m}{\int_0^{p-p_m} \frac{1}{\gamma_x} dp_s} \quad (19)$$

where

$$\begin{aligned} J &\equiv \int_0^1 \left(\frac{q_x}{q_1} - e^{-\frac{r_x}{q_1}} \right)^N d \frac{w_x}{w} \\ &= \frac{1}{1 - \epsilon_{av}} \int_0^1 \left(\frac{q_x}{q_1} - \epsilon \right)^N (1 - \epsilon)^{1-N} d \frac{x}{L} \quad (20) \end{aligned}$$

$$\gamma_K \equiv \frac{p - p_m}{\int_0^{p-p_m} \frac{1}{\gamma_x} dp_s} \quad (21)$$

In above equations, γ_K the average specific filtration resistance presented by Kozicki *et al.*⁷, J the correction factor by which γ_K should be multiplied to yield the true specific resistance γ , ϵ_{av} the average cake porosity, and L is the total thickness of cake.

In accordance with the so-called modern filtration

theory^{12,13,15,16}, q_x variation through cake may be negligibly small for filtrations of dilute slurries and low compressible materials as attempted in this study. Furthermore, the main objective of this study is to afford a useful mathematical tool for industrial practice for non-Newtonian filtration, and simplified equations of non-Newtonian filtration are presented in accordance with the conventional Ruth's concept for Newtonian filtration in which the filter medium is regarded as equivalent to a layer of cake¹⁰.

In accordance with Ruth's treatment of the total filtration resistance $= \alpha w + R_m = \alpha_{av}(w + w_m)$, the total resistance for non-Newtonian filtration can be written as $\gamma w + R_m = \gamma_{av}(w + w_m)$, where w_m is the fictitious cake mass¹⁰ which accounts for the medium resistance, and α_{av} and γ_{av} are the average specific filtration resistance for the cake and the medium and both are the functions of p , whereas α and γ are the functions of $(p - p_m)$. Under the conditions of the Ruth's treatment and the negligible q_x -variation, it is apparent from Eq. (20) that the correction factor becomes unity and the average specific resistance γ_{av} is

$$\gamma_{av} \equiv \frac{p}{\int_0^p \frac{1}{\gamma_x} dp_s} = \left\{ \frac{1 + \xi N}{(1 + \xi)N} \right\}^N \frac{p}{\int_0^p \frac{1}{\left\{ \frac{T \epsilon^2}{k S_o (1 - \epsilon)} \right\}^{1-N} \cdot \alpha_x} dp_s} \quad (22)$$

Figure 5 indicates the predicted values of average specific resistance based upon Eq. (22) and compression permeability cell measurements. It becomes apparent that the average specific resistances do vary remarkably with the change of flow behavior index N and are not susceptible to the pressure. This low susceptibility to pressure may be considered to come from the inherently low compressibility of slurry material used in this study.

In view of Eq. (18) of the filtration velocity, the filter medium resistance R_m for non-Newtonian filtration can be defined as

$$q_1^N \equiv \left(\frac{dv}{d\theta} \right)^N = \frac{g_c p_m}{K R_m} \quad (23)$$

Eliminating p_m in Eqs. (18) and (23), and solving for q_1 yields the filtration velocity equation of practical convenience analogous to Ruth's equation for Newtonian filtration in the form

$$q_1^N = \left(\frac{dv}{d\theta} \right)^N = \frac{g_c p}{K(\gamma w + R_m)} = \frac{g_c p}{K \gamma_{av}(w + w_m)} \quad (24)$$

where w is related to v by

$$w = \frac{\rho s}{1 - ms} v \quad (25)$$

In the above equations, s is the mass fraction of solids

Fig. 5 γ_{av} vs. N and p

in slurry, ρ the density of filtrate, and m is the ratio of wet to dry cake mass. Combining Eq. (24) with Eq. (25) and rearranging gives

$$\left(\frac{d\theta}{dv} \right)^N \equiv \left(\frac{1}{q_1} \right)^N = \frac{K \rho s \gamma_{av}}{g_c p (1 - ms)} \cdot (v + v_m) \quad (26)$$

