R^t	= pellet radius at time t	[cm]
r	= particle radius	[cm]
$(-r_A)$	= disappearance rate of component A	[mol/sec cm ²]
r_0	= initial particle radius	[cm]
s	= stoichiometric factor	[—]
T	= dimensionless time ($= sD_A C_{Ab} t / \rho_s r_0^2$)	[—]
t	= time	[sec]
u	= gas velocity	[cm/sec]
X	= dimensionless radial coordinate ($= R/R_0$)	[—]
X^t	= dimensionless pellet radius ($= R^t/R_0$)	[—]
Y	= dimensionless concentration ($= C_A/C_{Ab}$)	[—]
Z	= dimensionless particle radius (r/r_0)	[—]
ε	= void fraction	[—]
ε_0	= initial void fraction	[—]
μ	= viscosity of gas	[g/cm·sec]
ρ	= density of gas	[g/cm ³]
ρ_s	= density of solid	[mol/cm ³]

Literature Cited

- 1) Beckstand, L. W. and J. D. Miller: *Met. Trans.*, **8B**, 19

- (1977).
- 2) Chida, T. and T. Tadaki: *J. Chem. Eng. Japan*, **11**, 371 (1978).
- 3) Hashimoto, K. and P. L. Silveston: *AIChE J.*, **19**, 259 (1973).
- 4) "International Critical Tables", Vol. 5, p. 62, p. 64, McGraw-Hill, New York (1929).
- 5) "Kagaku-Benran (Kiso-Hen)", Maruzen, Tokyo (1975).
- 6) Lapidus, L.: "Digital Computation for Chemical Engineers", McGraw-Hill, New York (1962).
- 7) Petersen, E. E.: *AIChE J.*, **3**, 443 (1957).
- 8) Ramachandran, P. A. and J. M. Smith: *ibid.*, **23**, 353 (1977).
- 9) *idem*: *Chem. Eng. J.*, **14**, 137 (1977).
- 10) Surana, V. S. and I. H. Warren: *Trans. Inst. Min. Metall.*, **78**, 133 (1969).
- 11) Wakao, N. and J. M. Smith: *Chem. Eng. Sci.*, **17**, 825 (1962).
- 12) Warren, I. H. and G. I. D. Roach: *Trans. Inst. Min. Metall.*, **80**, 152 (1971).
- 13) Yoshida, K. and D. Kunii: *J. Chem. Eng. Japan*, **2**, 170 (1969).

OPTIMUM DESIGN FOR SOLIDS CIRCULATION SYSTEM WITH SHRINKING CORE KINETICS IN BOTH REACTOR AND REGENERATOR

SHOICHI KIMURA, TSUTAO OTAKE
AND OCTAVE LEVENSPIEL

*Department of Chemical Engineering,
Osaka University, Toyonaka 560*

The optimum volume ratio of reactor and regenerator which minimizes the total volume of the two is obtained for solids circulation systems where shrinking core kinetics hold for both gas-solid reaction and regeneration. For the general case the optimum volume ratio for a fixed flow rate of solids is represented graphically in terms of reaction and regeneration conditions. For a special case an approximate analytical solution for the minimization is obtained. The procedure to determine the optimum required volume for the reactor and that for the regenerator is also dealt with.

The effect of reaction and regeneration conditions and of solids flow rate on the optimum volumes is discussed.

Introduction

An earlier paper¹⁾ developed the performance equations for a system where solids are circulated between reactor and regenerator. In the reactor the solid combined with and removed a component from the gas stream; in the regenerator the captured component

was released and the solid was restored to its initial state. The reactor and the regenerator were taken as fluidized beds (mixed flow of solids) and gas-solid reaction and regeneration were both assumed to proceed inward from the surface of the particle (shrinking core model).

The removal of H₂S from a coal gasification stream by circulating iron oxide-iron sulfide particles is an example of such a system.

This paper finds the optimum volume ratio of reactor

Received October 5, 1978. Correspondence concerning this article should be addressed to S. Kimura. O. Levenspiel is at Dept. of Chem. Eng., Oregon State Univ., Corvallis, Oregon 97331 U.S.A.

and regenerator, the volume ratio which minimizes the total volume of reactor and regenerator for a given removal rate of the gaseous component.

