INDUSTRIAL EXPRESSION EQUATION FOR SEMI-SOLID MATERIALS OF SOLID-LIQUID MIXTURE UNDER CONSTANT PRESSURE

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Recent developments in expression techniques may emphasize the need for a simpler method which is easily applicable to industrial data analyses, as well as a more accurate method of analysis which may lead us to a firmer understanding of the detailed mechanisms of entire expression processes. This paper is mainly concerned with the development of a simplified equation for constant-pressure expression of thickened semi-solid materials. The equation is based on the so-called modified Terzaghi equation and also on expression experiments, and may be viewed as a semi-theoretical one. Graphical methods for determining a newly defined value of the consolidation behavior index ν and the modified consolidation coefficient C_e are also presented.

Introduction

Deliquoring of filter cakes^{2,16,18)} has become increasingly important in a broad range of fields such as sewage sludge treatment, juice recovery processing^{5,6)}, fermentation and various chemical process industries. Various types of equipment for expression are available. Hydraulic presses of the batch type have been used for centuries. For continuous operation, the screw press¹⁴⁾, the disc press, and various types of belt presses are now in common use.

Körmendy^{3,4)}, and Shirato and Murase^{7-13,15)} developed theories of expression in which a mechanical piston is used to squeeze liquid from solid-liquid mixtures. A number of commercial designs have incorporated pneumatically or hydraulically actuated diaphragms to compress wet filter cakes. Recent developments in expression techniques emphasize the need for an easier method for practical applications, as well as a more accurate, analytical method to improve still further the accuracy of predicting the whole process of expression. The main objective of this paper is to develop easier equations for practical expression operations. Graphical methods for determining a newly defined value of the consolidation behavior index ν and the so-called modified consolidation coefficient C_e are also discussed.

1. Theoretical Background

1.1 Basis of new method

In general, the original mixture to be expressed may

be thick and may appear to be a semi-solid or a thickened slurry. In this paper, the constant pressure expression of semi-solid materials will be discussed.

The so-called modified consolidation equation can be derived by combining the equations of flow and of continuity on the basis of several assumptions as^{8,14)}

$$\frac{\partial p_s}{\partial \theta_e} = C_e \frac{\partial^2 p_s}{\partial \omega^2} \quad (1)$$

In Eq. (1), p_s is the local solid compressive pressure, θ_e the consolidation time, ω a variable for indicating an arbitrary position, i.e. a volume of solids per unit sectional area, lying between the medium and the position, and C_e is the so-called modified consolidation coefficient defined by Eq. (2) and is assumed to be constant.

$$C_e \equiv \frac{g_o}{\mu \rho_s \alpha_{av} (-de/dp_s)} \quad (2)$$

where α_{av} is the average specific resistance of cake^{8,14)}, μ the viscosity of liquid, and ρ_s is the true density of solids.

The solution of Eq. (1) for constant-pressure expression of a uniform semi-solid material is^{8,10)}

$$U_e \equiv \frac{L_1 - L}{L_1 - L_\infty} = 1 - \sum_{N=1}^{\infty} \frac{8}{\pi^2 (2N-1)^2} \exp \left\{ -\frac{(2N-1)^2 \pi^2}{4} T_e \right\} \quad (3)$$

$$= 1 - \sum_{N=1}^{\infty} \frac{8}{\pi^2 (2N-1)^2} \exp \left\{ -\frac{(2N-1)^2 \pi^2 i^2 C_e}{4 \omega_0^2} \theta_e \right\} \quad (3')$$

where U_e is the consolidation ratio, L the thickness of compressed cake at θ_e , L_1 the initial thickness, L_∞ the final thickness of cake when $\theta_e = \infty$, i the number of drainage surfaces, ω_0 the total solid volume of cake per

Received April 28, 1978. Correspondence concerning this article should be addressed to M. Shirato. K. Atsumi is at Dept. of Chem. Eng., Shizuoka Univ., Hamamatsu 432.

unit sectional area, and T_c is the dimensionless time factor defined by the following equation.