Where v_m is the fictitious filtrate volume corresponding to the medium resistance defined by

$$v_m = \frac{1 - ms}{\rho s} \cdot w_m \quad (27)$$

On the assumption that m and γ_{av} are constant at a constant pressure filtration, integration of Eq. (26) results in

$$(v + v_m)^{(1+N)/N} = K_N (\theta + \theta_m) \quad (28)$$

where K_N is the non-Newtonian filtration coefficient under constant pressure and is defined by

$$K_N \equiv \frac{1 + N}{N} \cdot \left\{ \frac{g_c p (1 - ms)}{K \rho s \gamma_{av}} \right\}^{1/N} \quad (29)$$

and the fictitious filtration time θ_m corresponding to the medium resistance is defined by

$$\theta_m \equiv \frac{N}{1 + N} \left\{ \frac{K \rho s \gamma_{av}}{g_c p (1 - ms)} \right\}^{1/N} \cdot v_m^{(1+N)/N} = K_N^{-1} \cdot v_m^{(1+N)/N} \quad (30)$$

For filtration of slurries of dilute concentration and low compressible materials, $q_x \doteq q_1$ and $r_x \doteq 0$, and Eq. (13) can be written as

$$\left(\frac{dv}{d\theta} \right)^N \doteq \frac{g_c \partial p_L}{K \gamma_x \partial w_x} = - \frac{g_c \partial p_s}{K \gamma_x \partial w_x} = - \frac{g_c \partial p_s}{K \rho s \gamma_x (1 - \epsilon) \partial x} \quad (31)$$

Integrating Eq. (31) through a portion of cake thickness x and the total thickness L and combining yields

$$\frac{x}{L} = \frac{\int_{p_s}^p \frac{dp_s}{\gamma_x (1 - \epsilon)}}{\int_0^p \frac{dp_s}{\gamma_x (1 - \epsilon)}} = 1 - \frac{\int_0^{p_s} \frac{dp_s}{\gamma_x (1 - \epsilon)}}{\int_0^p \frac{dp_s}{\gamma_x (1 - \epsilon)}} \quad (32)$$

Fig. 6 p_s -distributions in cake

Fig. 7 Viscous characteristics of filtrate

Table 1 Predicted values of m

N	0.2	0.4	0.6	0.8	1.0
$P - P_m$					
500 G/cm ²	2.769	2.771	2.774	2.776	2.779
1000 "	2.712	2.715	2.718	2.721	2.725
3000 "	2.626	2.629	2.632	2.635	2.639
6000 "	2.575	2.578	2.581	2.584	2.588

It is interesting to note that ξ , K_0 and T , which are geometric constants of flow path, do not affect on p_s -distribution in cake as can be seen from Eqs. (32) and (14). Figure 6 represents p_s -distributions in cake calculated from Eq. (32) for various N values. The figure indicates that the solid compressive pressure in cake increases with decreasing the value N and consequently implies that the cake formed by non-Newtonian filtration are denser in cake structures than the cake by usual Newtonian filtration.

p_s -distributions being known, the porosity distribution and the average porosity ε_{av} can be determined, and the value of m can be calculated in accord with

$$m = 1 + \frac{\rho_s \varepsilon_{av}}{\rho_s (1 - \varepsilon_{av})} \quad (33)$$

Predicted values of m are tabulated for various values of pressures and flow behavior indexes in Table 1.

Variations of m are rather small for the changes in N and p due to the low compressibility of slurry material used.

3. Experimental Results and Discussions

Figure 7 illustrates the measurements of the viscous characteristics of filtrate by a cone-plate type rotational rheometer. The solid line in the figure represents the viscous characteristics for the actual filtrate and the dotted line is that for the original non-Newtonian solution not yet vitiated by solid materials. There exists some difference between the viscous characteristics of filtrate and the original non-Newtonian liquid. The difference may be due to adsorption of polymer in non-Newtonian liquid to some extent on solids surfaces and possibly to physico-chemical effects of a trace of some ions from solids¹⁷⁾. The values of N and K obtained by each measurement of the filtrate at four stages of a filtration run are also tabulated. The variations of both N and K of the filtrate are negligibly small, and it is safe to say that the viscous characteristics of filtrate are constant during a filtration run.

