NUMERICAL SOLUTIONS FOR COMBINED FREE AND FORCED LAMINAR CONVECTION IN HORIZONTAL RECTANGULAR DUCTS BY CONJUGATE GRADIENT METHOD

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A numerical solution using the conjugate gradient method is presented for the heat transfer problems of fully developed combined free and forced laminar convection with high Prandtl number fluid in horizontal rectangular ducts, where the wall heat flux is axially uniform and the wall temperature is peripherally uniform. The S. O. R. method frequently fails to obtain a stable solution for large values of $ReRaPr$. However, the proposed method gives a stable converged solution even when the values of $ReRaPr$ are large.

Based on the numerical calculation, the relationships of Nu/Nu_0 vs. $ReRaPr$ for various aspect ratios are presented in a graph and by the following equation:

$$Nu/Nu_0 = a(ReRaPr)^b$$

where 0 denotes pure forced convection, and a and b are related to the aspect ratio and $ReRaPr$. In addition, the streamlines and isothermals are obtained for various aspect ratios and $ReRaPr$.

Introduction

Many attempts have been made to solve the heat transfer problems of combined free and forced laminar convection, which are important in the design of industrial heat exchangers and nuclear power plants. However, most attempts were related to a circular tube. Recently, based on the numerical method by finite difference approximation, Cheng *et al.*¹⁾ and Ou *et al.*⁸⁾ presented the relationships between $ReRaPr$ and Nu in horizontal and inclined rectangular ducts, respectively. The authors⁷⁾ also presented these relationships for both horizontal and inclined arbitrary triangular ducts with the boundary condition of axially uniform wall heat flux and peripherally uniform wall temperature, using finite difference approximations in the triangular coordinates system. However, the numerical method by the S. O. R. and the A. D. I. algorithm failed to obtain a converged solution at a large value of $ReRaPr$.

In order to obtain stable solutions at a large value of $ReRaPr$, a numerical solution by use of the conjugate gradient method is proposed.

Numerical calculations are presented for the problems of fully developed laminar flow in a horizontal rectangular duct for high-Prandtl number fluids with the boundary condition of axially uniform wall heat flux and peripherally uniform wall temperature at any axial position.

From the comparison of the calculation results of the present method with those of the S. O. R. method, the wide applicability of the present method is confirmed.

Thus, the relationships of Nu vs. $ReRaPr$, the streamlines and isothermals have been calculated for various aspect ratios. Furthermore, the application of this method is discussed.

1. Basic Equation

Let's now consider steady fully developed laminar flow in a straight horizontal rectangular duct. The coordinates system is shown in Fig. 1. The boundary conditions are herein assumed to be axially uniform wall heat flux and peripherally uniform wall temperature at any axial position.

Using the assumptions of Boussinesq approximation, constant axial pressure gradient, and non-viscous dissipation, the governing equations by the

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rectangular coordinates are as follows:

Continuity equation

$$\partial u / \partial x + \partial v / \partial y = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v + g\beta(t - t_w) \quad (3)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w \quad (4)$$

Energy equation

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} + w \frac{\partial t}{\partial z} = \alpha \nabla^2 t \quad (5)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The boundary conditions at wall are expressed as follows:

$$u = v = w = t - t_w = 0 \quad \text{at wall}$$

For the case of high Prandtl number, Eqs. (1)–(5) can be simplified and rearranged in the following dimensionless forms.³⁾

$$(\partial^2 w^* / \partial x^{*2}) + (\partial^2 w^* / \partial y^{*2}) = -1 \quad (6)$$

$$(\partial^2 \omega^* / \partial x^{*2}) + (\partial^2 \omega^* / \partial y^{*2}) = -(ReRaPr/w_m^*)(\partial \theta^* / \partial x^*) \quad (7)$$

$$(\partial^2 \phi^* / \partial x^{*2}) + (\partial^2 \phi^* / \partial y^{*2}) = -\omega^* \quad (8)$$

$$(\partial^2 \theta^* / \partial x^{*2}) + (\partial^2 \theta^* / \partial y^{*2}) = u^*(\partial \theta^* / \partial x^*) + v^*(\partial \theta^* / \partial y^*) + w^* \quad (9)$$