$$T_c \equiv C_e i^2 \theta_c / \omega_0^2 \quad (4)$$

Although Eq. (3) converges for all θ_c -values, the convergence is very slow for small θ_c -values. Consequently, an alternative solution, which is more suitable than Eq. (3) for small θ_c -values, is derived as¹⁾

$$U_c = \sqrt{\frac{4}{\pi}} \left\{ \sqrt{T_c} + \sum_{N=1}^{\infty} (-1)^N \int_0^{T_c} \frac{\exp(-N^2/T_c)}{\sqrt{T_c}} dT_c \right\} \quad (5)$$

or

$$= \sqrt{\frac{4C_e i^2}{\pi \omega_0^2}} \left\{ \sqrt{\theta_c} + \sum_{N=1}^{\infty} (-1)^N \times \int_0^{\theta_c} \frac{\exp(-N^2 \omega_0^2 / C_e i^2 \theta_c)}{\sqrt{\theta_c}} d\theta_c \right\} \quad (5')$$

Equation (3) or (5) may be useful for a rough estimation of constant-pressure consolidation. However, they deviate enormously from experimental data, especially in the later stages of consolidation¹²⁾. For a more rigorous estimation of consolidation, a new consolidation Eq. (6), based upon the Terzaghi and Voigt element, was derived as^{12,15)}

$$\frac{\partial p_s}{\partial \theta_c} + \beta \eta \frac{\partial}{\partial \theta_c} \int_0^{\theta_c} (p_s - p_{s-1}) \exp\{-\eta(\theta_c - \tau)\} d\tau = C_e \frac{\partial^2 p_s}{\partial \omega^2} \quad (6)$$

Equation (6) takes into account not only the primary consolidation but also the so-called secondary consolidation effect of solids, and its solution can be represented as¹²⁾

$$\begin{aligned} U_c &= (1-B) \left[1 - \sum_{N=1}^{\infty} \frac{8}{\pi^2 (2N-1)^2} \exp\left\{-\frac{(2N-1)^2 \pi^2}{4} T_c\right\} \right] \\ &\quad + B \left\{ 1 - \exp\left(-\frac{\eta \omega_0^2}{i^2 C_e} T_c\right) \right\} \quad (7) \\ &= (1-B) \left[1 - \sum_{N=1}^{\infty} \frac{8}{\pi^2 (2N-1)^2} \right. \\ &\quad \times \exp\left\{-\frac{(2N-1)^2 \pi^2 i^2 C_e}{4 \omega_0^2} \theta_c\right\} \left. \right] + B \{1 - \exp(-\eta \theta_c)\} \quad (7') \end{aligned}$$

In the above equations, C_e is revised as

$$C_e \equiv \frac{g_c}{\mu \rho_s \alpha_{av} (1-B) (-de/dp_s)_E} \quad (8)$$

where $(-de/dp_s)_E$ represents the compressibility due to the primary consolidation, B , β and η are creep constants, and τ is the dummy variable for denoting an arbitrary consolidation time up to a given elapsed time θ_c . Equations (7) or (7') can describe accurately the whole consolidation process¹²⁾. However, to execute the numerical calculation of U_c vs. θ_c , the determination of three experimental values of B , η and C_e necessitates two graphical plots of experimental data¹²⁾, which may not be so convenient for industrial

usage.

Recently, Sivaram and Swamee¹⁷⁾ have proposed the following algebraic Equation (9) of consolidation ratio:

$$U_c = \frac{\sqrt{\frac{4}{\pi} T_c}}{\left\{ 1 + \left(\frac{4}{\pi} T_c \right)^{2.8} \right\}^{1/5.6}} \quad (9)$$

where the dimensionless time factor T_c is defined by Eq. (4). Sivaram and Swamee have succeeded in approximating Eq. (3) within an error of 3%. Equation (9) is very simple in its form, and needs the determination of C_e -value alone for calculating the whole consolidation process. However, the so-called secondary consolidation effect has been neglected, and their equation naturally deviates from experiments, especially for large values of θ_c , as well as Eq. (3).

1.2 New equation

Sivaram and Swamee¹⁷⁾ have empirically determined Eq. (9) on the basis of the following empirical facts, i. e. $U_c = \sqrt{(4/\pi) T_c}$ when $U_c < 0.53$ and U_c approaches asymptotically to 1 when $T_c \gg 1$.

