HEAT TRANSFER UNDER SOLIDIFICATION OF LIQUID ON AGITATED VESSEL WALL

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The heat transfer under solidification of lauric and capric acids on the cooling wall of an agitated vessel is studied, and the over-all and convective heat transfer coefficients, respectively, are measured and correlated with the impeller Reynolds number. The over-all heat transfer coefficient decreases sharply as depositing solid layer forms, but the convective heat transfer coefficient is well correlated by the already reported correlation equation independently of existence of solid layer. That is, the convective heat transfer coefficient in the vessel with solid layer coincides with that in the vessel without solid layer.

Introduction

In the chemical industry, the deposition of polymer at the cooling wall of the reactor makes hard to control the polymerization reaction, because the deposited layer has a high thermal resistance even if it is very thin.

This paper deals with the heat transfer under solidification of lauric and capric acids at the cooling wall of an agitated vessel, and with the model analysis of thermal resistance in the solid layer with fluid film resistance at the steady state.

1. Model Analysis

The mechanism of heat transfer at the cooling wall of agitated vessel is shown schematically in Fig. 1. The over-all and the convective thermal conductances, i. e., heat transfer coefficients, are defined as, respectively

$$Q = U(\pi DH)(T_b - T_w) \quad (1)$$

and

$$Q = h_L(\pi D_L H)(T_b - T_{MP}) \quad (2a)$$

$$Q = (\lambda_s/\delta)(\pi D_{in} H)(T_{MP} - T_w) \quad (2b)$$

where $D_L = D - 2\delta$ and $D_{in} = (D_L - D)/\ln(D_L/D)$. Combination of Eqs. (1) and (2a, b) gives the following relation

$$U = \frac{1}{(1/h_L)(D/D_L) + (\delta/\lambda_s)(D/D_{in})} = (1 - \alpha) \left(\frac{D_L}{D} \right) h_L \quad (3)$$

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where

$$\alpha = (T_{MP} - T_w)/(T_b - T_w)$$

2. Experimental Apparatus and Procedures

The details of the agitated vessel, 16 cm diameter and 16 cm depth, is shown in Fig. 2, where the cooling wall is made of stainless steel and upper and bottom walls are adiabatically made of plastic plates. The impeller used, 15.2 cm diameter and 15.2 cm height, is an anchor agitator made of brass.

The heat flux at the cooling wall is calculated from the heat generation of an electric heater in the vessel after steady states is established. The bulk temperature is obtained to average arithmetically the local temperatures measured by seven copper-constantan thermocouples, four of which are placed on the surface of the anchor agitator and the others are in the central part of the vessel, as shown in Fig. 2. The temperature of the cooling wall surface is indirectly measured by six thermocouples inserted into the vessel wall. Four of them are near the agitated fluid side and the rest near the cooling water side as shown in Fig. 2. The measured temperatures at both sides are averaged respectively and extrapolated to give the wall surface temperature.

The agitated fluids used are lauric and capric acids, which become amorphous solids at a temperature below the respective melting point. Water used as a test fluid, in order to examine the performances of the experimental apparatus.

In a set of experimental runs, the first run is started up by pouring the test fluid into the vessel, and heating

Table 1 Fluid properties

	Lauric acid	Capric acid	Water
Thermal conductivity [kcal/m·hr·°C]	0.140	0.121	0.533 (40°C) 0.548 (50°C) —
Specific heat [kcal/kg·°C]	0.38 (solid) 0.515	0.32 (solid) 0.440	0.999
Viscosity [g/cm·sec]	0.0688 (50°C) 0.0537 (60°C)	0.0434 (50°C) 0.0288 (70°C)	0.00656 (40°C) 0.00549 (50°C)
Density [g/cm³]	0.8744 (41.5°C) 0.8707 (50.25°C)	0.8884 (35.05°C) 0.8773 (50.17°C)	0.9922 (40°C) 0.9981 (50°C)
Melting point [°C]	43.20	31.00	0.0

by an electric heater at the specified heating rate. At the same time, the impeller is started to agitate the fluid at the specified rotational speed. Next the cooling water, kept at a temperature below the respective melting points, flows through the cooling jacket. After the rate of solidification approaches zero and the thermal condition reaches steady state, the temperatures at each point are measured. The second run is continued to vary the heating rate and/or the rotational speed and/or the cooling water temperature. The rotational speed is varied in the range 40 to 120 rpm by the vari-speed motor.

