have been interested in the vapor-liquid equilibria in systems containing water. The number of undiscovered combinations will be considerably reduced if experiments are intentionally made in order to fill blanks in Table 1, though all of them may not be discovered.

Nomenclature

<i>A</i>	= lowest-boiling component
<i>B</i>	= intermediate-boiling component
<i>C</i>	= highest-boiling component
<i>M</i>	= maximum azeotrope
<i>m</i>	= minimum azeotrope
<i>S</i>	= saddle-point azeotrope

S_1	= sum of indices of pure components
S_2	= sum of indices of binary azeotropes
S_3	= sum of indices of ternary azeotropes

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EFFECT OF DROP DIAMETER ON BREAKUP OF LIQUID DROP IN A SIMPLE SHEAR FIELD

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Dispersion operations in liquid-liquid systems are important in such separation processes as liquid-liquid extraction and ion exchange and in reaction processes such as emulsion polymerization. The mechanism of breakup of liquid drops has not yet been clarified sufficiently. Various factors seem to affect the breakup step of a liquid drop. The mechanism of breakup in a shear force is most important. Many studies were carried out by Taylor⁵⁾, Troza *et al.*⁶⁾, Bartok and Mason¹⁾, Cox²⁾ and Ito *et al.*⁴⁾ However, in these studies, the effect of drop size on breakup was only considered in the dimensionless shear rate G^* based on the linear theory. But the effects of the inertia terms cannot be neglected for large drops.

In the present research, the effect of drop diameter on breakup of liquid drops was investigated by introducing a dimensionless diameter. As the result it is shown that the dimensionless critical shear rate at burst is proportional to the square root of the dimensionless drop diameter in a range from 2.0×10^{-3} to 20.0×10^{-3} , which is equivalent to a range of real diameter from 0.5 to 5.0 mm.

1. Experimental Apparatus and Procedure

The schematic diagram of the apparatus is shown in Fig. 1. The apparatus used was similar to that used

by Flumerfelt¹⁾. A simple shear field without curvature was produced between two parallel belts moving in opposite direction. The endless belts were made from 2.0-mm rubber sheet. Each belt was tightly stretched over a driving roller and a subordinated roller. Supporting guide plates were installed to suppress the minute vibration of the belts. A variable-speed drive was separately connected to each sprocket.

As the fluid systems studied, castor oil was used for the continuous phase and various silicone oils were used for the dispersed phase. The viscosity ratio of the fluids of dispersed and continuous phases was changed in a range from 2.8×10^{-3} to 4.0. The interfacial tensions were measured by using the DuNoüy tensiometer. It was measured to be 4.0 dyne/cm for all

- ① Rotors attached to variable speed motor
- ② Belt (50 mm width)
- ③ Belt guide
- ④ Droplet

Fig. 1 Schematic diagram of apparatus

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combinations of castor oil and various silicone oils.

In the experiments, drops were placed between the belts with a micro syringe. The velocities of both belts were slowly accelerated. The deformation and breakup of the liquid drops were recorded photographically.

2. Experimental Results and Discussion

An example of the relation between the dimensionless critical shear rate at burst, G_B^* , and the dimensionless drop diameter, a^* , is shown in Fig. 2. The dimensionless drop diameter is defined as $a^* \equiv a/(\mu^2/\rho\sigma)$. Although Bartok and Mason²⁾ found in their experiments that dimensionless critical shear rate at burst was completely independent of drop diameter, a large dependence of dimensionless critical shear rate at burst on dimensionless drop diameter was observed in the present experiments. In their experiments the range of dimensionless drop diameter was from 4.5×10^{-7} to 35.7×10^{-7} , extremely small in comparison with those in this experiment, which ranged from 2.0×10^{-3} to 20.0×10^{-3} . In their experiments, the effect of the inertial terms of outer flow seemed to be neglected compared with the viscosity terms because of the extreme smallness of drop size, and the phenomena of drop breakup were explained well by an extension of the linear theory of drop deformation. In the present experiment, the effect of the non-linear terms cannot be neglected because of the comparative largeness of drop size. An attempt was made to correlate the relation between dimensionless critical shear rate at burst and dimensionless drop diameter by a power function of the latter, that is, $G_B^* = Aa^{*n}$. The relations are shown by the straight lines in Fig. 2. Furthermore, the relation between the power index, n , and the viscosity ratio, λ , is given in Fig. 3. In this figure, the lengths of error margin are given by subtracting the contribution factor of the regression line from unity, and the lengths are a measure of the reliability on the regression line. If a weighted average is calculated for the power indices n by taking the contribution factor into account, $\bar{n} = 0.56$ is obtained. However, for simplicity, when the relation between $G_B^*/\sqrt{a^*}$ and λ is plotted by assuming that $\bar{n} = 0.5$, a good correlation can be obtained as shown in Fig. 4.