Generally, the theoretical U_c -curve based upon Eq. (3) closely follows experimental observations for low values of time θ_c , whereas it deviates remarkably from experiments at the later stages of expression, as may be seen from Fig. 1. In view of the foregoing facts, the consolidation equation for constant-pressure expression of semisolid materials should be consistent with the following three conditions.

(1) For small θ_c -values, the effect of the so-called secondary consolidation may be ignored, and the following relation can be analytically deduced by reference to Eq. (5).

$$U_c = \sqrt{\frac{4}{\pi} T_c}, \quad T_c \ll 1 \quad (10)$$

(2) For large θ_c -values, U_c approaches asymptotically to unity, i. e.

$$U_c = 1, \quad T_c \gg 1 \quad (11)$$

(3) For intermediate values of time, U_c increases more slowly than the prediction calculated by Eqs. (5) or (9) because of the secondary consolidation effect.

In view of the above three conditions, one may obtain Eq. (12), which is analogous to Eq. (9). In Eq. (12), the constant value of 2.8 in Eq. (9) is replaced by ν in order to take the secondary consolidation effect into account.

$$U_c = \frac{\sqrt{4T_c/\pi}}{\{1 + (4T_c/\pi)^\nu\}^{1/2\nu}} \quad (12)$$

or

$$= \frac{\sqrt{K_e \cdot \theta_c}}{\{1 + (K_e \cdot \theta_c)^\nu\}^{1/2\nu}} \quad (12')$$

where

Fig. 1 U_c vs. $\sqrt{\theta_c}$

$$K_e \equiv \frac{4i^2 C_e}{\pi \omega_0^2} \quad (13)$$

In Fig. 2, U_c is plotted against $\sqrt{T_c}$ for various values of ν by using Eq. (12). The U_c vs. $\sqrt{T_c}$ relation in Eq. (12) is approximately equal to that in Eq. (3) when $\nu = 2.8$, and it is apparent from experiments and theories that $\nu \leq 2.8$. It may be also apparent from Figs. 1 and 2 that the so-called secondary consolidation effect can be taken into account, provided $\nu < 2.8$ in Eqs. (12) or (12'). ν has been introduced to represent the secondary effect and has been named the consolidation behavior index.

1) Determination of experimental values of K_e and C_e In accordance with Eq. (12'), plotting the experimental U_c -values against $\sqrt{\theta_c}$ may yield a straight line for small values of θ_c , as shown in Fig. 3, and its slope is squared to give the experimental value of K_e .

Substituting the values of K_e and the experimental conditions into Eq. (13) yields the value of C_e which is defined by Eqs. (13) and (4).

2) Determination of the value of consolidation behavior index ν The values of K_e or C_e being known, the experimental data of U_c vs. $\sqrt{\theta_c}$ as illustrated in Fig. 3 can be reduced into the dimensionless plots of U_c vs. $\sqrt{T_c}$, as shown in Figs. 4 and 5. Then the experimental value of consolidation behavior index ν can be well determined by comparing directly the calculated curves shown in Fig. 2 with experimental

Fig. 2 U_c vs. $\sqrt{T_c}$ diagram

Fig. 3 Determination of K_e

data.

2. Experimental Verification

To confirm the validity of the new method, constant-pressure expression experiments of semi-solid materials of Korean kaolin and 50 wt.% Mitsukuri-Gairome clay-50 wt.% Standard Super-Cel mixture were conducted by using several compression permeability cells⁸⁾.

In Table 1 the values of K_e , C_e and ν obtained from the new method are tabulated together with the experimental values of C_e , B and η based upon the previous Terzaghi-Voigt model¹²⁾. In Fig. 1, the solid lines represent the calculated results based upon Eq. (12'), whereas the chain and dotted lines are based upon Eq. (3') and Eq. (7'), respectively. It is apparent from the figure that, in spite of their mathematical simplicity, Eq. (12) or (12') are more favorable

