ANALYSIS OF SEMI-CONTINUOUS THICKENER USING A BATCH SETTLING CURVE

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Using the batch settling curve of a feed slurry instead of the solid flux curve, a graphical analysis for semi-continuous thickeners with cyclic bottom discharge is developed on the basis of the Kynch theory for the sedimentation of suspension and the Talmage and Fitch technique for the design of thickener area.

The analysis presented in this paper provides a method for predicting concentration distributions, the height of a discontinuity in concentration, and optimum operating conditions in semi-continuous thickeners. The values predicted by this method coincided well with experimental results for precipitated calcium carbonate slurries over a wide range of underloading or overloading operations.

Introduction

Settling tanks or continuous thickeners are used perhaps more extensively than any other unit operation for the separation of liquid and solids. Most previous studies of the operation and design of continuous thickeners have been based on the solid flux (mass of solids crossing unit area in unit time) vs. concentration curve which is obtained from a series of batch settling tests carried out on suspensions having concentrations ranging from that of the feed to that of the discharge.

Despite the fact that a thickener is often operated under unsteady state where the feed and overflow are continuous while the discharge of thickened sludge is discontinuous, the determination of thickener size, the so-called height and cross-sectional area for thickening, is based on steady state. It is evident that thickener height would not constitute a capacity-controlling factor if it were possible to distribute at the feed plane all the particles in the feed absolutely evenly. The height required is therefore the height needed to bring about an even distribution of the particles below the feed inlet or to operate the thickener under changing conditions. The relationship between thickener height and the operating time required for thickening must be revealed for operating semi-continuous thickeners with cyclic bottom discharge.

There are only a few papers describing the principles of semi-continuous thickeners on the basis of solid flux vs. concentration curve. Since there is a close relationship between the solid flux curve and the batch settling curve, the design and analysis of thickener operations may be simplified by using the batch settling curve which is easily obtained. The design of thickener area using the batch settling curve of the feed concentration was proposed by Talmage and Fitch, and was extended to analyze continuous thickener operations at steady state. However, no analysis of semi-continuous thickeners using the batch settling curve has yet appeared.

In this paper, the Kynch theory for sedimentation of suspensions and the Talmage and Fitch graphical technique have been extended to the analysis of semi-continuous thickeners. This analysis using a batch settling curve provides a method for designing and predicting semi-continuous thickener operations. Experiments were carried out with precipitated calcium carbonate slurries in water.

1. Batch Settling Curve and Kynch’s Theory

The batch settling curves of suspensions are conveniently divided into three types by initial concentration.

At high dilutions there exists no visual interface, and such a sedimentation of suspension is termed “free settling”. In this concentration range particles in suspension may settle individually. As solid concentration is increased and particles are crowded closer together, a sharp interface between settling suspension and clear liquid appears. This sedimentation is called “zone settling or hindered settling”. In this paper, the analysis of semi-continuous thickeners operated within this concentration range was only discussed. At yet higher concentrations the sedi-
The Kynch theory has been adopted for describing batch settling because it offers the only quantitative analysis of batch settling at the present time. A great many studies have accepted his assumption that "at any point in a suspension the velocity of fall of a particle depends only on the local concentration of particles". A batch settling curve plotting the height, \( H [\text{cm}] \), of the interface between suspension and liquid as a function of time, \( t [\text{min}] \), is shown in Fig. 1. Kynch showed that the concentration which exists at the suspension-liquid interface at an arbitrary time, \( t_a \), must be

\[
C_i = C_f H_f / H_i
\]  

(1)

and the upward velocity of propagation, \( U(c_i) \), of a layer of concentration, \( c_i \), must be (Fig. 1)

\[
U(c_i) = H_i / t_a
\]  

(2)

where \( c_i \) = concentration of suspension at interface, \( c_f \) = initial concentration of suspension, \( H_f \) = initial height of suspension and \( H_i \) = intercept on \( y \)-axis of a tangent drawn to settling curve at time, \( t_a \). The settling velocity of the interface, \( v(c_i) \), corresponding to \( c_i \), is determined by the slope of the tangent line. The intercept of the tangent, \( H_c \), is now in proportion to \( 1/c_i \) from Eq. (1). The mean sludge concentration, \( \bar{c}_s \), below the interface, \( H_s \), at time, \( t_a \), is

\[
\bar{c}_s = c_f H_f / H_s
\]  

(3)

2. Theory of Semi-continuous Thickener Operation

A semi-continuous thickener is a settling tank with a constant feed of suspension at the top, a constant discharge of clarified water at the top and periodic discharge of accumulated sludge at the bottom. The concentrations of feed suspension and mean discharge sludge are assumed to be unchanged.

The manner of defining an operating point for the semi-continuous thickener will be described. A suspension having a feed solid concentration, \( c_f \), is continuously fed at a volumetric flow rate, \( Q_f [\text{cm}^3/\text{min}] \), to the thickener of cross-sectional area, \( A [\text{cm}^2] \), without bottom discharge as shown in Fig. 2. All particles in the suspension being fed uniformly over the full area of the thickener settle downwards and a generally clear overflow zone is formed above the feed inlet. Below that a settling zone is formed and the particles accumulate at the bottom of the thickener, forming a sludge zone. The concentrations in these zones can be determined directly from the batch settling curve of the feed suspension.