Conclusion

The dimensionless critical shear rate at burst, G_B^* , is dependent not only on the viscosity ratio λ but also on the dimensionless drop diameter, a^* . A good correlation between $G_B^*/\sqrt{a^*}$ and λ can be obtained.

Nomenclature

A = regression coefficient defined by equation [—]
 a = diameter of undeformed droplet [cm]

Fig. 2 Variation of dimensionless critical shear rate at burst with dimensionless drop diameter

Fig. 3 Relation between power index n of regression line for G_B^* vs. a^* and viscosity ratio λ

Fig. 4 Relation between $G_B^*/\sqrt{a^*}$ and λ

a^* = dimensionless diameter of droplet
 $= (a\rho\sigma/\mu^2)$ [—]
 G = shear rate [1/sec]
 G^* = dimensionless shear rate $= (G\mu a/\sigma)$ [—]
 G_B^* = dimensionless critical shear rate value at

	burst	[—]
n	= power index of regression curve	[—]
$\bar{n}$	= mean value of n	[—]
λ	= viscosity ratio between dispersed and continuous phase (μ'/μ)	[—]
μ	= viscosity of continuous phase	[g/cm·sec]
μ'	= viscosity of dispersed phase	[g/cm·sec]
σ	= interfacial tension	[g/sec ²]
ρ	= density of continuous phase	[g/cm ³]

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THICKNESS OF LIQUID FALLING FILMS IN "CALMING ZONE" ON INCLINED PLATES

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The momentum integral equation taking into account the drag of air on a falling liquid film along inclined plates is solved. The velocity profile is assumed as laminar with the effect of air drag at the surface. The analysis predicts well the variation of film thickness with Re .

Film thickness variation has been studied numerically^{1,2)}, analytically for falling liquid films emerging from a flat slit^{2,7,8)} and free film flow at a vertical wall^{1,3,4)} and for non-Newtonian flow⁶⁾. In these studies the drag at air-liquid surface is not taken into account. Whitaker and Cerro⁵⁾ pointed out that the effect of gas-phase shear stress was negligible, after the simple reasoning based upon a standard boundary layer theory for the gas phase. However, the authors⁵⁾ measured the velocity profile in the liquid film by hydrogen bubble method and found that the drag was not negligible.

Equation (1) is a simplified form of the equation of motion,

$$-\frac{d}{dx} \int_0^h u^2 dy + u_s \frac{d}{dx} \int_0^h u dy + gh = \nu \left\{ \left(\frac{\partial u}{\partial y} \right)_w - \left(\frac{\partial u}{\partial y} \right)_s \right\} \quad (1)$$

and the continuity equation is

$$\int_0^h u dy = \frac{\nu}{4} Re \quad (2)$$

Equations (3) and (4) are dimensionless forms of

Eqs. (1) and (2), respectively.

$$-\frac{d}{dx_*} \int_0^{h_*} u_*^2 dy_* + h_* = \tau_{w*} - \tau_{s*} \quad (3)$$

$$\int_0^{h_*} u_* dy_* = \frac{Re}{4} \quad (4)$$

Equations (5), (6) and (7) are parts of the experimental results⁵⁾ under steady, fully developed flow conditions.

$$u_*(\eta) = \eta \left(u_{s*} + \frac{1}{2} h_*^2 \right) - \frac{1}{2} h_*^2 \eta^2 \quad (5)$$

$$-\tau_{s*} = \frac{K}{2(1-(1/2)K)} \tau_{w*} \quad (K=0.134) \quad (6)$$

$$h_* = Re^{1/3} \left/ \left(\frac{3}{4} - K \right) \right|^{1/3} \quad (7)$$

K indicates the degree of deviation from Nusselt's analysis. The assumption regarding velocity profile is modified as follows. Initial region ($0 \leq x_* \leq x_{1*}$) where a core region exists above boundary layer, and developed region ($x_{1*} \leq x_*$) where core does not exist, are settled. In initial region the boundary layer thickness $\delta_*(x_*)$ increases and the core depth $c_*(x_*)$ decreases with increasing x_* , and at $x_* = x_{1*}$, c_* becomes zero (see Fig. 1). For the initial region

$$u_*(x_*, y_*) = u_{c*}(x_*) \left\{ 2 \left(\frac{y_*}{\delta_*} \right) - \left(\frac{y_*}{\delta_*} \right)^2 \right\}, \quad 0 \leq y_* \leq \delta_* \quad (8)$$

$$u_* = u_{c*}(x_*), \quad \delta_* \leq y_* \leq \delta_* + c_* \quad (9)$$

$$u_* = n_{c*} \left\{ 2 \left(\frac{y_* - c_*}{\delta_*} \right) - \left(\frac{y_* - c_*}{\delta_*} \right)^2 \right\}, \quad \delta_* + c_* \leq y_* \leq h_* \quad (10)$$

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