Table 1 Values of K_e , C_e , ν , B and η

p [g-force/cm ²]	$p_s(e_{1,av})$ [g-force/cm ²]	K_e [sec ⁻¹]	C_e^* [cm ² /sec]	ν [—]	C_e^{**} [cm ² /sec]	B^{**} [—]	η^{**} [sec ⁻¹]
a) Korean kaolin							
13.0×10^3	4.0×10^3	10.40×10^{-4}	1.07×10^{-4}	1.50	1.24×10^{-4}	0.030	5.36×10^{-5}
56.5 "	43.1 "	5.16 "	1.07 "	0.65	1.56 "	0.350	5.13 "
165.3 "	87.4 "	11.60 "	2.00 "	0.90	2.07 "	0.107	5.60 "
160.9 "	43.1 "	8.12 "	1.55 "	1.50	2.31 "	0.035	5.10 "
b) 50 wt% Mitsukuri-Gairome clay-50 wt% Standard Super-Cel mixture							
4.0×10^3	1.0×10^3	4.62×10^{-3}	2.19×10^{-3}	0.70	2.32×10^{-3}	0.052	4.92×10^{-5}
14.0 "	10.0 "	3.91 "	2.80 "	0.32	3.80 "	0.368	4.75 "
15.0 "	10.0 "	4.96 "	3.49 "	0.37	4.00 "	0.273	4.72 "
20.0 "	10.0 "	6.30 "	2.46 "	0.45	2.89 "	0.162	5.03 "

* obtained by the present method

** obtained by the fitting method (the Terzaghi-Voigt model)¹²⁾ Fig. 4 U_c vs. $\sqrt{T_c}$ (Korean kaolin) Fig. 5 U_c vs. $\sqrt{T_c}$ (50 wt% Mitsukuri-Gairome clay-50 wt% Standard Super-Cel mixture)

to experimental results than Eq. (3) or (3'), and are fairly comparable, in their accuracy, to Eq. (7) or (7') based on the elaborate Terzaghi-Voigt model.

In Fig. 6, experimental ν -values for Korean kaolin and Mitsukuri-Gairome clay-Standard Super-Cel mixture are plotted against $p/p_s(e_{1,av})$, where p is the applied expression pressure, $e_{1,av}$ the average void ratio of the mixture at the beginning of consolidation, and $p_s(e_{1,av})$ is the solid compressive pressure where final equilibrium void ratio $e_{1,av}$ is attained.

In the same figure, the experimental values of the creep constant $B^{12,16)}$ are also shown. As may be seen from Fig. 6, the experimental values of ν are approximately constant for each material when $p/p_s(e_{1,av}) \geq 3-4$ and ν can be viewed as creep characteristics of the materials themselves, just as the value of B is the most important creep factor in Eqs. (7) or (7')^{12,16)}. ν reaches its maximum value 2.8 only when

the material exactly follows the Terzaghi-model.

Conclusion

To afford a useful mathematical tool for industrial analysis of constant-pressure expression of thickened semi-solid materials, a simplified semi-theoretical equation is presented on the basis of the so-called modified Terzaghi equation and expression experiments.

1) In spite of the clear simplicity of the new equation, the calculated results by using the new equation are fairly coincident with experimental results and also to the calculated results based upon the complicated Terzaghi-Voigt model.

2) On the basis of the new equation, graphical methods for determining the modified consolidation coefficient C_e and the newly defined consolidation behavior index ν are presented.

Fig. 6 ν and B vs. $p/p_s(e_{1,av})$

3) Generally, the consolidation behavior index $\nu \leq 2.8$. It can be safely concluded that the values of ν are constants concerning the creep characteristics of the materials themselves under the condition where $p/p_s(e_{1,av}) \geq 3-4$.

Acknowledgment

This work was supported by the Science Research Foundation of the Education Ministry, Japan, Grant No. 285211, and by Takeda Science Foundation, Osaka, Japan. The authors wish to acknowledge their support leading to the publication of this article.