For the measurement of solid layer thickness, the run is started in the same manner as mentioned above. After reaching steady state, the agitated fluid is quickly removed from the vessel and the solid layer remaining is peeled off from the wall, and its thickness is measured directly by slide calipers.

Another vessel is used additionally to examine the effect of vessel dimensions. The latter has 7 cm inner diameter and 7 cm depth, and the anchor agitator used is 6.2 cm diameter, 6.1 cm height and 1.0 cm width.

3. Fluid Properties

The properties of an agitated fluid are listed in **Table 1**, where thermal conductivity, specific heat and melting point were measured by authors, and the remaining data were quoted from International Critical Tables.

The thermal conductivity of liquid are measured by the transient response technique of a wire temperature at constant heating rate³⁾, and the specific heat of liquid is obtained from the temperature rising rate of the liquid in an adiabatic vessel with constant heating rate generator. The thermal conductivity of the solid is obtained from the measurement of solid layer thickness as shown in **Fig. 3**, where the thermal con-

Fig. 1 Schema of solid layer on cooling wall

Fig. 2 Agitated vessel with anchor agitator

Fig. 3 Relation between solid layer thickness and heat flux

ductivity is equal to the product of ordinate and abscissa. For a thicker solid layer than 2 mm, the thermal conductivity decreases gradually and the solid surface becomes very uneven.

4. Correlation of Experimental Data

First, the over-all heat transfer coefficient for lauric and capric acids is correlated as shown in **Fig. 4**, where the exponents of Prandtl number and viscosity ratio take constants of 1/3 and 0.14, respectively, according to the already reported result³⁾. This figure shows that the experimental data without solid layer lie on

Fig. 4 Correlation of over-all heat transfer coefficient with impeller Reynolds number

Fig. 5 Correlation of convective heat transfer Coefficient with impeller Reynolds number

Fig. 6 Effect of clearance on ϕ

the extrapolated line of water. However, the over-all heat transfer coefficient with solid layer decreases sharply as solid deposits on the cooling surface.

Secondly, the convective heat transfer coefficient, h_L , calculated from Eq. (3) is correlated by the usual dimensionless equation, i. e.,

* The exponent of viscosity ratio in Eqs. (4) and (5) is changed to 0.18 by Uhl, but in this experiment the effect of this difference can be neglected.

** The scraping heat transfer coefficient for lauric acid, $Pr \div 100$, is derived from the following equation³⁾ for the range $1700 < Re_d < 6000$.

$$Nu = 1.6(Re_d \cdot Pr)^{1/2}$$

$$\left(\frac{h_L D_L}{\lambda}\right) = \phi \left(\frac{d^2 N \rho}{\mu}\right)^a \left(\frac{c_p \mu}{\lambda}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \quad (4)$$

where the characteristic length in Nusselt number adopts the value of $D_L = D - 2\delta$ instead of the vessel diameter D . Figure 5 shows that the experimental data, except ones with thicker solid layer, well satisfy the following correlation equation for turbulent range by Uhl⁷⁾

$$\left(\frac{h_L D_L}{\lambda}\right) = 0.38 \left(\frac{d^2 N \rho}{\mu}\right)^{2/3} \left(\frac{c_p \mu}{\lambda}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \quad (5)^*$$

where the exponent "a" in Eq. (4) becomes 2/3 in the turbulent range. This figure means that the heat transfer coefficient with thin solid layer almost coincides with that without a solid layer in a vessel of $D - 2\delta$ diameter; that is, the convective heat transfer coefficient is expressed by Eq. (5) independently of the existence of a solid layer. But the convective heat transfer coefficient with the thicker solid layer appear to have a higher value, because the mechanism of heat transfer becomes scraping heat transfer with decreasing of the clearance between impeller tip and solid layer.

For the vessel with 7 cm inner diameter, the same results as Figs. 4 and 5 are obtained.

