	burst	[—]
n	= power index of regression curve	[—]
$\bar{n}$	= mean value of n	[—]
λ	= viscosity ratio between dispersed and continuous phase (μ'/μ)	[—]
μ	= viscosity of continuous phase	[g/cm·sec]
μ'	= viscosity of dispersed phase	[g/cm·sec]
σ	= interfacial tension	[g/sec ²]
ρ	= density of continuous phase	[g/cm ³]

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THICKNESS OF LIQUID FALLING FILMS IN "CALMING ZONE" ON INCLINED PLATES

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The momentum integral equation taking into account the drag of air on a falling liquid film along inclined plates is solved. The velocity profile is assumed as laminar with the effect of air drag at the surface. The analysis predicts well the variation of film thickness with Re .

Film thickness variation has been studied numerically^{1,2)}, analytically for falling liquid films emerging from a flat slit^{2,7,8)} and free film flow at a vertical wall^{1,3,4)} and for non-Newtonian flow⁶⁾. In these studies the drag at air-liquid surface is not taken into account. Whitaker and Cerro⁵⁾ pointed out that the effect of gas-phase shear stress was negligible, after the simple reasoning based upon a standard boundary layer theory for the gas phase. However, the authors⁵⁾ measured the velocity profile in the liquid film by hydrogen bubble method and found that the drag was not negligible.

Equation (1) is a simplified form of the equation of motion,

$$-\frac{d}{dx} \int_0^h u^2 dy + u_s \frac{d}{dx} \int_0^h u dy + gh = \nu \left\{ \left(\frac{\partial u}{\partial y} \right)_w - \left(\frac{\partial u}{\partial y} \right)_s \right\} \quad (1)$$

and the continuity equation is

$$\int_0^h u dy = \frac{\nu}{4} Re \quad (2)$$

Equations (3) and (4) are dimensionless forms of

Eqs. (1) and (2), respectively.

$$-\frac{d}{dx_*} \int_0^{h_*} u_*^2 dy_* + h_* = \tau_{w*} - \tau_{s*} \quad (3)$$

$$\int_0^{h_*} u_* dy_* = \frac{Re}{4} \quad (4)$$

Equations (5), (6) and (7) are parts of the experimental results⁵⁾ under steady, fully developed flow conditions.

$$u_*(\eta) = \eta \left(u_{s*} + \frac{1}{2} h_*^2 \right) - \frac{1}{2} h_*^2 \eta^2 \quad (5)$$

$$-\tau_{s*} = \frac{K}{2(1-(1/2)K)} \tau_{w*} \quad (K=0.134) \quad (6)$$

$$h_* = Re^{1/3} \left/ \left(\frac{3}{4} - K \right) \right|^{1/3} \quad (7)$$

K indicates the degree of deviation from Nusselt's analysis. The assumption regarding velocity profile is modified as follows. Initial region ($0 \leq x_* \leq x_{1*}$) where a core region exists above boundary layer, and developed region ($x_{1*} \leq x_*$) where core does not exist, are settled. In initial region the boundary layer thickness $\delta_*(x_*)$ increases and the core depth $c_*(x_*)$ decreases with increasing x_* , and at $x_* = x_{1*}$, c_* becomes zero (see Fig. 1). For the initial region

$$u_*(x_*, y_*) = u_{c*}(x_*) \left\{ 2 \left(\frac{y_*}{\delta_*} \right) - \left(\frac{y_*}{\delta_*} \right)^2 \right\}, \quad 0 \leq y_* \leq \delta_* \quad (8)$$

$$u_* = u_{c*}(x_*), \quad \delta_* \leq y_* \leq \delta_* + c_* \quad (9)$$

$$u_* = n_{c*} \left\{ 2 \left(\frac{y_* - c_*}{\delta_*} \right) - \left(\frac{y_* - c_*}{\delta_*} \right)^2 \right\}, \quad \delta_* + c_* \leq y_* \leq h_* \quad (10)$$