The experimental values of $(d\theta/dv)$ and $(d\theta/dv)^N$ for several runs are plotted with respect to v in Figs. 8 and 9. As expected from Eq. (26), $(d\theta/dv)^N$ vs. v shows a linear relationship except the initial stage of filtration. The deviation from the linear relationship being small, it can be concluded that Ruth's concept in which the medium resistance is regarded as equivalent to the fictitious cake¹⁰⁾ is a good first approximation for industrial non-Newtonian filtration.

From the abscissa intercepts of the straight lines in Fig. 9, v_m can be obtained. From the slopes of the lines, experimental values of average specific filtration resistances $(\gamma_{av})_{exp}$ can be calculated by Eq. (26) with the aid of m values. $(\gamma_{av})_{exp}$ thus obtained are tabulated in Table 2 together with the experimental values of p , N , K , Slopes and v_m , and the predicted values of γ_{av} and m . It can be seen from the table that the average specific filtration resistances γ_{av} decided by Eq. (22) on the basis of compression permeability cell measurements, are only 16% above than those determined experimentally, at the most. Since the pressure effects on γ_{av} are rather small in the case of this study, $(\gamma_{av})_{exp}$ vs. N are plotted in Fig. 10. The numerical values of 1000 to 3000 G/cm² written in the figure are the filtration pressure at which the constant pressure experiments are conducted. In Fig. 10, the theoretical values γ_{av} at $p=2000$ G/cm² are also illustrated by the solid line. As may be seen from the figure, it is apparent that the average specific resistance is affected markedly by a little variation in the flow behavior index N .

The time-volume relations which are determined by

Table 2 Filtration characteristics values under various conditions

p [G/cm ²]	N [—]	K [dyne·sec ^{N} /cm ²]	Slope [sec ^{N} /cm ^{$N-1$}]	v_m [cm]	$(\gamma_{av})_{exp}$ [cm ² - N /g]	γ_{av} [cm ² - N /g]	$\{\gamma - (\gamma_{av})_{exp}\} / \gamma$ × 10 ² [%]	m [—]
1000	0.483	8.75	2.489	4.16	2.03 × 10 ⁶	2.22 × 10 ⁶	8.6	2.716
1500	0.427	11.72	1.351	5.75	1.24 × 10 ⁶	1.37 × 10 ⁶	9.5	2.683
1500	0.437	10.72	1.475	5.38	1.48 × 10 ⁶	1.51 × 10 ⁶	2.2	2.683
1500	0.498	6.93	1.560	3.85	2.42 × 10 ⁶	2.77 × 10 ⁶	12.6	2.684
2000	0.404	17.58	1.192	5.74	0.98 × 10 ⁶	1.14 × 10 ⁶	14.7	2.660
2000	0.504	6.558	1.338	6.27	2.94 × 10 ⁶	3.10 × 10 ⁶	5.2	2.661
2500	0.435	12.60	0.983	7.39	1.41 × 10 ⁶	1.62 × 10 ⁶	13.2	2.643
3000	0.472	9.83	0.928	7.35	2.05 × 10 ⁶	2.42 × 10 ⁶	15.5	2.630

Fig. 8 $(d\theta/dv)$ vs. v

Fig. 9 $(d\theta/dv)^N$ vs. v

Fig. 10 $(\gamma_{av})_{exp}$ vs. N

Fig. 11 Filtrate volume per unit area vs. time

theoretical calculations and filtration experiments are compared in Fig. 11.

