r_0	= radial distance from centerline to interface	[cm]
U	= characteristic velocity	[cm/sec]
u	= tangential velocity component	[cm/sec]
u_s	= tangential velocity component at interface	[cm/sec]
v	= normal velocity component	[cm/sec]
x	= coordinate along interface	[cm]
y	= coordinate normal to interface	[cm]
y^*	= value of y , above which $u=0$ is assumed	[cm]
α	= arbitrary constant, $\equiv 1/2$	[—]
$\beta, \gamma, \varepsilon$	= parameters, defined by Eqs. (18) to (20)	[—]
δ	= parameter, defined by Eq. (17)	[cm]
η	= similarity variable, defined by Eq. (6)	[—]
θ	= angle of interface	[rad]
κ	= longitudinal curvature	[1/cm]
ξ	= variable, defined by Eq. (7)	[—]
ϕ	= normalized concentration of solute, $\equiv (\omega_{AS} - \omega_A)/(\omega_{AS} - \omega_{A\infty})$	[—]
ϕ_0	= first order approximation of ϕ	[—]

ω_A = mass fraction of solute

[—]

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CREEP CONSTANTS IN EXPRESSION OF COMPRESSIBLE SOLID-LIQUID MIXTURES

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Introduction

Increasing interest in filter-cake dewatering^{1,3,9)} has been shown in industry. In expression of solid-liquid mixtures, dewatering proceeds rather rapidly at the beginning of expression and becomes slow with elapsed time. Especially in the later period, deliquoring proceeds at a very slow speed which is mainly dependent upon the creep effects of the solid materials in the mixture. In recent studies³⁻⁶⁾ of constant-pressure expression, it has been shown that the whole process of expression of solid-liquid mixtures can be accurately analysed by introducing the empirical creep constants B and η on the basis of an elaborate consolidation theory based upon the so-called Terzaghi-Voigt combined model. For a better understanding of the intrinsic nature of expression mechanisms based upon the Terzaghi-Voigt model, further experimentation on the creep constants of B and η may be needed.

The objective of this paper is the experimental in-

vestigation of the effects on the creep constants B and η of operating factors in expression operations.

1. Experiments

For studying the changes in the values of B and η due to applied pressure p and initial average void ratio $e_{1,av}$ of expression materials, expression experiments with both filter cakes and semi-solid materials are conducted under constant-pressure conditions. Three kinds of materials, Korean kaolin, 50 wt. % Mitsukuri Gairome clay-50 wt. % Standard Super-Cel mixture, and Solka Flocc (BW-200), are used, their properties being tabulated in Table 1. Compression permeability cells (6.0 and 5.4 cm inside diameter) are used, and the decrease in thickness ($L_1 - L$) at a time θ_e is measured by dial gauges. The expression pressure used in this work ranges from 0.4 up to 196 Kg-force/cm².

2. Experimental Results and Discussion

Consolidation mechanisms occur in two phenomena of void decrease^{2,4-8)}. In view of the fact that the decrease in a local void ratio consists of the so-called primary and secondary consolidation and that the rheological constitution can approximate to the

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Table 1 Materials used

Material	Korean kaolin	50wt. % Mitsukuri Gairome clay-50wt. % Standard Super-Cel Mixture	Solka Floc (BW-200)
True density of solid, ρ_s [g-mass/cm ³]	2.59	2.35	1.58
Equilibrium void ratio, e [—]	$4.52 - 0.369 \ln p_s$ ($600 \leq p_s \leq 10000$)	$5.01 - 0.340 \ln p_s$ ($200 \leq p_s \leq 12000$)	$9.94 - 0.938 \ln p_s$ ($500 \leq p_s \leq 10000$)
Specific resistance, α [cm/g-mass]	$1.24 \times 10^{12} + 1.76 \times 10^9 p_s^{0.922}$ ($600 \leq p_s \leq 10000$)	$5.14 \times 10^{10} + 1.65 \times 10^8 p_s^{0.766}$ ($200 \leq p_s \leq 12000$)	$3.74 \times 10^8 + 5.90 \times 10^5 p_s^{1.02}$ ($500 \leq p_s \leq 10000$)
Initial average void ratio, $e_{1,av}$ [—]	0.576 to 1.74	1.90 to 2.54	0.595 to 5.54
Applied expression pressure, p [g-force/cm ²]	1.3×10^8 to 1.96×10^5	1.3×10^8 to 2.0×10^4	4.0×10^2 to 1.96×10^5
Average value of creep constants, B [—]	0.030	0.053	0.075
η [sec ⁻¹]	5.16×10^{-5}	4.76×10^{-5}	3.47×10^{-5}