1. The Fractional Conversion of Solids Leaving Reactor and Regenerator

To simplify the models and language consider, in the same manner as before, that particles are blackened in the reactor; whitened in the regenerator. The blackening reaction and the whitening reaction may be expressed as

blackening reaction:

whitening reaction:

Taking the H_2S removal system as an example, A is for H_2S , W for iron oxide, B for iron sulfide, and G for oxygen in the air.

Thus the reactor is called the "blackener" and the regenerator the "whitener". **Figure 1** illustrates the solids circulation system with all its associated symbols. Particles leaving the blackener and the whitener have a varying number of concentric rings of one or other material according to their history (the past residence times in the blackener and the whitener). **Figure 2** schematically shows the variation of rings according to the length of residence time in the blackener and in the whitener.

The progress of reaction for shrinking core kinetics is assumed to be controlled by the first-order chemical reaction. Thus, based on unit surface of reacting front, the rate of reaction is given by

$$-r'' = k_s \bar{C} \quad (3)$$

where $\bar{C}$ is the average concentration of the gaseous reactant in the beds and k_s is the rate constant for the surface reaction. The advancement of the reacting front is then expressed in terms of the time to convert a particle with radius R from one material to the other one, τ , by

$$t/\tau = 1 - (r_G/R) \quad (4)$$

where r_G is the radial position of the reacting front in a particle. Time to blacken an all-white particle, τ_B , is given by³⁾

$$\tau_B = \rho_W R / \alpha k_{sB} \bar{C}_A \quad (5)$$

where ρ_W is the molar density of white solid and k_{sB} is the blackening reaction rate constant based on unit surface area of the reacting front.

We assume for the blackening reaction that whenever an advancing black front reaches a black region in a particle it immediately jumps across the region and continues advancing through the next white region. The whitening reaction also has jumps across the white region.

Fig. 1 Sketch of a solids circulation system with all its associated symbols

Fig. 2 Variation of concentric rings of solid B or W according to length of residence time

Applying exponential RTD of mixed flow for solid particles, the volumetric fractions of black solid in the particles leaving the blackener and the whitener, $\bar{X}_{bB}$ and $\bar{X}_{bW}$, are then given, as described in detail elsewhere¹⁾, by

$$\bar{X}_{bB} = 3 \int_0^1 (1-l)^2 P_{bB}(l) dl \quad (6)$$

$$\bar{X}_{bW} = 3 \int_0^1 (1-l)^2 P_{bW}(l) dl \quad (7)$$

where $P_{bB}(l)$ and $P_{bW}(l)$ are probabilities of finding black at location l in a particle leaving the blackener and the whitener, given by the following¹⁾, respectively:

$$P_{bB}(l) = P_{bW}(l) + \frac{(\theta_B - \theta_W)^2 E}{[\theta_B - \theta_W E]^2} \quad (8)$$

$$P_b(l) = \frac{\theta_B \theta_W (\theta_B - \theta_W) [E - \theta_W^2 E + \theta_W^2 E^2]}{[\theta_B - \theta_W E]^2} \quad (9)$$

where

$$E = \exp [-(\theta_B - \theta_W) l] \quad (10)$$

l is the non-dimensional distance from the particle

surface, defined as:

$$l=1-(r/R) \quad (11)$$

and the characteristic operating constants, θ_B for the blackener and θ_W for the whitener, are defined as:

$$\theta = \frac{\text{time required for complete conversion of a fresh particle in the blackener (or in the whitener)}}{\text{mean residence time in the blackener (or in the whitener)}} = \frac{\tau}{\bar{t}} \quad (12)$$

2. Optimum Design

2.1 Derivation of equations to be solved

Consider the blackening reaction taking place in the blackener, which is represented by Eq. (1). A material balance for the gaseous component captured by the circulating solids in the blackener gives (see Fig. 1)

$$\left(\begin{array}{c} \text{(stoichiometric)} \\ \text{coefficient} \end{array} \right) \left(\begin{array}{c} \text{gaseous A captured} \\ \text{in the blackener} \end{array} \right) = \left(\begin{array}{c} \text{(increase of black solid)} \\ \text{in the circulating particles} \end{array} \right)$$

or in terms of conversion of gaseous A, X_A , and the difference of fractional conversion of black solid, $\bar{X}_{bB} - \bar{X}_{bW}$, which is the overall conversion of solids in the system,

$$\alpha F_{A0} X_A = F_s \rho_W (\bar{X}_{bB} - \bar{X}_{bW})$$

or

$$\bar{X}_{bB} - \bar{X}_{bW} = \alpha F_{A0} X_A / F_s \rho_W \quad (13)$$

where F_{A0} is molar feed rate of gaseous A into the blackener. Since solid density of particles varies according to the fraction of black solid, we here define the flow rate of solids, F_s , as net volume of solids circulated per unit time.

