INTRODUCTION

Increasing interest in filter-cake dewatering has been shown in industry. In expression of solid-liquid mixtures, dewatering proceeds rather rapidly at the beginning of expression and becomes slow with elapsed time. Especially in the later period, deliquoring proceeds at a very slow speed which is mainly dependent upon the creep effects of the solid materials in the mixture. In recent studies of constant-pressure expression, it has been shown that the whole process of expression of solid-liquid mixtures can be accurately analyzed by introducing the empirical creep constants \( B \) and \( \eta \) on the basis of an elaborate consolidation theory based upon the so-called Terzaghi-Voigt combined model. For a better understanding of the intrinsic nature of expression mechanisms based upon the Terzaghi-Voigt model, further experimentation on the creep constants of \( B \) and \( \eta \) may be needed.

The objective of this paper is the experimental investigation of the effects on the creep constants \( B \) and \( \eta \) of operating factors in expression operations.

1. Experiments

For studying the changes in the values of \( B \) and \( \eta \) due to applied pressure \( p \) and initial average void ratio \( e_{1,AV} \) of expression materials, expression experiments with both filter cakes and semi-solid materials are conducted under constant-pressure conditions. Three kinds of materials, Korean kaolin, 50 wt. % Mitsukuri Gairome clay-50 wt. % Standard Super-Cel mixture, and Solka Floe (BW-200), are used, their properties being tabulated in Table 1. Compression permeability cells (6.0 and 5.4 cm inside diameter) are used, and the decrease in thickness \((L_x-L)\) at a time \( t_G \) is measured by dial gauges. The expression pressure used in this work ranges from 0.4 up to 196 Kg-force/cm\(^2\).

2. Experimental Results and Discussion

Consolidation mechanisms occur in two phenomena of void decrease. In view of the fact that the decrease in a local void ratio consists of the so-called primary and secondary consolidation and that the rheological constitution can approximate to the

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CREEP CONSTANTS IN EXPRESSION OF COMPRESSIBLE SOLID-LIQUID MIXTURES

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| \( r_0 \) | radial distance from centerline to interface [cm] |
| \( U \) | characteristic velocity [cm/sec] |
| \( u \) | tangential velocity component [cm/sec] |
| \( u_\theta \) | tangential velocity component at interface [cm/sec] |
| \( v \) | normal velocity component [cm/sec] |
| \( x \) | coordinate along interface [cm] |
| \( y \) | coordinate normal to interface [cm] |
| \( y^* \) | value of \( y \), above which \( u=0 \) is assumed [cm] |
| \( \alpha \) | arbitrary constant, \( \equiv \frac{1}{2} \) [-] |
| \( \beta, \gamma, \varepsilon \) | parameters, defined by Eqs. (18) to (20) [-] |
| \( \delta \) | parameter, defined by Eq. (17) [cm] |
| \( \eta \) | similarity variable, defined by Eq. (6) [-] |
| \( \theta \) | angle of interface [rad] |
| \( \xi \) | longitudinal curvature [1/cm] |
| \( \phi \) | normalized concentration of solute, \( \equiv (\omega_{A,B}-\omega_A)/(\omega_{A,B}-\omega_{A,W}) \) [-] |
| \( \phi_0 \) | first order approximation of \( \phi \) [-] |

\( \omega_A \) = mass fraction of solute [\(-\)]

Literature Cited


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Table 1 Materials used

<table>
<thead>
<tr>
<th>Material</th>
<th>Korean kaolin</th>
<th>Mitsukuri Gaorome clay-50wt. % Standard Super-Cel Mixture</th>
<th>Solka Floc (BW-200)</th>
</tr>
</thead>
<tbody>
<tr>
<td>True density of solid, $\rho_s$ [g-mass/cm$^3$]</td>
<td>2.59</td>
<td>2.35</td>
<td>1.58</td>
</tr>
<tr>
<td>Equilibrium void ratio, $e_0$ [—]</td>
<td>(600 $\leq$ $p_s$ $\leq$ 10000)</td>
<td>(200 $\leq$ $p_s$ $\leq$ 12000)</td>
<td>(500 $\leq$ $p_s$ $\leq$ 10000)</td>
</tr>
<tr>
<td>Specific resistance, $\alpha$ [cm$^3$/g-mass]</td>
<td>$1.24 \times 10^{12}$ + $1.76 \times 10^9 p_s^{0.933}$</td>
<td>$5.14 \times 10^{12}$ + $1.65 \times 10^9 p_s^{1.765}$</td>
<td>$3.74 \times 10^{12}$ + $5.90 \times 10^9 p_s^{1.09}$</td>
</tr>
<tr>
<td>Initial average void ratio, $e_{av}$ [-]</td>
<td>0.576 to 1.74</td>
<td>1.90 to 2.54</td>
<td>0.595 to 5.54</td>
</tr>
<tr>
<td>Applied expression pressure, $P$ [g-force/cm$^2$]</td>
<td>$1.3 \times 10^5$ to $1.96 \times 10^5$</td>
<td>$1.3 \times 10^4$ to $2.0 \times 10^4$</td>
<td>$4.0 \times 10^2$ to $1.96 \times 10^5$</td>
</tr>
<tr>
<td>Average value of creep constants, $B$ [-]</td>
<td>$0.030$</td>
<td>$0.053$</td>
<td>$0.075$</td>
</tr>
<tr>
<td>$\gamma$ [sec$^{-1}$]</td>
<td>$5.16 \times 10^{-5}$</td>
<td>$4.76 \times 10^{-5}$</td>
<td>$3.47 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Terzaghi-Voigt model as formerly reported$^{4-6}$, the consolidation process for filter-cake expression can be represented by$^{4-6}$

$$U_c = \frac{L_1 - L}{L_1 - L_{wo}} \left[ 1 - \exp \left( - \frac{\pi^{0.5} C_r \theta_c}{4 \omega_0^2} \right) \right] + B \left[ 1 - \exp \left( - \gamma \theta_c \right) \right]$$

