THE ONSET CONDITIONS OF OSCILLATORY MOTION OF SINGLE GAS BUBBLES RISING IN VARIOUS LIQUIDS

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Many studies have been made about the behaviors of single gas bubbles ascending in various liquids, that is, the rising velocities or the shapes of single bubbles^{2,4,8,9)}. Single bubbles rise in a straight line when the bubble diameter is very small, and with increase of the bubble diameter, the path of the gas bubble turns to be zigzag or spiral.

On the condition of onset of zigzag or spiral motion, Saffman⁶⁾ and Hartunian and Sears³⁾ carried out experimental and theoretical work. Saffman⁶⁾ suggested that the zigzag motion was caused by the interaction between a periodic oscillation of the wake and the instability of the motion near the front of the bubble, and deduced that the value of the critical Weber number We_c at which instability appeared was 1.03. But this criterion does not agree with experimental results.

Hartunian and Sears³⁾ suggested that the interaction between surface tension and hydrodynamic pressure could result in instability of the bubble motion, and on a basis of the existing data and their own experimental results, they proposed that there were two distinct criteria for the instability:

1) for the impure and somewhat more viscous liquids, a critical Reynolds number $Re_c=202$;

2) for the pure, relatively low viscous liquids, a critical Weber number $We_c=3.18$.

But in their work the relations between the onset conditions of oscillation and physical properties of liquids are not clear.

The objects of this study are to determine experimentally the onset conditions of oscillation of single gas bubbles rising in various liquids and to propose a more general criterion for the onset conditions of oscillation.

1. Experimental

The experimental apparatus was almost the same one used for the previous work⁹⁾. The bubble column made of pyrex glass of 80 mm in diameter was installed on a brass base by sealing with purified mercury. In order to prevent the liquid from contamination, the

liquid was in contact only with glass, metal and purified mercury. Glass square tank was set to reduce the optical distortion in case of photographing.

The desired volume of air was injected with a micro-syringe, which was previously calibrated, into the bubble generation device and the air bubble was formed slowly from the glass nozzle in the liquid.

When the bubble ascended about 15 cm above the glass nozzle, the bubble was photographed against a black background by using a stroboscopic flash bulb placed above the liquid surface. The frequency of flash was 2600–3000 cycles/min.

10 bubbles of the same equivalent spherical diameter were photographed and the average rising velocity was determined. When the paths of more than 5 bubbles of the same equivalent spherical diameter taken on photographs were zigzag, its diameter and average rising velocity were considered as the critical diameter and the critical rising velocity.

Table 1 shows the liquids used for experiments and their physical properties were measured. The organic liquids used for experiments were reagent grade, and the distilled water was used in preparing each aqueous solution.

2. Experimental Results and Discussions

With the increase of the bubble diameter, the ascending path of single gas bubbles changes from a straight line to an oscillatory path. The experimental results, that is, the critical diameter d_c , rising velocity U_c , Reynolds number Re_c and Weber number We_c at which oscillatory motion appears are also shown in Table 1.

It is known that the motion of bubble ascending in a liquid is influenced considerably by the purity of liquid^{4,9)}. But, since the quantitative representation of the purity of liquid is not yet clear, whether the liquid used was pure or not was judged from the comparison of the shape of C_D vs. Re curve obtained in this experiment with that of the literatures^{4,8)}. As the result, the liquids used for this experiment except distilled water are considered to belong to the purified system. Distilled water showed an intermediate behavior between the purified system and the contaminated system.

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Table 1 Physical properties of liquids and critical values of onset of bubble oscillation

Liquid	Temp. [°C]	ρ [g/cm ³]	μ [cP]	σ [dyne/cm]	M [—]	d_c [cm]	U_c [cm/sec]	Re_c [—]	We_c [—]
Distilled water	21.0	0.998	0.981	72.5	2.39×10^{-11}	0.171	22.8	397	1.22
Methanol	22.2	0.784	0.618	22.5	1.60×10^{-10}	0.124	29.1	458	3.66
60% Methanol in water	22.3	0.894	1.50	31.3	1.81×10^{-9}	0.250	21.6	321	3.31
30% Methanol in water	22.2	0.951	1.69	41.0	1.22×10^{-9}	0.250	23.4	329	3.17
46% Glycerine in water	22.0	1.115	4.50	68.1	1.14×10^{-8}	0.292	25.5	185	3.11
48% Glycerine in water	20.0	1.110	5.19	67.7	2.07×10^{-8}	0.319	26.0	178	3.54
50% Glycerine in water	23.0	1.125	5.21	66.2	2.21×10^{-8}	0.325	25.9	182	3.70
52% Glycerine in water	19.0	1.132	6.51	67.2	5.13×10^{-8}	0.363	26.6	168	4.33
Isoamyl alcohol	22.4	0.809	4.11	23.0	2.82×10^{-7}	0.350	19.8	136	4.80
60% Soybean oil in hexane	21.3	0.825	5.33	25.4	5.85×10^{-7}	0.337	21.0	110	4.85

Fig. 1 Relation between Re_c and M for purified and contaminated systems

To clarify the relation between the condition of onset of oscillation and the physical properties of liquids, Re_c and dimensionless group consisted of physical properties of liquid M are plotted in Fig. 1. Re_c is correlated within an accuracy of $\pm 15\%$ for the purified system as follows,

$$Re_c = 9.0 M^{-0.173} \quad (1)$$

Re_c are plotted against M using the literature data which are considered to belong to the purified system and they are well correlated with Eq. (1) as shown in Fig. 1.