We can also relate the flow rate, F_s , bed volumes, V_B and V_W , and mean residence times of circulating solids, $\bar{t}_B$ and $\bar{t}_W$, by

$$V = V_B + V_W = F_s (\bar{t}_B + \bar{t}_W) \quad (14)$$

where V_B and V_W are net volumes of solids in the blackener and in the whitener, respectively.

In practice there will always be some upper limit to the allowable circulation rate of solids. So let us minimize the total volume of blackener and whitener for a given circulation rate of solids and for a given conversion of the gaseous reactant A, or minimize V for fixed F_s and $F_{A0} X_A$. From Eq. (13) and Eq. (14) we see that this is equivalent to

$$\text{minimize } \Phi = \bar{t}_B + \bar{t}_W \text{ for fixed } \bar{X}_{bB} - \bar{X}_{bW} \quad (15)$$

Our concern is to obtain the volume ratio of blackener and whitener which minimizes Φ in Eq. (15). Hence let us here introduce a new variable, σ , which is related to the volume ratio, and express Φ in terms of this variable. Thus

$$\sigma = \frac{\theta_W}{\theta_B} = \left(\frac{\tau_W}{\tau_B} \right) \left(\frac{\bar{t}_B}{\bar{t}_W} \right) = \left(\frac{\tau_W}{\tau_B} \right) \left(\frac{V_B}{V_W} \right) \quad (16)$$

After some algebraic manipulation using Eq. (12) and Eq. (16) Φ is given by

$$\Phi = \frac{\tau_B \sigma + \tau_W}{\theta_B \sigma} \quad (17)$$

To keep the condition in Eq. (15) that $\bar{X}_{bB} - \bar{X}_{bW}$ is fixed, σ and θ_B must be correlated with each other, since $\bar{X}_{bB} - \bar{X}_{bW}$ is given in terms of σ and θ_B by Eqs. (6) to (9) as

$$\bar{X}_{bB} - \bar{X}_{bW} = 3 \int_0^1 \frac{(1-l)^2 (1-\sigma)^2 E}{[1-\sigma E]^2} dl = \frac{3\lambda}{\theta_B \sigma} \quad (18)$$

where

$$\lambda = 1 - \frac{2}{\theta_B} \ln \frac{1-\sigma \exp[-\theta_B(1-\sigma)]}{1-\sigma} - 2(1-\sigma)(1-\xi) \quad (19)$$

$$\xi = \int_0^1 \frac{l}{1-\sigma E} dl \quad (20)$$

Using the correlation between σ and θ_B conditioned by Eq. (18), Φ of Eq. (17) to be minimized turns out to be a single variable function with respect to σ .

Consequently, the equation to be solved reduces from Eq. (15) to the following:

$$\left[\begin{array}{l} \frac{d\Phi}{d\sigma} = 0 \\ \text{:conditioned by} \end{array} \right. \quad (21)$$

$$\left[\begin{array}{l} \frac{\lambda}{\theta_B \sigma} = \frac{1}{3} (\bar{X}_{bB} - \bar{X}_{bW}) \end{array} \right. \quad (22)$$

Expressing the solution of Eq. (21) and Eq. (22) as σ_{opt} , the optimum volume ratio of blackener and whitener is given by

$$\left(\frac{V_W}{V_B} \right)_{opt} = \frac{\tau_W / \tau_B}{\sigma_{opt}} \quad (23)$$

2.2 Graphical representation of the optimum volume ratio of blackener and whitener

Having derived the equation to be solved, we here show the practical procedure to obtain the optimum volume ratio and represent it graphically.