$$u^* = \partial \phi^* / \partial y^* \quad (10)$$

$$v^* = -\partial \phi^* / \partial x^* \quad (11)$$

where

$$x^* = x/D_e, \quad y^* = y/D_e, \quad u^* = u/(\alpha/D_e),$$

$$v^* = v/(\alpha/D_e), \quad w^* = w/(-)(D_e^4/\mu)(\partial p/\partial z),$$

$$\theta^* = (t - t_w)/(-)(D_e^4/\mu)(1/\alpha)(\partial p/\partial z)(\partial t_m/\partial z),$$

$$Ra = g\beta D_e^4(\partial t_m/\partial z)/\alpha\nu, \quad Re = D_e w_m/\nu, \quad Pr = \nu/\alpha \quad (12)$$

The boundary conditions at wall can be also rewritten as

$$u^* = v^* = w^* = \theta^* = \phi^* = 0 \quad (13)$$

2. Finite-Difference Approximations by Rectangular Coordinates System

The finite-difference approximations of Eqs. (6)–(9) by the rectangular coordinates system are expressed as follows:

$$w_{i,j}^* = A\{w_{i-1,j}^* + w_{i+1,j}^* + \gamma^2(w_{i,j-1}^* + w_{i,j+1}^*) + h^{*2}\} \quad (14)$$

$$\omega_{i,j}^* = A\{\omega_{i-1,j}^* + \omega_{i+1,j}^* + \gamma^2(\omega_{i,j-1}^* + \omega_{i,j+1}^*) + (h^* ReRaPr/2w_m^*)(\theta_{i+1,j}^* - \theta_{i-1,j}^*)\} \quad (15)$$

$$\phi_{i,j}^* = A\{\phi_{i-1,j}^* + \phi_{i+1,j}^* + \gamma^2(\phi_{i,j-1}^* + \phi_{i,j+1}^*) + h^{*2}\omega_{i,j}^*\} \quad (16)$$

Fig. 1 Rectangular coordinates system

$$\theta_{i,j}^* = A\{(1 + 0.5h^*u_{i,j}^*)\theta_{i-1,j}^* + (1 - 0.5h^*u_{i,j}^*)\theta_{i+1,j}^* + \gamma^2(1 + 0.5k^*v_{i,j}^*)\theta_{i,j-1}^* + \gamma^2(1 - 0.5k^*v_{i,j}^*)\theta_{i,j+1}^* - h^{*2}w_{i,j}^*\} \quad (17)$$

where

$$A \equiv 1/2(1 + \gamma^2), \quad h^* = h/D_e, \quad k^* = k/D_e$$

The central difference approximation forms of Eqs. (10) and (11) are

$$u_{i,j}^* = (1/2k^*)(\phi_{i,j+1}^* - \phi_{i,j-1}^*) \quad (18)$$

$$v_{i,j}^* = -(1/2h^*)(\phi_{i+1,j}^* - \phi_{i-1,j}^*) \quad (19)$$

Let's now define $f_{i,j}(\theta^*)$ by the following equation:

$$f_{i,j}(\theta^*) = (1 + 0.5h^*u_{i,j}^*)\theta_{i-1,j}^* + (1 - 0.5h^*u_{i,j}^*)\theta_{i+1,j}^* + \gamma^2(1 + 0.5k^*v_{i,j}^*)\theta_{i,j-1}^* + \gamma^2(1 - 0.5k^*v_{i,j}^*)\theta_{i,j+1}^* - 2(1 + \gamma^2)\theta_{i,j}^* - h^{*2}w_{i,j}^* \quad (20)$$

Then the finite difference approximation of Eq. (9) or (17) is expressed as

$$f_{i,j}(\theta^*) = 0 \quad (21)$$

Applying the method of Gosman *et al.*³⁾ to the boundary conditions of vorticity, the following equations can be obtained:

$$\omega_{i,1}^* = -\{3(\phi_{i,2}^* - \phi_{i,1}^*)/k^{*2} + \omega_{i,2}^*/2\} \quad (22)$$

$$\omega_{i,n+1}^* = -\{3(\phi_{i,n}^* - \phi_{i,n+1}^*)/k^{*2} + \omega_{i,n}^*/2\} \quad (23)$$

$$\omega_{1,j}^* = -\{3(\phi_{2,j}^* - \phi_{1,j}^*)/h^{*2} + \omega_{2,j}^*/2\} + (1/16) \times (ReRaPr/w_m^*)(-\theta_{3,j}^* + 4\theta_{2,j}^* - 3\theta_{1,j}^*)h^* \quad (24)$$

