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MASS TRANSFER TO A ROTATING SPHERE IN A STREAM

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The local and average mass transfer coefficients to a rotating sphere were measured by electrochemical method, when the angle α between the axis of rotation and the direction of the uniform stream were both 45° and 90° . For measurements of the local mass transfer rates, circular isolated electrodes of 2 mm dia. were used, and the time averaged values and the variations of the local mass transfer rates were obtained by processing the signals from each isolated electrode during some revolutions. Moreover, on the basis of the measurements of average mass transfer coefficients, new empirical equations were obtained with which they have been correlated well for $\alpha=0^\circ$, 45° and 90° .

Introduction

The study on heat and mass transfer to a rotating sphere in a stream is important in the analysis of heat or mass transfer between the circumferential fluid and the particles suspended in it³⁾. Several studies have been reported on heat or mass transfer to/from a rotating sphere placed in a uniform stream. From a hydrodynamic point of view, these studies should be divided into two classes. When the angle α between the uniform stream and the axis of rotation of the sphere is equal to zero, the velocity profile around the sphere shows axially symmetry. On the other hand, when $\alpha \neq 0^\circ$, the velocity profile is completely three dimensional so that it becomes very difficult to study it theoretically. In the case of $\alpha=0^\circ$, Tanaka and Tago⁵⁾ suggested a semi-empirical equation for the average mass transfer coefficients, which could correlate the experimental results well. Also the present authors²⁾ obtained the local mass transfer rates by solving the laminar boundary layer equations, and the numerical results showed them in good agreement with the measurements. On the other hand, for $\alpha=$

90° , Noordsij and Rotte⁴⁾, and Tanaka and Tago⁵⁾ measured the average Sherwood numbers, and moreover Eastop¹⁾ obtained the average Nusselt numbers. However all these investigations have been no more than the measurements of average heat or mass transfer coefficients.

In the present work, the local mass transfer rates of a sphere rotating in a uniform stream were measured for $\alpha=90^\circ$ and 45° by means of the so-called electrochemical method. At the same time, also the average mass transfer coefficients were obtained to provide some discussions about the effects of α .

1. Experimental Apparatus and Procedure

The experimental apparatus is diagrammatically shown in **Fig. 1(a)**. Almost all the parts except for the test section are the same as that in our previous study²⁾. The test section used for $\alpha=45^\circ$ is illustrated in **Fig. 1(b)**.

In the previous investigation²⁾, the local mass transfer rates to an isolated electrode were stationary during a revolution of the sphere, since the velocity profile around the sphere was symmetric about the axis of rotation. However, because α is not zero in the present work, the velocity profile becomes three

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dimensional and then the rate of local mass transfer is considered to vary during a revolution. Thus it is necessary to determine the location of the isolated electrode during a revolution. **Figure 2** show the device for this purpose. A disk is attached at the end of the rotating shaft in which a hole is drilled. A phototransistor and luminous diode are set in the manner that the hole is put between them. When the disk is rotated, a pulse—called “pulse-1”—is generated during one revolution of the sphere. The location of the isolated electrode is determined by referring “pulse-1”. On the rotating disk also thirty-six holes are drilled along a different circumference, which is located at the inner part of the hole for “pulse-1”, to produce thirty-six pulses of the same interval per revolution. These pulses are named as “pulse-2” and used as triggering pulses to sample the AD-converted values.

Preliminary experiments were performed in which signals from the isolated electrode (i.e. the voltage drops in a standard resistor) were recorded by an electromagnetic oscillograph. The experiments, however, suggested the following two problems:

1) Since the separation point of the velocity profile around the sphere was located at about 76° from the forward stagnation point*, almost all the isolated electrodes passed through the wake region during one revolution, and the mass transfer rates in this region exhibited considerable fluctuation.

2) As will be mentioned later, there existed conditions under which Sh_l became oscillating over the whole area of the sphere.