We_c and Re_c of the foregoing data for the purified system are plotted in Fig. 2 as proposed by Hartunian *et al.*⁽³⁾. We_c is correlated within an accuracy of $\pm 25\%$ as follows,

$$We_c = 21.5 Re_c^{-0.32} \quad (2)$$

By the above comparison, it can be said that Eq. (1) or (2) is applicable for wider range of M , that is, $8.4 \cdot 10^{-11} < M < 5.9 \cdot 10^{-7}$ than the criterion suggested by Hartunian *et al.*⁽³⁾.

The relations of Re_c and M for contaminated system

Fig. 2 We_c vs. Re_c for purified system. The keys are same as in Fig. 1

Fig. 3 Relation between Re_c and M for water

in the existing literatures are also plotted in Fig. 1. The criterion proposed by Hartunian *et al.*⁽³⁾, that is, $Re_c = 202$ is applicable to the liquids of the contaminated system independent of M in the range $3.3 \cdot 10^{-11} < M < 5.5 \cdot 10^{-8}$.

Water is a peculiar liquid in the sense that it is very sensitive to contamination. It has been pointed out that the rising velocities of single bubbles ascending in the water scatter considerably⁽¹⁰⁾.

In Fig. 3 Re_c for water obtained in this experiment is plotted against M together with other researchers' data. Though some points are well represented by Eq. (1), and points for tap water and contaminated water lie near the line $Re_c = 202$, it can be said that the values of Re_c scatter between two lines.

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Nomenclature

C_D	= drag coefficient of bubble, $4dg/3U^2$	[—]
d	= equivalent spherical diameter of bubble	[cm]
g	= gravitational acceleration	[cm/sec ²]
M	= dimensionless parameter, $g\mu^4/\rho\sigma^3$	[—]
Re	= Reynolds number, $dU\rho/\mu$	[—]
U	= terminal velocity of bubble	[cm/sec]
We	= Weber number, $dU^2\rho/\sigma$	[—]
μ	= liquid viscosity	[g/cm·sec]
ρ	= liquid density	[g/cm ³]
σ	= liquid surface tension	[dyne/cm]

<Subscript>

c = critical

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MOTION OF DROPS IN NON-NEWTONIAN FLUID SYSTEMS AT LOW REYNOLDS NUMBER

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In previous work⁹⁾, motion and mass transfer of fluid spheres in non-Newtonian fluid systems were analyzed for the high Reynolds number region. The proposed theory agreed well with experimental data available in the literature. The purpose of this paper is to investigate motion of drops in non-Newtonian fluid systems for the low Reynolds number region. A semiempirical equation for the drag coefficient of drops in non-Newtonian fluid systems is presented.

For characterization of the non-Newtonian behavior of fluids, the power law model is used.

$$\tau_{ij}^i = -K \left(\frac{1}{2} \dot{\Delta}_k^m \dot{\Delta}_k^m \right)^{(n-1)/2} \dot{\Delta}_j^i \quad (1)$$

We consider that the drag coefficient of drops in non-Newtonian fluid systems is analogous to that in Newtonian fluid systems.

For the low Reynolds number region, the drag coefficient of drops in Newtonian fluid systems is given as^{2,8)}

$$C_{DN} = \frac{24}{Re_N} \frac{2X_N + 3}{3X_N + 3} \quad (2)$$

where

$$Re_N = \rho d U / \mu \quad \text{and} \quad X_N = \mu / \mu'$$

When the Reynolds number and the viscosity ratio in Eq. (2) are respectively replaced with those in power law fluid systems given by

$$Re_{NN} = \rho d^n U^{2-n} / K \quad (3)$$

and

$$X_{NN} = \left(\frac{KU^{n-1}}{d^{n-1}} \right) / \left(\frac{K'U'^{n'-1}}{d'^{n'-1}} \right) \quad (4)$$

we consider that the drag coefficient of drops in power law fluid systems is approximately predicted by Eq. (2). The Reynolds number in power law fluid systems as defined Eq. (3) is often used to discuss motion of non-Newtonian fluid^{6,12)}. The viscosity ratio defined as Eq. (4) is used by Marrucci *et al.*³⁾ and Shiotsuka and Kawase¹⁰⁾. When motion of Newtonian drops in non-Newtonian fluids is discussed, Eq. (4) becomes $K/\mu' \cdot (U/d)^{n-1}$ which was used by some investigators⁶⁾. Of course, when both external and internal fluids are Newtonian fluids, Eqs. (3) and (4) become the Reynolds number and the viscosity ratio in Newtonian fluid systems defined as Eq. (2), respectively.

The relationship of the ratio of the drag coefficient of drops in non-Newtonian fluid systems to that in Newtonian fluid systems and the non-Newtonian behavior of external fluids is shown in Fig. 1. A

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