Nomenclature

B	= creep constant representing the ratio of consolidation by the so-called secondary consolidation to the total consolidation	[—]
C_e	= modified consolidation coefficient	[cm ² /sec]
e	= local void ratio	[—]
$e_{1,av}$	= average void ratio of mixture at the beginning of consolidation	[—]
g_c	= conversion factor	[dyne/g-force]
i	= number of drainage surfaces	[—]
K_e	= constant defined by Eq. (13)	[sec ⁻¹]
L	= thickness of compressed mixture at time θ_c	[cm]
L_1	= initial thickness of mixture	[cm]
L_∞	= final thickness of mixture when $\theta_c = \infty$	[cm]
p_s	= local solid compressive pressure	[g-force/cm ²]
$p_{s,1}$	= local solid compressive pressure when $\theta_c = 0$	[g-force/cm ²]
$p_s(e_{1,av})$	= solid compressive pressure where the final equilibrium void ratio $e_{1,av}$ is attained	[g-force/cm ²]
T_c	= consolidation time factor	[—]
U_c	= average consolidation ratio	[—]
α_{av}	= average specific resistance of cake	[cm/g]
β	= creep constant	[—]
η	= creep constant	[sec ⁻¹]
θ_c	= consolidation time	[sec]
μ	= viscosity of liquid	[g/cm·sec]
ν	= consolidation behavior index	[—]
ρ_s	= density of solids in mixture	[g/cm ³]
τ	= dummy variable for representing an arbitrary	

	consolidation time up to a given elapsed time θ_c	[sec]
ω	= variable for indicating an arbitrary position in cake, i.e. volume of solids per unit sectional area	[cm]
ω_0	= total solid volume of mixture per unit sectional area	[cm]

Literature Cited

- 1) De Jager, E. M., M. Van Den Tempel and P. De Bruyne: *Proc. Royal Aca. Amsterdam Ser.*, **B66**, 1 (1963).
- 2) Emmett, R. C., Jr. and D. A. Dahlstrom: *Chem. Eng. Progr.*, **68**, No. 1, 58 (1972).
- 3) Körmeny, I.: *J. Food. Sci.*, **29**, 631 (1964).
- 4) Körmeny, I.: *Acta Alimentaria*, **1**, 315 (1972).
- 5) Schwartzberg, H. G.: Proceedings of the Second Pacific Chem. Eng. Congress (PACHEC '77), Vol. 1, 675 (1977).
- 6) Schwartzberg, H. G., J. R. Rosenau and G. Richardson: *AIChE Symp. Ser.*, **73**, No. 163, 177 (1977).
- 7) Shirato, M., H. Kato, T. Murase and M. Shibata: *J. Fermentation Technology (Japan)*, **43**, 255 (1965).
- 8) Shirato, M., T. Murase, H. Kato and S. Fukaya: *Kagaku Kōgaku*, **31**, 1125 (1967).
- 9) Shirato, M., T. Murase, M. Negawa and T. Senda: *J. Chem. Eng. Japan*, **3**, 105 (1974).
- 10) Shirato, M., T. Murase, H. Kato and S. Fukaya: *Filtration and Separation*, **7**, 277 (1970).
- 11) Shirato, M., T. Murase, M. Negawa and H. Moridera: *J. Chem. Eng. Japan*, **4**, 263 (1971).
- 12) Shirato, M., T. Murase, A. Tokunaga and O. Yamada: *J. Chem. Eng. Japan*, **7**, 229 (1974).
- 13) Shirato, M., T. Murase and T. Aragaki: *Dechema Monographien*, Nr. 1432-1451, Bd. 74, 9 (1974).
- 14) Shirato, M., T. Murase, N. Hayashi, K. Miki, T. Fukushima, T. Suzuki, N. Sakakibara and T. Tajima: *Kagaku Kōgaku Ronbunshu*, **3**, 303 (1977).
- 15) Shirato, M., T. Murase, N. Hayashi and T. Fukushima: *J. Chem. Eng. Japan*, **10**, 154 (1977).
- 16) Shirato, M., T. Murase and N. Hayashi: Proceedings of the Second Pacific Chem. Eng. Congress (PACHEC '77), Vol. 1, p. 667 (1977).
- 17) Sivaram, B. and K. Swamee: *J. Japanese Soc. Soil Mech. Found. Eng.*, **17**, 48 (1977).
- 18) Tiller, F. M. and H. Risbud: First Pacific Chemical Engineering Congress, AIChE-Soc. Chem. Engrs., Japan, Vol. 1, 80 (1972).