3. Computational Procedure of Conjugate Gradient Method

As is well known,³⁾ converged solutions by the S. O. R. method of Eqs. (14)–(19) and (22)–(24) may be obtained when the following conditions are satisfied: i) the sum of the absolute value of the coefficient for θ^* in Eq. (17) must be less than or equal to unity at every node of the mesh; ii) this sum must be less than unity at least on one mesh node; and iii) the value of the coefficient for θ^* must not vary too greatly from one cycle of iteration to another.

When $ReRaPr$ increases, however, the above con-

Fig. 2 Flowchart

dition cannot be satisfied because the values of $u_{i,j}^*$ and $v_{i,j}^*$ increase.

Thus, the S. O. R. method failed to obtain converged solutions, as suggested in the previous paper^{1,7)}.

To obtain a stable solution for the larger values of $ReRaPr$, the conjugate gradient method of Fletcher and Reeves²⁾ is herein used.

Let's now define the objective function $\phi(\theta^*)$ as follows:

$$\phi(\theta^*) = \sum_{j=2}^n \sum_{i=2}^m \{f_{i,j}(\theta^*)\}^2 \quad (25)$$

where

$$m = n/2 + 1$$

The value of $\phi(\theta^*)$ must be zero when the calculation has converged. However, $\phi(\theta^*)$ has greater value than zero, since the actual calculation is carried out by a finite number of significant figures.

Generally, $\phi(\theta^*)$ may have a single peak near a solution. Based on the consideration mentioned above, it will be concluded that the solution of Eqs. (14)–(19) with the associated boundary conditions, Eqs. (22)–(24), can be reduced to the following non-linear optimization problems:

$$\phi(\theta^*) \rightarrow \min.$$

subject to Eqs. (14)–(16), (18), (19)

and (22)–(24)

The flowchart of the conjugate gradient method for minimization of $\phi(\theta^*)$ is shown in Fig. 2.

Additional explanation is given as follows.

i) The calculation of $w_{i,j}^*$ and w_m^* . The values of $w_{i,j}^*$ are obtained by solving Eq. (14) by use of the S. O. R. method¹⁾ for convenience, though analytical solutions for the rectangular duct were presented by

Timoshenko *et al.*⁹⁾. Then, the value of w_m^* can be calculated by Simpson's rule.

ii) Calculation of initial values for $\theta_{i,j}^*$. In the algorithm of the conjugate gradient method, the choice of initial values for $\theta_{i,j}^*$ is important in obtaining a solution of good accuracy. The algorithm will be more successful¹⁰⁾ if the initial values of $\theta_{i,j}^*$ do not lie in a region where the function is ill-behaved. Therefore, it is hoped that the initial values for $\theta_{i,j}^*$ are laid as near as possible to the point giving the minimum of $\phi(\theta^*)$.

iii) The calculation of $u_{i,j}^*$ and $v_{i,j}^*$. In the calculation of $g, f_{i,j}(\theta^*)$ and $\phi(\theta^*)$, the values of $u_{i,j}^*$ and $v_{i,j}^*$ must be obtained as functions of $\theta_{i,j}^*$.

Substituting $\theta_{i,j}^*$ into Eqs. (15) and (24), the set of Eqs. (15), (16) and (22)–(24) can be solved by using the S. O. R. method^{1,7)}. Thus, $u_{i,j}^*$ and $v_{i,j}^*$ can be obtained from Eqs. (18) and (19).

iv) The calculation method of g . The gradient vector g with respect to $\phi(\theta^*)$ is defined by the following equation:

$$g = \frac{\partial \phi(\theta^*)}{\partial \theta_{k,l}^*} = \left\{ \frac{\partial \phi}{\partial \theta_{2,2}^*}, \frac{\partial \phi}{\partial \theta_{3,2}^*}, \dots, \frac{\partial \phi}{\partial \theta_{m,2}^*}, \frac{\partial \phi}{\partial \theta_{2,3}^*}, \frac{\partial \phi}{\partial \theta_{3,3}^*}, \dots, \frac{\partial \phi}{\partial \theta_{m,3}^*}, \dots, \frac{\partial \phi}{\partial \theta_{2,n}^*}, \frac{\partial \phi}{\partial \theta_{3,n}^*}, \dots, \frac{\partial \phi}{\partial \theta_{m,n}^*} \right\} \quad (26)$$