In such cases where Sh_l shows different values during every revolution, the results, which are obtained by only one record during a revolution, are not considered to be meaningful. Then in order to get useful results, it is necessary to record the signals during several revolutions and to process them statistically. In the present work, a hybrid computer equipped with AD-converters were used to process the data. **Figure 3** shows the block diagram for the data processing. The signals from the isolated electrodes were amplified and recorded by a data recorder. At the same time, the two kinds of pulses mentioned above were also recorded in its other channels. After the signals were recorded during about 300 revolutions, they were reproduced and put into the computer to calculate the time averaged Sh_l and its coefficient of variation. The computer used was FACOM U-200 in Data Processing Center in Kyoto University.

The experiments were carried out at three kinds of stream Reynolds numbers for rotational Reynolds numbers between 500 and 55000. The physical prop-

* The detail about the separation point is referred to the previous investigation²⁾.

Fig. 1 Schematic diagram of experimental apparatus

Fig. 2 Assembly of rotating sphere and devices for determination of location of isolated electrode

Fig. 3 Block diagram of signal processing

Table 1 Properties of the electrolytes

Ferricyanide conc.	0.005	[mol/l]
Ferrocyanide conc.	0.005	[mol/l]
Potassium hydroxide conc.	0.5	[mol/l]
Density	1.024	[g/cm ³]
Viscosity	0.00897	[g/cm·sec]
Diffusivity of ferricyanide ion	7.45×10^{-6}	[cm ² /sec]

erties of electrolyte are summarized in **Table 1**.

2. Results and Discussions

2.1 The local Sherwood number: Sh_l

Figures 5–10 show examples of the profile of time averaged Sh_l and their coefficients of variations, C_v . C_v is defined as the following equation.

Fig. 4 Coordinate system

Fig. 5 Profile of Sh_i and C_v for $\alpha=90^\circ$

$$C_v = \sqrt{\frac{\sum_i (Sh_i)_i^2}{N} - \left(\frac{\sum_i (Sh_i)_i}{N} \right)^2} \bigg/ \frac{\sum_i (Sh_i)_i}{N} \quad (1)$$

where $(Sh_i)_i$ is Sh_i at the i -th sampling (i.e. the i -th revolution), and the total sampling number N is about 300. In the following discussions, the time averaged Sh_i ($= \sum_i (Sh_i)_i / N$) is denoted for convenience as Sh_i .

In order to prescribe the location of the isolated electrode, two angles, θ and φ , are introduced as illustrated in Fig. 4. θ is the angle between the axis of rotation and the line connecting the isolated electrode to the center of the sphere. φ is the circumferential angle, which is measured in the direction of rotation from the plane that contains the axis of rotation and is parallel to the free stream velocity. The total number of isolated electrodes used were nine for $\alpha=90^\circ$ from $\theta=0^\circ$ to 90° , and fifteen for $\alpha=45^\circ$

Fig. 6 Profile of Sh_i and C_v for $\alpha=90^\circ$

from $\theta=0^\circ$ to 157° , which were located in approximately the same interval of 11° .

Figures 5-7 show the measurements for $\alpha=90^\circ$ and $\theta=34^\circ$. The isolated electrode used here passed partly the wake region during one revolution. Figures 5(a) and (b) show the profile of Sh_i and C_v , respectively, at $Re_t=3860$. For comparison Sh_i in quiescent fluid are also presented on the ordinate in Fig. 5(a). At $Re_r=0$ (i.e. no revolution), Sh_i becomes minimum at the nearly symmetric position in each side of φ . The positions correspond to the separation points of the velocity profile over the still sphere. As the sphere begins to rotate, even if Re_r is still very small, the separation point in the positive side of φ vanishes. At the same time, the values of C_v over the wake region in the positive side of φ become gradually small as seen in Fig. 5(b). This can be interpreted as resulting from the stabilization of flow due to the revolution. On the other hand, another separation point in the negative side of φ is progressively shifted in the positive φ -direction and the minimum value of Sh_i is gradually increased as Re_r becomes larger. However this separation point also disappears at $Re_r \approx 2000$. At $Re_r=0$, Sh_i is maximum at $\varphi=0^\circ$ and rapidly decreases up to the separation point. As Re_r increases this maximum point moves in the positive φ -direction and the profile of Sh_i becomes nearly flat as Re_r becomes larger than 3470. As shown in Fig. 5(b), the region, where the laminar boundary layer is