(1)

and for semi-solid material expression by$^{4-6}$

$$U_c = (1 - B) \left[ 1 - \sum_{N=1}^{8} \frac{8}{(2N-1)^{0.5}} \exp \left( - \frac{(2N-1)^{0.5} C_r \theta_c}{4 \omega_0^2} \right) \right] + B \left[ 1 - \exp \left( - \gamma \theta_c \right) \right]$$

(2)

where $U_c$ is the average consolidation ratio at a consolidation time $\theta_c$, $L_1$ the initial thickness when $\theta_c = 0$, $L$ the thickness at a time $\theta_c$, $L_{wo}$ the final thickness when $\theta_c = \infty$, $C_r$ the modified consolidation coefficient, $i$ the number of drainage surfaces, $\omega_0$ the total solid volume per unit sectional area, $\gamma$ the empirical constant with respect to the deliquoring rate due to creep effects, and $B$ is the empirical constant defined by

$$B = (L_1 - L_{wo})/L_1$$

(3)

In Eq. (3), $(L_1 - L_{wo})$ indicates the maximum liquid volume deliquored by secondary consolidation.

Generally, the time rates of secondary consolidation are much smaller than those of primary consolidation. In view of the fact$^{4-6}$ that $\gamma \ll C_r (i \pi /2 \omega_0^2)$, Eqs. (1) and (2) become approximately Eq. (4) when $\theta_c \gg 0$

$$U_c = 1 - B \cdot \exp (- \gamma \theta_c)$$

(4)

and the values of $B$ and $\gamma$ can be graphically determined from experimental data in the later stages of $\ln (1 - U_c)$ vs. $\theta_c$, as illustrated in Fig. 1. As may be seen from Fig. 1, the experimental values of $B$ depend upon both the applied pressure $p$ and the initial average void ratio $e_{av}$, whereas the values of $\gamma$ are substantially constant.

The experimental $B$ values are shown for various $p/p_c(e_{av})$ values in Fig. 2, where $p_c(e_{av})$ denotes the solid compressive pressure which yields the equilibrium void ratio of compressed cake $e_{av}$ in compression-permeability cell measurements. It can be seen from the figure that $B$ is substantially constant for $p/p_c(e_{av})$ larger than a limiting value. In Fig. 3, $p/p_c(e_{av})$ vs. $p$ for filter cake expression of three materials are shown when expressions are conducted.
Fig. 3 Theoretical \( p_p(e_{1,av}) \) values of filter cakes

under the same pressure as the filtration pressure. The \( p_p(e_{1,av}) \) values for filter-cake expression in industrial practice are larger than about 3 or 4, as may be seen from Fig. 3. Therefore, it can be safely concluded that the values of both \( B \) and \( \eta \) are approximately constant for practical expression operations, and they can be viewed as the expression characteristics of the materials themselves. The experimental values of \( B \) and \( \eta \) are also tabulated in Table 1.

Acknowledgment

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Nomenclature

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\begin{align*}
B &= \text{creep constant defined by Eq. (3)} \quad [-] \\
C_e &= \text{consolidation coefficient} \quad [\text{cm}^2/\text{sec}] \\
e_{1,av} &= \text{initial average void ratio of mixture at} \\
e_{\infty} &= \text{final equilibrium void ratio} \quad [-] \\
i &= \text{number of drainage surfaces} \quad [-] \\
L &= \text{thickness of compressed cake at a time } \theta_e \quad [\text{cm}] \\
L_0 &= \text{initial thickness of original cake when } \theta_e = 0 \quad [\text{cm}] \\
L_\infty &= \text{final thickness of compressed cake when } \theta_e = \infty \quad [\text{cm}] \\
p &= \text{applied expression pressure} \quad [\text{g-force/cm}^2] \\
p_s &= \text{local solid compressive pressure} \quad [\text{g-force/cm}^2] \\
p_s(e_{1,av}) &= \text{solid compressive pressure where the equilibrium void ratio of compressed cake } e_{1,av} \text{ is attained} \quad [\text{g-force/cm}^2] \\
U_c &= \text{average consolidation ratio} \quad [-] \\
\alpha &= \text{local specific resistance of cake} \quad [\text{cm/g-mass}] \\
\eta &= \text{creep constant} \quad [\text{sec}^{-1}] \\
\theta_e &= \text{consolidation time} \quad [\text{sec}] \\
\rho_s &= \text{true density of solid} \quad [\text{g-mass/cm}^3] \\
\omega_0 &= \text{total solid volume in original mixture per unit sectional area} \quad [\text{cm}] 
\end{align*}
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Literature Cited