Accordingly, differentiation with respect to $\theta_{k,l}^*$ of Eq. (25) gives

$$g = \frac{\partial \phi(\theta^*)}{\partial \theta_{k,l}^*} = 2 \sum_{j=2}^n \sum_{i=2}^m f_{i,j}(\theta^*) (\partial f_{i,j}(\theta^*) / \partial \theta_{k,l}^*) \quad (27)$$

where $\partial f_{i,j}(\theta^*) / \partial \theta_{k,l}^*$ can be expressed from Eq. (20) as follows:

$$\begin{aligned} \frac{\partial f_{i,j}(\theta^*)}{\partial \theta_{k,l}^*} = & 0.5h^* u_{i,j}'^* (\theta_{i-1,j}^* - \theta_{i+1,j}^*) + 0.5k^* \gamma^2 v_{i,j}'^* \\ & \times (\theta_{i,j-1}^* - \theta_{i,j+1}^*) + (1 + 0.5h^* u_{i,j}'^*) \theta_{i-1,j}'^* \\ & + (1 - 0.5h^* u_{i,j}'^*) \theta_{i+1,j}'^* \\ & + \gamma^2 (1 + 0.5k^* v_{i,j}'^*) \theta_{i,j-1}'^* \\ & + \gamma^2 (1 - 0.5k^* v_{i,j}'^*) \theta_{i,j+1}'^* - 2(1 + \gamma^2) \theta_{i,j}'^* \end{aligned} \quad (28)$$

where the prime means differentiation with respect to $\theta_{k,l}^*$

$$u_{i,j}'^* = \frac{\partial u_{i,j}^*}{\partial \theta_{k,l}^*}, v_{i,j}'^* = \frac{\partial v_{i,j}^*}{\partial \theta_{k,l}^*}, \theta_{i,j}'^* = \frac{\partial \theta_{i,j}^*}{\partial \theta_{k,l}^*}$$

and

$$\begin{cases} \theta_{i,j}'^* = 1 & \text{at } i=k \text{ and } j=l \\ \theta_{i,j}'^* = 0 & \text{at } i \neq k \text{ or } j \neq l \end{cases} \quad (29)$$

The values of $u_{i,j}'^*$ and $v_{i,j}'^*$ in Eq. (28) can be obtained as follows.

First, the finite-difference approximations for $\omega_{i,j}^*$, $\phi_{i,j}^*$, $u_{i,j}^*$, $v_{i,j}^*$ and $\theta_{i,j}^*$ in Eqs. (15), (16), (18) and (19) and in the associated boundary conditions of Eqs. (22)–(24) are converted with differential expressions

with respect to $\theta_{k,l}^*$, i. e., $\omega_{i,j}^*$, $\phi_{i,j}^*$, $u_{i,j}^*$, $v_{i,j}^*$ and $\theta_{i,j}^*$, respectively.

Secondly, the conditions of Eq. (29) are applied to evaluating θ^{*} . The set of equations obtained by the above operation can be solved by use of the S. O. R. method. The results of $u_{i,j}^*$ and $v_{i,j}^*$ are independent of the values of $\theta_{i,j}^*$.

The results of $u_{i,j}^*$ and $v_{i,j}^*$ should be taken in four-dimensional variables for saving C. P. U. time if the machine has a great memory capacity.

Otherwise, these results should be stored temporarily in the file domain of supplementary memory equipment.