Fig. 7 Profile of Sh_i and C_v for $\alpha=90^\circ$

Fig. 8 Profile of Sh_i and C_v for $\alpha=45^\circ$

considered to cover and then C_v is small, is transferred to positive φ -direction and in the remaining parts C_v grows with increase of Re_r . At $Re_r \approx 3470$, C_v becomes large over the whole portion of φ . This is interpreted in that the large disturbances are caused as the result of the interference between the uniform stream and the velocity due to the rotation. As will be mentioned

Fig. 9 Profile of Sh_i and C_v for $\alpha=45^\circ$

Fig. 10 Profile of Sh_i and C_v for $\alpha=45^\circ$

later, this values of Re_r (≈ 3470) nearly correspond to that at which the average Sherwood number becomes minimum. When Re_r grows larger, Sh_i increases rapidly and C_v again decreases. This is considered due to the fact that the velocity field around the rotating sphere is approaching to the condition where the rotation is predominant. In fact, Re_r being greater than 12700, Sh_i becomes nearly equal to the value that is obtained in the quiescent fluid (i.e. $Re_t=0$). Figures 6 and 7 represent the results at $Re_t=2060$ and 10000, respectively. The changes in the shape of the profile of Sh_i with Re_r are qualitatively the same as

Fig. 11 Experimental results of Sh_{av} of a rotating sphere for $\alpha=90^\circ$ and 45°

that in Fig. 5. However, the displacement rate of the maximum point of Sh_i is slower as Re_t increases and the lower limit at which Sh_i matches the value for $Re_t=0$ becomes larger. Especially at $Re_t=10000$, within the limit of the present experiment, Sh_i does not completely coincide with the Sh_i at $Re_t=0$.

Figures 8–10 show the results for $\alpha=45^\circ$ and $\theta=56^\circ$ at $Re_t=3880$, 2080 and 10300 , respectively. The whole changes of profile of Sh_i with Re_r are almost similar to that for $\alpha=90^\circ$. However, C_v decreases monotonously over the whole area of the sphere as Re_r increases.

The experimental results of Sh_i obtained by each isolated electrodes are integrated over the whole area of the sphere to calculate the average Sherwood numbers. The calculated values are in good agreement with that measured within the accuracy of $\pm 5\%$ at various Re_t and Re_r for both $\alpha=90^\circ$ and 45° .

2.2 The average Sherwood number: Sh_{av}

The measurements on Sh_{av} for $\alpha=90^\circ$ and 45° are

* When $\alpha=0^\circ$, Sh_{av} shows no minimum value. So that in this case $(Re_r)_{min}$ is defined as Re_r at which Sh_{av} becomes larger than that for $Re_r=0$.

Fig. 12 Relationship between Re_t and $(Re_r)_{min}$

Fig. 13 Effect of α on Sh_{av}

shown in Figs. 11(a) and (b), respectively. In the figures, the data plotted on the ordinate refer to Sh_{av} at $Re_r=0$. Here Sh_{av} reduces gradually as Re_r increases at constant Re_t . After being minimum, Sh_{av} grows again. The ratio of reduction of Sh_{av} is generally larger as Re_t is small, and for the same Re_t the ratio is slightly larger for $\alpha=90^\circ$ than that for $\alpha=45^\circ$. Let Re_r at which Sh_{av} becomes minimum expresses as $(Re_r)_{min}$. The relationship between $(Re_r)_{min}$ and Re_t is shown in Fig. 12 and is represented as the following equation.

$$(Re_r)_{min} = 0.195 \times 10^{1.05\alpha} Re_t^{(1.12-0.258\alpha)} \quad (2)$$

(α : radian)