Terzaghi-Voigt model as formerly reported^{2,4-6}, the consolidation process for filter-cake expression can be represented by⁴⁻⁶

$$U_c \equiv \frac{L_1 - L}{L_1 - L_\infty} = (1-B) \left\{ 1 - \exp \left(-\frac{\pi^2 i^2 C_e}{4\omega_0^2} \theta_c \right) \right\} + B \{ 1 - \exp(-\eta \theta_c) \} \quad (1)$$

and for semi-solid material expression by⁴⁻⁶

$$U_c = (1-B) \left[1 - \sum_{N=1}^{\infty} \frac{8}{\pi^2 (2N-1)^2} \exp \left\{ -\frac{(2N-1)^2 i^2 \pi^2 C_e}{4\omega_0^2} \theta_c \right\} \right] + B [1 - \exp(-\eta \theta_c)] \quad (2)$$

where U_c is the average consolidation ratio at a consolidation time θ_c , L_1 the initial thickness when $\theta_c=0$, L the thickness at a time θ_c , L_∞ the final thickness when $\theta_c=\infty$, C_e the modified consolidation coefficient, i the number of drainage surfaces, ω_0 the total solid volume per unit sectional area, η the empirical constant with respect to the deliquoring rate due to creep effects, and B is the empirical constant defined by

$$B = (L_1 - L_\infty)_c / (L_1 - L_\infty) \quad (3)$$

In Eq. (3), $(L_1 - L_\infty)_c$ indicates the maximum liquid volume deliquored by secondary consolidation. Generally, the time rates of secondary consolidation are much smaller than those of primary consolidation. In view of the fact⁴⁻⁶ that $\eta \ll C_e(i\pi/2\omega_0)^2$, Eqs. (1) and (2) become approximately Eq. (4) when $\theta_c \gg 0$

$$U_c \doteq 1 - B \cdot \exp(-\eta \theta_c) \quad (4)$$

and the values of B and η can be graphically determined from experimental data in the later stages of $\ln(1-U_c)$ vs. θ_c as illustrated in Fig. 1. As may be seen from Fig. 1, the experimental values of B depend upon both the applied pressure p and the initial average void ratio $e_{1,av}$, whereas the values of η are substantially constant. The experimental B values are shown for various $p/p_s(e_{1,av})$ values in Fig. 2, where $p_s(e_{1,av})$ denotes

Fig. 1 $\ln(1-U_c)$ vs. θ_c Fig. 2 B vs. $p/p_s(e_{1,av})$

the solid compressive pressure which yields the equilibrium void ratio of compressed cake $e_{1,av}$ in compression-permeability cell measurements. It can be seen from the figure that B is substantially constant for $p/p_s(e_{1,av})$ larger than a limiting value. In Fig. 3, $p/p_s(e_{1,av})$ vs. p for filter cake expression of three materials are shown when expressions are conducted

Fig. 3 Theoretical $p/p_s(e_{1,av})$ values of filter cakes

under the same pressure as the filtration pressure. The $p/p_s(e_{1,av})$ values for filter-cake expression in industrial practice are larger than about 3 or 4, as may be seen from Fig. 3. Therefore, it can be safely concluded that the values of both B and η are approximately constant for practical expression operations, and they can be viewed as the expression characteristics of the materials themselves. The experimental values of B and η are also tabulated in Table 1.

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Nomenclature

B	= creep constant defined by Eq. (3)	[—]
C_e	= consolidation coefficient	[cm²/sec]
$e_{1,av}$	= initial average void ratio of mixture at	

	the beginning of consolidation	[—]
e_∞	= final equilibrium void ratio	[—]
i	= number of drainage surfaces	[—]
L	= thickness of compressed cake at a time θ_c	[cm]
L_1	= initial thickness of original cake when $\theta_c=0$	[cm]
L_∞	= final thickness of compressed cake when $\theta_c=\infty$	[cm]
p	= applied expression pressure	[g-force/cm²]
p_s	= local solid compressive pressure	[g-force/cm²]
$p_s(e_{1,av})$	= solid compressive pressure where the equilibrium void ratio of compressed cake $e_{1,av}$ is attained	[g-force/cm²]
U_c	= average consolidation ratio	[—]
α	= local specific resistance of cake	[cm/g-mass]
η	= creep constant	[sec⁻¹]
θ_c	= consolidation time	[sec]
ρ_s	= true density of solid	[g-mass/cm³]
ω_0	= total solid volume in original mixture per unit sectional area	[cm]

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