Accordingly, g can be calculated from Eqs. (27) and (28) by substituting the values of $u_{i,j}^*$ and $v_{i,j}^*$ and the results of $u_{i,j}^*$ and $v_{i,j}^*$ obtained from item iii).

v) The modification of θ^* The dimensionless temperature vector θ^* is corrected by

$$\theta^{(\nu)*} = \theta^{(\nu-1)*} + \lambda_{opt}^{(\nu-1)} d^{(\nu-1)} \quad (30)$$

where

$$d^{(1)} = -g^{(1)} \text{ at initial step} \quad (31)$$

$$d^{(\nu)} = -g^{(\nu)} + \frac{g^{(\nu)} g^{(\nu-1)}}{g^{(\nu-1)T} g^{(\nu-1)}} d^{(\nu-1)} \text{ at general routine} \quad (32)$$

The vector $d^{(1)}$ in Eq. (31) implies the steepest descent direction, and the vector $d^{(\nu)}$ in Eq. (32) implies the conjugate gradient direction. In Eq. (30), $\lambda_{opt}^{(\nu-1)}$ is obtained by linear search to minimize $\phi(\theta^{(\nu-1)*} + \lambda d^{(\nu-1)})$ with respect to λ . Quadratic or cubic interpolation is appropriate in the present problem since this problem has a large number of variables⁴⁾. Quadratic interpolation is used in this paper.

vi) The convergence criterion It is observed from Eqs. (15), (16), (18) and (19) that the absolute errors of u^* and v^* occurring from the small errors included in the solutions increase as $ReRaPr$ increases. Therefore, the value of $\phi(\theta^*)$ in the solutions becomes large for large values of $ReRaPr$. Thus it is difficult to estimate previously the minimum of $\phi(\theta^*)$. Therefore, the calculation is continued till it satisfies the convergence criterion by Jacoby *et al.*⁴⁾, that is, the computation end if $\phi(\theta^{(\nu-1)*}) - \phi(\theta^{(\nu)*}) < \varepsilon_1$ and $\|\lambda_{opt}^{(\nu-1)} d^{(\nu-1)}\| < \varepsilon_2$, where ε_1 and ε_2 are selected as 10^{-12} and 10^{-5} , respectively.

4. Discussion

Table 1 shows the values of Nu in the square duct for the various $ReRaPr$ calculated by the S. O. R. and the proposed method where the number of divisions, n , is taken to be 8. The initial values of $\theta_{i,j}^*$ for $ReRaPr > 7 \times 10^5$ are assigned by the solutions obtained from $ReRaPr = 0$, whereas for $ReRaPr > 7 \times$

Table 1 Comparison of the calculation results of the proposed method with the SOR method for a square duct ($n=8$)

Run no	$ReRaPr$	SOR method	Nu Proposed method
1	0	3.549	
2	10^4	3.581	3.586
3	5×10^4	4.058	4.056
4	10^5	4.548	4.546
5	2×10^5	5.167	5.167
6	4×10^5	5.888	5.886
7	7×10^5	6.538	6.539
8	10^6	diverge	6.982
9	2×10^6		8.342
10	4×10^6		10.044
11	6×10^6		11.199
12	10^7		13.115

Table 2 The influence of the number of divisions n on Nu in pure forced convection

γ	$n=8$	$n=20$	$n=32$	Marco <i>et al.</i>
1	3.5491	3.6060	3.6241	3.607949
2.0	4.0374	4.1256	4.1465	4.123303

10^5 the initial values of $\theta_{i,j}^*$ are assigned by the solutions just obtained from the previous run. The calculated values of Nu by both methods, as presented in Table 1, are very close to each other in the region of $ReRaPr$ where the converged solution can be obtained by the S. O. R. method. Table 1 also demonstrates that the proposed method provides a stable solution for the region of $ReRaPr$ where a converged solution cannot be obtained by the S. O. R. method.

Generally, the accuracy of the solution depends on the number of divisions n . For pure forced convection, $ReRaPr=0$, the values of Nu are listed for various γ and n in Table 2 in comparison with those by the analytical method of Marco and Han⁸⁾. The solutions for $n > 20$ agree fairly well with those by the analytical method. For $\gamma=0.5$, 1.0 and 2.0, the relations of Nu/Nu_0 vs. $ReRaPr$ obtained by three methods are given in Fig. 3. The method by Cheng *et al.*¹⁾ failed to obtain a converged solution for $ReRaPr > 10^5$, where their dimensionless forms in Eq. (6)–(12) are slightly different from those of this paper. In these relations, the curves become nearly straight lines in some regions of $ReRaPr$.