When Re_t is about 3800, $(Re_r)_{min}$ is about 3500 and 3200 for $\alpha=90^\circ$ and 45° , respectively, which nearly coincide with Re_r where Sh_i becomes minimum. As pointed out by Tanaka and Tago⁵⁾ and in our previous work,²⁾ Sh_{av} for $\alpha=0^\circ$ showed no tendency to decrease as Re_r increased. The systematic comparison of Sh_{av} for these three cases of α is illustrated in Fig. 13 at $Re_t \approx 2000$ and 10000 . It should be noted that for $\alpha=90^\circ$ and $Re_t=2000$, Sh_{av} becomes smaller near $(Re_r)_{min}$ than that at $Re_t=0$. This phenomenon has

been indicated at first by Noordsij and Rotte⁴⁾. For $\alpha=90^\circ$ and at large Re_t , Sh_{av} cannot coincide with the value for $Re_t=0$ even at large Re_r within the limit of this study.

Tanaka and Tago⁵⁾ suggested following semi-empirical equation to correlate Sh_{av} for $\alpha=0^\circ$ and 90° .

$$(Sh_{av}-1)/Sc^{1/3}Re_t^{0.5}=\{A\pm B(a\omega/U_\infty)^{1.5}\}^{1/3} \quad (3)$$

(+: $\alpha=0^\circ$, -: $\alpha=90^\circ$)

where the constants A and B are obtained from the empirical equations of Sh_{av} for $Re_r=0$ and $Re_t=0$, respectively.

In the present experiments $A=0.185$ and $B=0.093$ in contrast to $A=0.0589$ and $B=0.0166$ obtained by Tanaka and Tago^{5)*}. As mentioned in our previous work²⁾, Eq. (3) could correlate the experimental results for $\alpha=0^\circ$ satisfactorily. The comparison between the experimental results for $\alpha=90^\circ$ and that calculated by Eq. (3) is represented in Fig. 14. The values of $a\omega/U_\infty$ where $(Sh_{av}-1)/Sc^{1/3}Re_t^{1/2}$ becomes minimum are quite different between the measurements and the calculations under the various conditions, especially for large Re_t . It seems difficult to express the experimental results satisfactorily by Eq. (3). It is clear from Figs. 11(a) and (b) that the curves joining Sh_{av} at each Re_t are nearly similar to one another. Then these curves would be fitted by some adequate transpositions. The following equation is empirical one under such consideration.

$$\frac{Sh_{av}-1}{Sc^{1/3}Re_t^{(0.67-0.0388\alpha)}}=(0.088+0.049\alpha-0.00242\alpha^2) \times \{Re_r/(Re_r)_{min}\}^{(0.48+0.0675\alpha)} \quad (4)$$

(for $1.0 \leq Re_r/(Re_r)_{min} \leq 60$)

$$\frac{Sh_{av}-1}{Sc^{1/3}Re_t^{(0.5+0.07\alpha)}}=(0.58-0.45\alpha+0.151\alpha^2) \times \{Re_r/(Re_r)_{min}\}^{-0.051\alpha} \quad (5)$$

(for $0.1 \leq Re_r/(Re_r)_{min} \leq 1.0$)

where α is represented as radian in these equations. The comparisons between the measurements and the calculations by Eqs. (4) and (5) are shown in Figs. 15 (a) to (c). As a whole, the measurements are correlated well with Eqs. (4) and (5). However, especially for $\alpha=0^\circ$ and 45° , there are some scattering in the range of $Re_r/(Re_r)_{min}$ between 1.0 and 3.0.

The theoretical analysis on the local mass transfer coefficient of the rotating sphere is a problem for a future paper.

Conclusion

The local Sherwood numbers Sh_l during a revolution of a rotating sphere in a uniform stream were measured for $\alpha=45^\circ$ and 90° by means of electro-

* The detail discussion about the constants is referred to in the previous work²⁾.