5. Discussion

As depositing a solid layer on a cooling surface, the apparent vessel diameter, D_L , and the clearance, $(D - d)/2 - \delta$, decrease to affect the numerical constant of 0.38 in Eq. (5). Examining the effect of the clearance, the correlation constant in Eq. (4), ϕ , is plotted against the ratio of impeller diameter to apparent vessel diameter, as shown in Fig. 6. This figure shows that the effect of the clearance on ϕ does not appear to be significant in the range of $0.87 < (d/D_L) < 0.98$. At the same time, two correlation curves of ϕ for the vessel without solid layer are expressed in Fig. 6, where the one is derived from the analogy relation by the authors²⁾

$$\phi = 0.15 \frac{(\beta D_L / d)^2}{\{\eta(\beta D_L / d) \ln(D_L / d)\}^{1/3}} \quad (6)$$

where

$$\eta = 1 + \exp[-10\{(D_L / d) - 1\}] \quad (7)^{1)}$$

and the another derived from the Table by Uhl⁷⁾. The experimental data appear to be correlated by these curves within experimental accuracy and to approach the scraping heat transfer coefficient** as d/D approaches unity. That is, this figures shows again that the convective heat transfer coefficient in the vessel with solid layer coincides with that in the vessel without solid layer, whose diameter equal to the apparent diameter, D_L .

In order to eliminate the clearance effect from the

Fig. 7 Relationship between j -factor and modified Reynolds number

correlation expression, the j -factor and modified Reynolds number are adopted in Fig. 7. The experimental data are expectedly well correlated by the following correlation equation derived from the analogy relation by authors^{2,4)}

$$j_H \left(\frac{1.4d}{\beta D_L} \right) = 0.121 \left(\frac{L' v_\theta \rho}{\mu} \right)^{-1/3} \quad (8)$$

where the characteristic length for the proximity impeller must be corrected by the factor given by Eq. (7), i. e.,

$$L' = L \cdot \eta = \left\{ \frac{D_L}{2} \ln \left(\frac{D_L}{d} \right) \right\} \cdot \eta \quad (9)^{1)}$$

Moreover, in this figure the data for the vessel of $D=7$ cm are plotted at the same time and also correlated well by Eq. (8). On the other hand, the experimental data well satisfy the correlation equation by Uhl also, but deviate from that of Nishikawa *et al.*^{6,7)}. However, this figure is not conclusive that both Eq. (5) by Uhl and Eq. (8) by authors are the best for anchor agitator, because the experimental data are near the transition range from turbulent to laminar, and because the extrapolation of analogy relation based on "law of the wall" in fully turbulent vessel²⁾ derives a higher heat transfer coefficient than that by Eq. (8) as shown in Fig. 7.

6. Concluding Remarks

Heat transfer under solidification of lauric and capric acids on the cooling wall of an agitated vessel is studied, and the over-all and convective heat transfer coefficients, respectively, are measured and correlated with the impeller Reynolds number. The over-all heat transfer coefficient decreases sharply as the solid layer deposits, but the convective heat transfer coefficient is well correlated by the already reported correlation equation independent of the existence of a solid layer. That is, the convective heat transfer coefficient in the vessel with a solid layer coincides

with that in the vessel without solid layer.

This experiment is useful as an experimental technique setting the condition of constant wall temperature at a complex geometrical wall in convective heat transfer.

Nomenclature

a	= power of Reynolds number	[—]
c_p	= specific heat	[cal/g·°C]
d	= impeller diameter	[cm]
D	= vessel diameter	[cm]
D_{in}	= logarithmic mean diameter ($= (D - D_L) / \ln(D/D_L)$)	[cm]
D_L	= apparent vessel diameter ($= D - 2\delta$)	[cm]
h_L	= convective heat transfer coefficient	[cal/cm ² ·sec·°C]
H	= vessel height	[cm]
j_H	= j -factor ($= (h_L / \rho c_p v_\theta) \cdot Pr^{2/3}$)	[—]
L	= characteristic length	[cm]
L'	= corrected characteristic length for proximity impeller by Eq. (9)	[cm]
N	= rotational speed	[sec ⁻¹]
Q	= heat transfer rate	[cal/sec]
Re_d	= $d^2 N \rho / \mu$	[—]
Re_G	= $L' v_\theta \rho / \mu$	[—]
T_b	= bulk temperature	[°C]
T_{MP}	= melting point	[°C]
T_w	= wall surface temperature	[°C]
U	= over-all heat transfer coefficient	[cal/cm ² ·sec·°C]
v_θ	= characteristic velocity	[cm/sec]
α	= parameter defined by $(T_{MP} - T_w) / (T_b - T_w)$	[—]
β	= correction factor defined by $2 \cdot \ln(D/d) / (D/d - d/D)$	[—]
δ	= solid layer thickness	[cm]
η	= correction factor defined by Eq. (7)	[—]
λ	= thermal conductivity of liquid	[cal/cm·sec·°C]
λ_s	= thermal conductivity of solid	[cal/cm·sec·°C]
μ	= viscosity at bulk temperature	[g/cm·sec]
μ_w	= viscosity at wall temperature	[g/cm·sec]
ρ	= density of liquid	[g/cm ³]
ϕ	= correlation constant in Eq. (4)	[—]