Conclusions

Constant pressure filtration theory for the power-law non-Newtonian fluid is developed on the basis of extended Rabinowitsch-Mooney equation. Several analytical relations, i.e. the fundamental flow equation, the newly defined average specific filtration resistance,

the distribution equation for solid compressive pressure in cake, and the constant pressure filtration equation are derived. All the equations presented in this paper reduce to the conventional filtration equations for Newtonian fluids by putting N as unity and K as the viscosity μ of Newtonians.

Numerical methods are also suggested for evaluating the non-Newtonian filtration characteristics by using compression permeability cell measurements. Filtration experiments are carried out under the conditions of various constant pressures and various viscous characteristics of a power-law non-Newtonian fluid, and it becomes apparent that the methods based upon compression permeability data are valid for non-Newtonian filtration.

In addition, it is shown that average specific resistance varies markedly with a change in the value of the flow behavior index N of the power-law fluid and denser cake are formed from non-Newtonian filtration of pseudo-plastic fluid than the cakes formed from usual Newtonian filtration.

It should be noted that the present study is confined to a slurry material of relatively low compressibility and further work on highly compressible ones are required for rigorous analysis of non-Newtonian filtration.

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Nomenclature

a, b	= geometric constants depending on cross-sectional shape of flow path	[—]
e	= local voidage	[—]
g_c	= conversion factor	[dyne/G]
J	= correction factor defined by Eq. (20)	[—]
K	= fluid consistency index	[dyne·sec ^{N} /cm ²]
K_N	= coefficient of constant pressure filtration	[cm ^{1+N} /sec]
K_o	= geometric constant defined by Eq. (7)	[—]
k	= Kozeny's constant, i.e., 5.0	[—]
L	= cake thickness	[cm]
m	= mass ratio of wet to dry cake	[—]
N	= flow behavior index	[—]
p	= applied filtration pressure	[G/cm ²]
p_L	= local hydraulic pressure	[G/cm ²]
p_m	= pressure loss across the filter medium	[G/cm ²]
p_s	= local solid compressive pressure	[G/cm ²]
q_1	= filtration velocity	[cm ³ /cm ² ·sec]
q_x	= local apparent velocity of filtrate	[cm ³ /cm ² ·sec]
R_m	= filter medium resistance	[cm ² ·sec]
r_H	= hydraulic radius	[cm]
r_x	= local migration rate of solid	[cm ³ /cm ² ·sec]
S_0	= effective specific surface of solid	[cm ² /cm ³]
s	= mass fraction of solid in slurry	[—]
T	= tortuosity	[—]
U_e	= velocity in actual sinuous flow path	[cm/sec]
U_x	= relative velocity of filtrate to solids along	

	with the direction of filter cake depth	[cm/sec]
v	= filtrate volume per unit area	[cm ³ /cm ²]
v_m	= fictitious filtrate volume per unit area	[cm ³ /cm ²]
w	= dry cake mass per unit area	[g/cm ²]
w_m	= fictitious dry cake mass per unit area corresponding to R_m	[g/cm ²]
w_x	= dry cake mass per unit area in distance x from the medium	[g/cm ²]
x	= distance from the medium	[cm]
x'	= length of actual sinuous flow path	[cm]
α_x	= conventional local specific filtration resistance for Newtonian fluid	[cm/g]
γ	= average specific filtration resistance defined by Eq. (19)	[cm ^{2-N} /g]
γ_K	= average specific filtration resistance presented by Kozicki <i>et al.</i> and defined by Eq. (21)	[cm ^{2-N} /g]
γ_x	= local specific filtration resistance	[cm ^{2-N} /g]
$\dot{\gamma}$	= shear rate	[1/sec]
ε	= local porosity	[—]
ε_{av}	= average porosity	[—]
θ	= filtration time	[sec]
θ_m	= fictitious filtration time corresponding to R_m	[sec]
ξ	= geometric constant defined by Eq. (8)	[—]
ρ	= density of filtrate	[g/cm ³]
ρ_s	= true density of solid	[g/cm ³]
τ	= shear force	[G/cm ²]
τ_w	= shear force at the wall of flow path	[G/cm ²]

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