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and $u_{o*}(x_*)$ is derived from the balance of gravitational and kinetic energies, as follows:

$$u_{o*}(x_*)^2 = u_{o*}^2 + 2x_* \quad (11)$$

For the developed region,

$$u_*(x_*, y_*) = u_{o*}(x_*) \left\{ 2 \left(\frac{y_*}{\delta_*} \right) - \left(\frac{y_*}{\delta_*} \right)^2 \right\}, \quad 0 \leq y_* \leq h_* \quad (12)$$

Assuming that the ratio τ_{s*}/τ_{w*} at any position is constant $(1-\alpha)$, Eq. (13) is written, and α in each region is given by Eq. (14) (for initial region) and Eq. (15) (for developed region).

$$\tau_{w*}(x_*) - \tau_{s*}(x_*) = \alpha \tau_{w*}(x_*) \quad (13)$$

$$(h_* - c_*)/\delta_* = \alpha \quad (14)$$

$$h_*/\delta_*' = \alpha \quad (15)$$

Putting these velocity profiles into Eq. (4), one obtains Eqs. (16) and (17) for initial region and developed region, respectively.

$$c_*(x_*) = \frac{Re}{4u_{o*}(x_*)} - \alpha^2 \left(1 - \frac{\alpha}{3} \right) \cdot \delta_*(x_*) \quad (16)$$

$$u_{o*}(x_*) = \frac{Re}{4\alpha(1 - (1/3)\alpha) \cdot h_*(x_*)} \quad (17)$$

With the initial condition $\delta_* = 0$ at $x_* = 0$, Eqs. (18), (19) and (20) are derived as

$$h_* = \frac{Re}{4u_{o*}} + \alpha \left\{ 1 - \alpha \left(1 - \frac{1}{3}\alpha \right) \right\} \delta_* \quad (18)$$

$$\delta_* = \sqrt{3(u_{o*} - u_{o*}^0 u_{o*}^{-9})^{1/2}} \quad (19)$$

$$u_{o*} = \sqrt{u_{o*}^2 + 2x_*} \quad \text{for } 0 \leq x_* \leq x_{1*} \quad (20)$$

Equations (21), (22) and (23) are also obtained from Eqs. (3), (12), (15) and (17).

$$x_* + c = \frac{u_{o*}}{a} \left\{ \frac{1}{2} \ln \frac{(h_* - u_{o*})^2}{h_*^2 + h_* u_{o*} + u_{o*}^2} + \sqrt{3} \tan^{-1} \frac{2h_* + u_{o*}}{\sqrt{3}u_{o*}} \right\} \quad (21)$$

$$u_{o*} = \left\{ Re \alpha / 2 \left(1 - \frac{1}{3}\alpha \right) \right\}^{1/3} \quad (22)$$

$$a = -24\alpha \left(1 - \frac{1}{3}\alpha \right) / Re \left(\frac{4}{3} - \alpha + \frac{1}{5}\alpha^2 \right) \quad \text{for } x_* \geq x_{1*} \quad (23)$$

The critical value x_{1*} is obtained with Eqs. (16) and (19), with the value of which integration constant c in Eq. (21) is determined. To present the film thickness h_* as a function of x_* , Re and inclination angle of the plate θ , u_{o*} should be eliminated with Eqs. (24), (25), (26) and (27), as shown in Fig. 1.

$$Q = u_o h_o \quad (24)$$

$$u_o = \sqrt{2g_x h''} \quad (25)$$

$$h' - h_o \cos \theta = h'' \quad (26)$$

$$h' = (Q/317)^{3/2} \quad (27)$$

Equation (27) is the Francis-Weir equation.

α in Eq. (13) stands for the ratio of τ_{s*} to τ_{w*} .