Literature Cited

- 1) Hiraoka, S., I. Yamada, K. Suzumura, A. Kawai, A. Isono and R. Ito: *Bull. Nagoya Inst. Tech.*, **27**, 243 (1975).
- 2) Hiraoka, S.: *Kagaku Kogaku no Shimpō* no. 9 "netsu process kogaku", Maki-shoten press (1975).
- 3) Mizushima, T., R. Ito, Y. Ikuta, S. Hiraoka, Y. Konaka and J. Watanabe: *Kagaku Kōgaku*, **31**, 1208 (1967).
- 4) Mizushima, T., R. Ito and S. Hiraoka: 5th CHISA Congress B1. 4 (1975).
- 5) Nagata, S.: "Mixing principle and applications", Kodansha-A Halsted Press Book (1975).
- 6) Nishikawa, M., N. Kamata and S. Nagata: *Kagaku Kogaku Ronbunshu*, **1**, 466 (1975).
- 7) Uhl, V. W. and J. B. Gray; "Mixing theory and practice", Academic Press (1966).
- 8) Uhl, V. W.; 3th CHISA Congress F 5.3 (1969).

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SHORT COMMUNICATIONS

AN EMPIRICAL FORMULA OF POWER CONSUMPTION IN RELATION TO CIRCULATION FLOW RATE IN AGITATED VESSELS

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There are many theories and much experimental work for power correlation. But these have never paid attention on total circulation flow in relation to power input. Recently we proposed a simple empirical formula for this problem based on flow similarity and dimensional consideration^{1,2)}. Confirmation of the empirical relation is attempted here.

1. Basic Relation

From the forgoing work³⁾, the following Eq. (2) is reduced, which means that, together with Eq. (1), the torque is not proportional to Q_d^2 but that Tg_e/Q_d^2 increases as Q_d decreases between similar impellers as shown in Fig. 1.

$$Tg_e = \rho \bar{\Delta T} Q_d \quad (1)$$

$$\frac{\bar{\Delta T}}{Q_d} = \frac{C_D}{4(1+\alpha)^2} \left(\frac{b}{b'} \right)^2 \left(\frac{1}{n_p b} \right) \left[\frac{(1-V'_{ei})^2}{16X^2} + \frac{1-2V'_{ei}}{2X} - 1 \right] \quad (2)$$

where

$$X = (1/4)(1 - V'_{ei} + V'_{ei} \ln V'_{ei}) \quad (3)$$

Let us assume for simplicity but without loss of generalized impellers of different diameters under baffled condition. Q_d decreases with decreasing diameter, but $\bar{\Delta T}$ relatively increases, as shown in Fig. 1. The decrease of torque with decreasing diameter is not

great. This is because the discharge flow from the smaller impeller induces more additional circulation flow owing to the higher level of $\bar{\Delta T}$ on the discharge flow⁴⁾. Then it is expected that the angular momentum of total circulation flow on the surface on which induced action of discharge flow has been completed, $\bar{\Delta T}_{(r=r_s)}$, tends not only to be similar between a small and a large impeller, but to be in simple functional relation in comparison with $\bar{\Delta T}$. This consideration is partly supported by experimental results in flow similarity; that is, the circulation flow field is almost similar in the region II, III and IV in Fig. 2, and similar parameters of total circulation flow rate and tank diameter^{1,2)}. Assuming flow similarity, expression for $\bar{\Delta T}_{(r=r_s)}$ corresponding to Eq. (2) would be

$$\bar{\Delta T}_{(r=r_s)} = C_D \frac{Q}{D} f_1 \quad (4)$$

where f_1 is some minor correction function. The conservation equation of angular momentum on the cylindrical surface $r=r_s$ is

$$Tg_e = \rho \bar{\Delta T}_{(r=r_s)} Q f_2 \quad (5)$$

where f_2 is a correction function for the momentum transfer due to Reynolds stress and/or laminar shear stress and would mainly be a function of d/D . If the previously proposed relation³⁾ is a good approximation of power correlation for turbulent region, the product $f_1 f_2$ is reduced to Eq. (6) and power number is given by Eq. (7) from Eqs. (4) and (5).

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