Taking the total derivative of Eq. (17) with respect to σ and applying Eq. (21) give the equation for σ to satisfy together with Eq. (22) as

$$\left(\frac{\sigma \Psi}{(\theta_B / \sigma) - \Psi} \right)_{opt} = \frac{\tau_W}{\tau_B} \quad (24)$$

where Ψ is the total derivative of θ_B with respect to σ and is given by utilizing Eq. (22),

$$\begin{aligned} \Psi &= - \frac{d\theta_B}{d\sigma} \\ &= \left\{ \frac{\theta_B^2}{2} \left(1 - \frac{\sigma}{1-\sigma} \right) \left(1 - \frac{\Delta \bar{X}_b \theta_B}{3} \right) \right. \\ &\quad \left. - \frac{\theta_B}{1-\sigma} + \frac{1+\sigma \theta_B}{\sigma(1-\sigma)} \ln \frac{1-\sigma \exp[-\theta_B(1-\sigma)]}{1-\sigma} \right\} \\ &\quad / \left\{ \sigma \theta_B - \frac{\Delta \bar{X}_b}{2} \theta_B^2 \sigma - \ln \frac{1-\sigma \exp[-\theta_B(1-\sigma)]}{1-\sigma} \right\} \quad (25) \end{aligned}$$

and

$$\Delta \bar{X}_b = \bar{X}_{bB} - \bar{X}_{bW} \quad (26)$$

Since θ_B is related to σ for a fixed $\bar{X}_{bB} - \bar{X}_{bW}$ by Eq. (22), Eq. (24) may be solved to obtain σ_{opt} for a given τ_W/τ_B .

Give σ and evaluate τ_W/τ_B by the left-hand side of Eq. (24). This σ is then equivalent to σ_{opt} corresponding to the evaluated value of τ_W/τ_B . Using the sets of σ_{opt} and τ_W/τ_B obtained in this manner, we can construct a chart to represent the optimum volume ratio of blackener and the whitener in terms of τ_W/τ_B by Eq. (23).

Let us here briefly consider the characteristics of σ_{opt} for two special cases. When $\tau_W = \tau_B$, $\sigma_{opt} = 1$ and then $(V_W/V_B)_{opt} = 1$. This can be shown simply by evaluating Ψ of Eq. (25) for $\sigma = 1$ and substituting it into Eq. (24). When $\bar{X}_{bB} - \bar{X}_{bW} = 1$, $\sigma_{opt} = \sqrt{\tau_W/\tau_B}$ and then $(V_W/V_B)_{opt} = \sqrt{\tau_W/\tau_B}$ for this case. This will be discussed in a later section.

To make full use of these advantageous characteristics, the optimum volume ratio divided by the square root of τ_W/τ_B , $(V_W/V_B)_{opt}/\sqrt{\tau_W/\tau_B}$, is plotted against $\bar{X}_{bB} - \bar{X}_{bW}$ with τ_W/τ_B as a parameter in Fig. 3. Figure 3 shows that $(V_W/V_B)_{opt}/\sqrt{\tau_W/\tau_B}$ can be approximated to be unity when τ_W/τ_B approaches unity. This means that equal volume is the optimum when $\tau_W \cong \tau_B$. We see also that $(V_W/V_B)_{opt}$ can be approximated to be $\sqrt{\tau_W/\tau_B}$ for any value of τ_W/τ_B when $\bar{X}_{bB} - \bar{X}_{bW}$ is close to unity. We have this regime when the circulating solids are fully utilized and the solids leaving the blackener are almost blackened and those leaving the whitener are almost whitened.

Once we know the reaction conditions in the blackener and the whitener, τ_W/τ_B , the optimum volume ratio is directly obtained from Fig. 3 after evaluating the value of $\bar{X}_{bB} - \bar{X}_{bW}$ by Eq. (13) with the given conversion of the gaseous reactant and the given flow rate of circulating solids.

3. Discussion

The optimum volume ratio of the blackener and the whitener is now available to determine the volume of each. In this chapter we deal with the procedure to obtain the optimum required volume of the blackener and that of the whitener assuming mixed flow of gas.

To take into consideration that the overall reaction rate in the bed is dependent on the extent of conversion of solids in the bed, we introduce the effectiveness factor for gas-solid reaction. The effectiveness factor will then conveniently be used to express the performance equation for mixed flow, which relates the required volume to the operating conditions.