Fig. 14 Comparison between experimental results of Sh_{av} and Tanaka's equation

Fig. 15 Comparison between experimental results of Sh_{av} and Eqs. (4) and (5)

chemical method using circular isolated electrodes. As Re_r increases, the shape of the profile of Sh_l changes from that in stream predominant region to that in rotation predominant one, and at the boundary between them Sh_l fluctuates violently.

Sh_l for each isolated electrode was integrated over the whole area of the rotating sphere to calculate the average values. The calculations were in good agreement with Sh_{av} measured within an accuracy of $\pm 5\%$ under various conditions.

New empirical equations for Sh_{av} , Eqs. (4) and (5), were suggested for various α between 0° and 90° .

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Nomenclature

a	= radius of sphere	[cm]
C_v	= coefficient of variation	[—]

$\mathcal{D}$	= diffusivity	[cm ² /sec]	ω	= angular velocity	[sec ⁻¹]
k_{av}	= average mass transfer coefficient	[cm/sec]	ν	= kinetic viscosity	[cm ² /sec]
k_l	= local mass transfer coefficient	[cm/sec]			
N	= total sampling number	[—]			
Re_r	= rotational Reynolds number ($a^2\omega/\nu$)	[—]			
$(Re_r)_{min}$	= rotational Reynolds number at the minimum value of Sh_{av}	[—]			
Re_t	= stream Reynolds number (aU_∞/ν)	[—]			
Sc	= Schmidt number	[—]			
Sh_{av}	= average Sherwood number ($ak_{av}/\mathcal{D}$)	[—]			
Sh_l	= local Sherwood number ($ak_l/\mathcal{D}$)	[—]			
U_∞	= free stream velocity	[cm/sec]			
α	= angle of inclination	[deg. or rad.]			
φ	= meridian angle (in Fig. 4)	[—]			
θ	= longitudinal angle (in Fig. 4)	[—]			

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A NEW METHOD OF CONVERGENCE FOR SOLVING REACTING DISTILLATION PROBLEMS

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A new correction factor (η) for reaction rate has been defined. It is analogous to θ for correcting molal flow rates in the multi- θ method of convergence. Then a new calculation method with two such kinds of the independent variables as θ and η has been developed for solving reacting distillation problems at the steady state and at the unsteady state.

Convergence is obtained without difficulty in the case where vapor liquid equilibrium ratios may be formulated as a function of temperature only. However, in the case of calculations based on experimental data in the esterification systems, the values of $\Delta\eta$ and $\Delta\theta$ resulted from the Newton Raphson's method have some time a large number. Therefore the correction method for these by use of arctangent function has been proposed in order to avoid divergence.

This new calculation method is called "Multi θ - η method of convergence".

Introduction

The multi θ method of convergence has been proposed for solving multi stage distillation problems at steady state and at unsteady state by Holland⁴⁻⁸.

This method has been developed by use of a new correction factor (η) for the reaction rate, η is analogous to θ for correcting the molal flow rate in the multi θ method of convergence. A new method with such two kinds of independent variables as θ and η has been developed for solving reacting distillation problems.

This method is called the "Multi θ - η method of convergence".

This method is not only useful for steady state

problems, but also it is applicable for unsteady state problems by use of the implicit method.

1. Calculation Fundamentals

Specifications for the molal flow rate of distillate, waste and over-flow, the molal holdup of liquid, and liquid compositions are given by the multiplier θ_j as shown in Eq. (1) through Eq. (5)

$$B_{co} = \theta_{-1}(B/D)_a D_{co} \quad (1)$$

$$b_{ico} = \theta_{-1}(b_i/d_i)_a d_{ico} \quad (2)$$

$$u_{jico} = \theta_j(u_{ji}/d_i)_a d_{ico} \quad (3)$$

$$U_{jco} = \sum_{i=1}^c u_{jico} \quad (4)$$

$$x_{ji} = (u_{ji}/U_j)_{co}, \quad y_{ji} = K_{ji}(u_{ji}/U_j)_{co} \quad (5)$$

In order to derive the component material balance for the whole tower, the holdup of the vapor on each

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