For the regions of straight-line representation of Nu/Nu_0 vs. $ReRaPr$, Nu/Nu_0 can be described by the equation

$$Nu/Nu_0 = a(ReRaPr)^b \quad (33)$$

where a and b are constants given by Table 3.

The variation of slope of the straight lines in Fig. 3 is explained by the streamline and isothermal patterns.

Fig. 3 Nu/Nu_0 vs. $ReRaPr$ for various aspect ratios γ in horizontal rectangular ducts

Fig. 4 Streamlines and isotherms for $\gamma=1$ and $ReRaPr=10^5$

Fig. 5 Streamlines and isotherms for $\gamma=1$ and $ReRaPr=10^7$

Figure 4 shows the streamlines and isotherms for $\gamma=1$ and $ReRaPr=10^5$. All of these lines are roughly elliptical. On the other hand, when $ReRaPr$ increases these lines deform from the elliptical, as shown in

Table 3 The values of a and b

γ	$Nu/Nu_0 = a (ReRaPr)^b$		Region of $ReRaPr$
	a	b	
2	0.2148	0.1435	$10^5 \sim 10^6$
	0.1262	0.1819	$10^6 \sim 10^7$
1	0.1775	0.1706	$6 \times 10^4 \sim 10^5$
	0.1304	0.1928	$10^5 \sim 2 \times 10^7$
	0.1062	0.2059	$2 \times 10^7 \sim 10^8$
0.5	0.1284	0.1889	$10^5 \sim 4 \times 10^7$
	0.0746	0.2211	$4 \times 10^7 \sim 10^8$

Fig. 6 Streamlines and isotherms for $\gamma=1$ and $ReRaPr=10^8$

Fig. 7 Streamlines and isotherms for $\gamma=2$ and $ReRaPr=10^7$

Fig. 5. Furthermore, when $ReRaPr$ approaches 10^8 , the lines shown in Fig. 6 deform remarkably, and the center of circulation for ψ^* is closer to the vertical wall. In addition, local circulation of opposite direction appears near the bottom wall, and local circulation for θ^* appears near the vertical wall. A similar phenomenon as in Fig. 6 is shown in Fig. 7, where $\gamma=2$ and $ReRaPr=10^7$. But when γ decreases the deformation is relaxed, as shown in Fig. 8. Therefore, the slope of straight line for $\gamma=0.5$ in Fig. 3 does not vary up to $ReRaPr=4 \times 10^7$. Consequently, the variation of slope in Fig. 3 is explained by the deformations and the appearance of local circulation for ψ^* and θ^* . In

the streamlines and isothermals for $\gamma=2$ and $ReRaPr=6 \times 10^7$ shown in Fig. 9, the main circulation with two centers and the local circulation with a center near the bottom wall are found for ψ^* , while local circulation near the vertical wall is found in the main circulation for θ^* . Therefore, deviation from the straight line at $\gamma=2$ and $ReRaPr > 10^7$ shown in Fig. 3 may be caused by those complex circulations.

It is observed from Figs. 4-9 that the gradients of ψ^* and θ^* become steeper near the bottom and the vertical wall as $ReRaPr$ increases. Therefore, the calculated results by $n=20$ may contain some errors in large values of $ReRaPr$.

Concluding Remarks

In combined free and forced laminar convection in horizontal rectangular ducts, a numerical solution using the conjugate gradient method was presented for heat transfer problems with high-Prandtl number fluid, where the wall heat flux is axially uniform and the wall temperature is peripherally uniform.

The S. O. R. method fails to obtain a converged solution for large values of $ReRaPr$, whereas the conjugate gradient method provides stable converged solutions even when the values of $ReRaPr$ are large.

From the numerical results, the relationships of Nu/Nu_0 vs. $ReRaPr$ for the various γ are shown in Fig. 3 and by Eq. (33).

Based on the streamline and isothermal patterns for various γ and $ReRaPr$ shown in Figs. 4-9, the variation of slope of the straight lines in Fig. 3 were explained. Though three centers of circulation appeared for $\gamma=2$ and $ReRaPr=6 \times 10^7$, further investigation of this interesting phenomenon are needed.

This method is widely applicable to the problems of flow and heat transfer in flow passages of other geometrical configurations.