Further, an approximate analytical solution for the minimization will be discussed.

Fig. 3 Optimum volume ratio for given blackening and whitening reaction conditions

3.1 Effectiveness factor for gas-solid reaction

We know that the overall reaction rate of solids is lowered proportionally to the decrease of surface area of the reacting fronts. To measure how much the reaction rate is lowered because of the decrease of reacting front area, define the effectiveness factor as

$$\eta = \frac{\text{surface area of the outermost reacting front of particles}}{\text{exterior surface area of particles}} \quad (27)$$

This quantity depends only on the area of presently advancing outermost reacting front and is not affected by the extent of conversion of solid inside of the front.

Using the density function for the location of the outermost reacting front of particles leaving the blackener, $q_B(l)$, we can express the effectiveness factor for the reaction in the blackener as

$$\eta_B = \int_0^1 (1-l)^2 q_B(l) dl \quad (28)$$

$q_B(l)$ has been obtained in the earlier paper¹¹:

$$q_B(l) = \frac{\theta_B(\theta_B - \theta_W)^2 E}{[\theta_B - \theta_W E]^2} + \frac{(\theta_W - \theta_B)\delta(l-1)}{\theta_W - \theta_B \exp[-(\theta_W - \theta_B)]} \quad (29)$$

After substituting Eq. (29) into Eq. (28) and rearranging we find that

$$\eta_B = \frac{\lambda}{\sigma} \quad (30)$$

We can also obtain the effectiveness factor for the whitening reaction, η_W , and, following the same procedure as described above, find that

$$\eta_W = \int_0^1 (1-l)^2 q_W(l) dl = \lambda \quad (31)$$

where $q_W(l)$ is the density function for the location of

Fig. 4 Effectiveness factor for reaction in the blackener under optimum condition

the outermost whitening front of particles leaving the whitener (see Kimura *et al.*¹¹⁾).

We can now relate the effectiveness factor with the overall fractional conversion of solids in the system. The former focuses attention on the advancing surface of concentric rings and the latter on the total number of rings (see Fig. 2). Thus using Eq. (18), Eq. (30) and Eq. (31) gives

$$\bar{X}_{bB} - \bar{X}_{bW} = \frac{3}{\theta_B} \eta_B = \frac{3}{\theta_W} \eta_W \quad (32)$$

For a single reactor with continuous feed of fresh solids we find an analytical expression for the effectiveness factor by taking the limit of Eq. (30) as $\theta_W = 0$, that is, $\sigma = 0$, that

$$\eta_B = 1 - \frac{2}{\theta_B} + \frac{2}{\theta_B^2} [1 - \exp(-\theta_B)] \quad (33)$$

In this case the overall fractional conversion of the particles leaving the reactor is obtained by insertion of Eq. (33) into Eq. (32) and with $\bar{X}_{bW} = 0$ as

$$\bar{X}_{bB} = \frac{3}{\theta_B} - \frac{6}{\theta_B^2} + \frac{6}{\theta_B^3} [1 - \exp(-\theta_B)] \quad (34)$$

This relation was obtained in somewhat different ways by Yagi *et al.*⁴⁾ for this special case.

Making use of Eq. (32) together with Eq. (22) and Eq. (24), we can calculate the effectiveness factor for the optimum value of σ . Thus σ_{opt} for a given τ_W/τ_B and θ_B corresponding to this σ_{opt} give the effectiveness factor under the optimum condition in terms of τ_W/τ_B . The effectiveness factor for the reaction in the blackener is expressed as $(\eta_B)_{opt}$ and is shown in Fig. 4. Figure 4 tells us that $(\eta_B)_{opt}$ is strongly dependent on the value of $\bar{X}_{bB} - \bar{X}_{bW}$. Since the flow rate of circulating solids, F_s , is directly related to $\bar{X}_{bB} - \bar{X}_{bW}$ by Eq. (13), this means that the flow rate of circulating

solids has significant influence on the effectiveness factor. To get a high value of effectiveness factor, a high solids flow rate is preferable as long as economically and mechanically acceptable.