Fig. 1 Schematic of flow model considered

Fig. 2 Film thicknesses as function of x_* ($\theta = 9^\circ$)

Fig. 3 Comparison of experimental and theoretical film thicknesses at a point

$\tau_{s*} = 0$ when $\alpha = 1$. Some examples of $h_*(x_*)$, for $\alpha = 1.0718$ and for $\alpha = 1$, are shown in Fig. 2. α is not a sensitive parameter for film thickness when the value of x_* is small. The curves have maximum at $x_* \sim 10^2$ ($x \sim 1$ cm). This fact agrees qualitatively with the experimental observation that a falling film has a rising near the entrance. In Fig. 3, film thickness variation at $x_* \sim 10^4$ is compared with direct measurements⁵⁾ by pointer and gauge method.

Nomenclature

c	= core region depth	[cm]
c_*	= $c(g \sin \theta)^{1/3} / \nu^{2/3}$	[—]
h	= liquid film thickness	[cm]
h_*	= $h(g \sin \theta)^{1/3} / \nu^{2/3}$	[—]
K	= parameter in Ref. 5	[—]
Q	= $u \cdot h$	[cm ² /sec]
Re	= $4Q/\nu$	[—]

u	= fluid velocity	[cm/sec]
u_*	= $u/(\nu g \sin \theta)^{1/3}$	[—]
$u_{\infty*}$	= defined by Eq. (22)	[—]
x	= distance from the entrance	[cm]
x_*	= $x(g \sin \theta)^{1/3}/\nu^{2/3}$	[—]
y	= coordinate axis	[cm]
y_*	= $y(g \sin \theta)^{1/3}/\nu^{2/3}$	[—]
α	= parameter defined by Eq. (13)	[—]
δ	= boundary layer thickness	[cm]
δ_*	= $\delta(g \sin \theta)^{1/3}/\nu^{2/3}$	[—]
η	= y/h	[—]
θ	= inclination of plate to horizon	[deg]
ν	= kinematic viscosity of liquid	[cm ² /sec]
τ	= shear stress	[g/cm·sec ²]
τ_*	= $\tau/\rho(\nu g \sin \theta)^{2/3}$	[—]
<Subscripts>		
c	= core region	

o	= at entrance ($x=0$)
w	= at wall
1	= critical position

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MASS TRANSFER INTO LAMINAR GAS STREAMS IN WETTED-WALL COLUMNS — COCURRENT GAS-LIQUID FLOW WITH CIRCULATION IN GAS PHASE —

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In a previous paper²⁾, the effect of gas and liquid flow rates on mass transfer rate in laminar gas streams in wetted-wall columns with cocurrent gas-liquid flow was studied theoretically and experimentally. An analytical solution was obtained for the average gas-phase Sherwood number Sh_G as a function of the gas-phase Graetz number Gz_G and the dimensionless interfacial gas velocity U . This analysis, however, is not applicable to the case of $U > 2$ where circulation exists in the gas phase.

The purpose of this short paper is to present an approximate analytical solution for the present case and to confirm this approximate solution experimentally.

1. Theory

Consider a gas and a liquid flowing cocurrently in a vertical wetted-wall column. It is assumed that the gas and liquid streams are laminar and fully developed. The equations of motion for gas and liquid streams have been solved in a previous paper¹⁾ and the fol-

lowing expression giving the velocity profile for the gas has been obtained:

$$u = u_m(2 - U) - 2u_m(1 - U)(r/r_i)^2 \quad (1)$$

with

$$U = u_i/u_m \quad (2)$$

The values of the average gas velocity u_m , the interfacial gas velocity u_i and the radius of the gas passage r_i can be calculated from the equations given in the previous paper¹⁾ as functions of the gas-phase and liquid-phase Reynolds numbers, Re_G and Re_L , and the physical properties of the gas and liquid.

In cocurrent flow of the gas and liquid, the dimensionless interfacial velocity U is always positive and increases with decreasing Re_G and increasing Re_L . The flow pattern in the gas phase depends upon the value of U . When U is equal to zero, the flow situation of the gas corresponds to single-phase flow in a circular tube, and when U is equal to unity, the gas flow corresponds to plug flow with a flat velocity profile. Further, when U is equal to two, the gas velocity at the column axis is zero, and when U is larger than two, circulation exists in the gas phase,

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