Remarks

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Nomenclature

a	= width of rectangular duct	[m]
b	= height of rectangular duct	[m]
d	= direction vector for correction of θ^* given by Eqs. (31) and (32)	[—]
D_e	= equivalent diameter ($= 2ab/(a+b)$)	[m]
f	= function defined by Eq. (20)	[—]
g	= acceleration of gravity	[m/hr ²]
$\mathbf{g}$	= gradient vector defined by Eq. (26)	[—]
h, k	= grid space in x and y directions	[m]
m	= $n/2+1$	
n	= number of divisions in x and y

Fig. 8 Streamlines and isothermals for $\gamma=0.5$ and $ReRaPr=10^7$

Fig. 9 Streamlines and isothermals for $\gamma=2$ and $ReRaPr=6 \times 10^7$

	directions	[—]
p	= static pressure	[kg/mhr ²]
Pr	= Prandtl number	[—]
Ra	= Rayleigh number, $g\beta D_e^4(\partial t_m/\partial z)/\alpha\nu$	[—]
Re	= Reynolds number, $D_e w_m/\nu$	[—]
t	= temperature of bulk flow	[°C]
u, v, w	= velocity components in x, y and z directions	[m/hr]
x, y, z	= rectangular coordinates	[m]
α	= thermal diffusivity	[m ² /hr]
β	= coefficient of thermal expansion	[1/°C]
γ	= aspect ratio, a/b	[—]
ϵ_1, ϵ_2	= convergence criterion	[—]
θ	= $t - t_w$	[°C]
λ_{opt}	= parameter in Eq. (30)	[—]
μ	= viscosity	[kg/mhr]
ν	= kinematic viscosity	[m ² /hr]
ρ	= density	[kg/m ³]
ϕ	= objective function defined by Eq. (25)	[—]
ϕ_{min}	= converged value of ϕ	[—]
ψ	= stream function	[m ² /hr]
ω	= vorticity	[1/hr]

<Subscripts>

i, j	= space subscripts of grid point in a and b directions
k, l	= space subscripts of grid point in a and b directions

- b directions
 m = mean value at cross-section
 w = value at wall
 0 = condition for pure forced convection

<Superscripts>

- * = dimensionless form defined by Eq. (12)
 $'$ = differentiation with respect to $\theta_{k,l}^*$
 ν = ν -th convergence step

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KINETICS OF ABSORPTION OF NO INTO AQUEOUS Na_2SO_3 SOLUTIONS CONTAINING Fe(III)-EDTA-Na AND $\text{Fe}_2(\text{SO}_4)_3$

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The rates of absorption of NO into aqueous Na_2SO_3 solutions containing Fe(III)-EDTA-Na and/or $\text{Fe}_2(\text{SO}_4)_3$ as catalysts were measured at temperatures from 8 to 45°C using a baffled agitated vessel with a flat gas-liquid interface. The experimental results were analyzed by chemical absorption theory and it was found that the reaction is first-order with respect to NO and zero-order with respect to Na_2SO_3 for solutions containing Fe(III)-EDTA-Na alone or both Fe(III)-EDTA-Na and $\text{Fe}_2(\text{SO}_4)_3$. The first-order rate constants for both catalyzed solutions were calculated and correlated as functions of temperature, ionic strength of the solution and concentrations of Fe(III)-EDTA-Na and $\text{Fe}_2(\text{SO}_4)_3$.

Introduction

The removal of NO from flue gases emitted from stationary sources, such as power plants, by liquid absorbents is an important absorption process for control of air pollution. Several absorbents and absorption processes have been proposed for the removal of NO and some of them are in the development stage for practical application. Absorption of NO from flue gases into aqueous Na_2SO_3 solutions containing Fe(III)-EDTA-Na (ferric monosodium

ethylenediaminetetraacetate)²⁾ is one such absorption process.

The present work was undertaken to clarify the kinetics of the reaction between NO and the above solutions and to study the effect of adding $\text{Fe}_2(\text{SO}_4)_3$ to the above solutions.

1. Experimental

Absorption experiments were carried out in an agitated vessel with a flat gas-liquid interface. The agitated vessel was 12.3 cm I. D. and was the same as that used in the previous work^{5,8)}. The stirring speeds of the liquid and gas stirrers were 200 and 500 rpm,

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