3.2 Volumes of blackener and whitener based on the optimum volume ratio

Making use of the optimum volume ratio and the effectiveness factor, we can now obtain the optimum required volume of the blackener and that of the whitener.

Assuming mixed flow of gas, the average concentration of the gaseous reactant in Eq. (3) is equal to the outlet one, $C_{A0}(1 - X_A)$. Then a material balance for the blackener with respect to the gaseous reactant A gives

$$(\text{gaseous A captured in the blackener}) = (\text{surface reaction rate}) \times \left(\frac{\text{total exterior surface area}}{\text{of particles in the bed}} \right) (\text{effectiveness factor})$$

or

$$F_{A0} X_A = k_{sB} C_{A0} (1 - X_A) \left(\frac{3V_B}{R} \right) \eta_B \quad (35)$$

When the system is designed so that the optimum volume ratio holds for the blackener and the whitener, η_B in Eq. (35) can be replaced with $(\eta_B)_{opt}$, and then V_B in Eq. (35) is the optimum required volume for the blackener, $(V_B)_{opt}$. Thus Eq. (35) may be rewritten with some manipulation as

$$(V_B)_{opt} = \frac{F_{A0} X_A \tau_B}{3\rho_W (\eta_B)_{opt}} \quad (36)$$

After calculating τ_B and τ_W with the given conditions, the effectiveness factor, $(\eta_B)_{opt}$, can be evaluated for the calculated value of τ_W/τ_B from Fig. 4. The optimum required volume for the whitener is then obtained by this $(V_B)_{opt}$ and the optimum volume ratio, $(V_W/V_B)_{opt}$, which is given in Fig. 3. Thus

$$(V_W)_{opt} = (V_B)_{opt} \cdot (V_W/V_B)_{opt} \quad (37)$$

Since Fig. 4 shows that an increase in τ_W/τ_B gives a lower effectiveness factor, Eq. (36) and Eq. (37) imply that reaction conditions should be selected so that the whitening reaction is considerably faster than the blackening reaction.

3.3 Approximate analytical solution for $\bar{X}_{bB} - \bar{X}_{bW} \cong 1$

Consider a special case for the optimum volume ratio of Fig. 3, that is, the optimum volume ratio for $\bar{X}_{bB} - \bar{X}_{bW} \cong 1$. We encounter this regime when the system is operated under the condition that the mean residence times of solids are larger than the times required for complete conversion of fresh particles, thus $\bar{t}_B > \tau_B$ and $\bar{t}_W > \tau_W$ or $\theta_B < 1$ and $\theta_W < 1$. For this case we can obtain an approximate analytical solution for the optimization.

Expand λ of Eq. (19) in a Taylor series in the neighborhood of $\theta_B = 0$. Taking terms up to the second order with respect to θ_B , λ is approximated as

$$\lambda = \frac{\sigma \theta_B}{3} \left[1 - \frac{(1+\sigma)\theta_B}{4} \right] \quad (38)$$

Substituting Eq. (38) into Eq. (19) gives θ_B in terms of σ as

$$\theta_B = \frac{4(1-\Delta\bar{X}_b)}{1+\sigma} \quad (39)$$

With this approximate form for θ_B , Eq. (17) to be minimized is expressed in terms of σ as

$$\Phi = \frac{\tau_B}{4(1-\Delta\bar{X}_b)} \frac{(\sigma+1)(\sigma+\tau_W/\tau_B)}{\sigma} \quad (40)$$

Applying Eq. (40) to Eq. (21) finally gives σ_{opt} as

$$\sigma_{opt} = \sqrt{\frac{\tau_W}{\tau_B}} \quad (41)$$

Inserting into Eq. (23), the optimum volume ratio of blackener and whitener reduces to

$$\left(\frac{V_W}{V_B} \right)_{opt} = \sqrt{\frac{\tau_W}{\tau_B}} \quad (42)$$

Equation (42) is identical to the expression for a catalyst deactivation and regeneration system in which solids are circulated between reactor and regenerator, both of which are fluidized beds²⁾.

For operation in this regime, the procedure to obtain the optimum required volumes for the blackener and the whitener is simplified as follows. Substituting Eq. (38) and Eq. (39) into Eq. (30), the effectiveness factor, $(\eta_B)_{opt}$, becomes

$$(\eta_B)_{opt} = \frac{4(1-\Delta\bar{X}_b)\Delta\bar{X}_b}{3(1+\sqrt{\tau_W/\tau_B})} \quad (43)$$

Substituting into Eq. (36) and Eq. (37) and rearranging with Eq. (13) give the optimum volumes as

$$(V_B)_{opt} = \frac{F_s \tau_B}{4(1-\alpha F_{A0} X_A / F_s \rho_W)} \left(1 + \sqrt{\frac{\tau_W}{\tau_B}} \right) \quad (44)$$

$$(V_W)_{opt} = \frac{F_s \tau_B}{4(1-\alpha F_{A0} X_A / F_s \rho_W)} \left(1 + \sqrt{\frac{\tau_W}{\tau_B}} \right) \sqrt{\frac{\tau_W}{\tau_B}} \quad (45)$$

Conclusion

For a solids circulation system where shrinking core kinetics hold for both gas-solid reaction and regeneration the optimum volume ratio of reactor and regenerator which minimizes the total volume of the two was obtained in terms of the reaction and regeneration conditions for a fixed flow rate of circulating solids and for a given removal rate of a gaseous component.

The effectiveness factor for gas-solid reaction was introduced and conveniently used to calculate the optimum volumes of the reactor and the regenerator.

The results characterized for this system became identical in special cases to those for a single reactor system with continuous solids feed or for a catalyst

deactivation and regeneration system.

Nomenclature

C_{A0}	= molar concentration of gaseous reactant A in the gas stream entering the blackener	[mol/m ³]
$\bar{C}$	= average concentration of a gaseous reactant in a bed	[mol/m ³]
F_{A0}	= molar feed rate of a gaseous reactant A into the blackener	[mol/h]
F_s	= flow rate based on net volume of solids circulated per unit time between the blackener and the whitener	[m ³ /h]
k_s	= surface reaction rate constant	[m/h]
k_{sB}	= blackening reaction rate constant based on unit surface area	[m/h]
l	= nondimensional distance from the exterior surface of a particle	
R	= radius of a particle	[m]
r	= radial position within a particle	[m]
$-r''$	= rate of reaction based on unit surface of a reaction front	[mol/m ² ·h]
t	= time	[h]
$\bar{t}_B, \bar{t}_W$	= mean residence times of solids in the blackener and in the whitener	[h]
V	= total net volume of solids in the system	[m ³]
V_B, V_W	= net volumes of solids in the blackener and in the whitener, respectively	[m ³]
X_A	= conversion of a gaseous reactant A in the blackener	
$\bar{X}_B, \bar{X}_{BW}$	= volumetric fraction of black solid in the particles leaving the blackener and the whitener, respectively	
α, β	= stoichiometric coefficients for the reaction represented by Eq. (1) and Eq. (2)	
η_B, η_W	= effectiveness factors for the blackening reaction and for the whitening reaction, respectively	
ρ_W	= molar density of white solid	[mol/m ³]
θ_B, θ_W	= operating characteristics of the blackener and the whitener, defined as $\tau_B/\bar{t}_B$ and $\tau_W/\bar{t}_W$, respectively	
τ_B, τ_W	= time required for complete conversion of a entirely white particle and of a entirely black particle, respectively	[h]
σ	= $\theta_W/\theta_B = (\tau_W/\tau_B)(\bar{t}_B/\bar{t}_W) = (\tau_W/\tau_B)(V_B/V_W)$	
$\delta(l-1)$	= Dirac delta function occurring at $l=1$	
Φ	= $\bar{t}_B + \bar{t}_W$	[h]

Literature Cited

- 1) Kimura, S., T. J. Fitzgerald and O. Levenspiel: submitted to *I&EC Process Design Dev.*
- 2) Kunii, D. and O. Levenspiel: "Fluidization Engineering", John Wiley & Sons, N.Y. (1970).
- 3) Levenspiel, O.: "Chemical Reaction Engineering", John Wiley & Sons, N.Y. (1972).
- 4) Yagi, S. and D. Kunii: *Chem. Eng. Sci.*, **16**, 364, 372, 380 (1961).

(Presented at the 12th Autumnal Meeting of The Soc. of Chem. Engrs., Japan, at Okayama, Oct. 1978.)