In a continuous thickener with continuous underflow at steady state, the feed flow velocity is \( Q_f / A \), the underflow velocity is \( Q_u / A \) and the overflow velocity is \( Q_o / A \). Consequently,

\[
Q_f / A = Q_u / A + Q_o / A
\]  

(4)

Each flow velocity can be expressed with \( \alpha, \beta, \) and \( \gamma \), which are obtained by the slopes of \( H_j P, RP' \), and \( H_j P' \) on the batch settling graph as shown in Fig. 3. The angles \( \alpha, \beta \) and \( \gamma \) show the flow rate of feed, overflow and discharge respectively for continuous thickener operation. A point, \( P' \), may be called the operating point expressing the operating conditions for the continuous thickener.

In a semi-continuous thickener without discharge, the underflow velocity, \( Q_u / A \), is zero. Therefore, after the thickener has been filled with slurry, Eq. (4) becomes

\[
Q_f / A = Q_o / A
\]  

(5)

Then the operating point shifts from the point, \( P' \), for the continuous thickener to a point, \( P \), for the semi-continuous thickener, because \( \beta \) is zero. Opera-
tion of the semi-continuous thickener, which can be classified for convenience into three cases, critical, under-, and over-loading operations, according to the value of feed solid flux, \( c_f Q_f/A \), will be discussed by the use of the operating point in the following sections.

2.1 Critical-loading operation

A critical-loading operation is defined as a maximum solid load which is fed to a semi-continuous thickener without bottom discharge when the overflow concentration, \( c_o \), is zero. This also defines the maximum capacity of the thickener.

In this case the solid particle balance in the settling zone must be as follows:

\[
c_f Q_f/A = c_f v(c_f)
\]  

Therefore, the operating point, \( P \), can be determined as shown in Fig. 4(a). In Fig. 4(a) the slope of the straight line indicating \( Q_f/A \) must be the same as the slope corresponding to the settling velocity, \( v(c_f) \), from Eq. (6). At a given time, \( t_o \), the depth and the concentration distribution of the accumulated sludge zone can be predicted using the batch settling curve of the feed concentration as follows:

Initially the feed concentration, \( c_f \), corresponding to the initial height, \( H_f \), of the batch settling curve is arbitrarily fixed on the abscissa in Fig. 4(b). A discontinuity between feed concentration, \( c_f \), and a critical concentration, \( c_s \), which was termed by Kynch a "first-order discontinuity", is firstly propagated upwards at a velocity, \( U(c_s) \), given by the slope of the line joining the critical settling point, \( S \), to a point, \( R \), in Fig. 4(a) where \( c_s \) is the concentration of the interface of settling at the critical point. The height of the discontinuity, therefore, can be estimated at time, \( t_o \), as

\[
H_d = U(c_s) t_o
\]  

The mean concentration, \( \bar{c}_s \), of the sludge zone and any concentration, \( c^*(c^*>c_s) \), in the sludge zone can be graphically determined from Eqs. (1) and (3) as shown in Fig. 4(b). It is recognized that the concentration, \( \bar{c}_s \), is independent of operating time and depth of the settling zone. Making use of these procedures the concentration distributions in the semi-continuous thickener can be simply determined in Fig. 4(b).

The critical-loading operation is a transition state varying from underloading operation to overloading operation and is the most fundamental operation for designing the semi-continuous thickener.

2.2 Underloading operation

An underloading operation is defined as the operation in which feed solids are less than the maximum capacity of the semi-continuous thickener, and needless to say the overflow concentration is zero.

In the underloading operation, the concentration of the settling zone is assumed to be \( c_o \) and, instead of Eq. (6), the solid particle balance in the settling zone must be

\[
c_f v(c_f) > c_f Q_f/A = c_f v(c_f), \quad c_f > c_o
\]  

According to Eq. (8), the slope of the line joining the points \( P \) and \( H_f \) in Fig. 5(a) represents the feed flow velocity, \( Q_f/A \), which must be lower than that corresponding to the settling velocity, \( v(c_f) \), at \( c_f \). In this case the critical concentration having the fastest upward velocity of propagation is obtained from the intercept, \( H_o \), on \( H \) axis, of the tangent from \( S \) to the settling curve and the concentration of the settling zone is estimated by \( H_o \), on the assumption that the settling velocity at \( c_s \) is equal to \( v(c_s) \). This procedure is permissible in practice because the actual settling velocity in the settling zone is nearly equal to that of the feed concentration. In Fig. 5(a), the point, \( H_o \), which is the height of a layer of the mean sludge concentration after any time, \( t_o \), can be given by

\[
H_o = U(c_s) t_o
\]  

where \( U(c_s) \) is the upward velocity of the layer with the concentration, \( c_s \), given by the slope of the line joining the points \( R \) to \( W \) and the tangent from \( H_s \) touches the settling curve at the point, \( W \).
By using this method, the concentration distribution in the semi-continuous thickener in underloading operation at the time, \( t_a \), can be obtained as shown in Fig. 5(b).

Since a practical semi-continuous thickener is usually operated at underloading, the concept of the underloading operation must be understood for exact design and operation.

### 2.3 Overloading operation

If a feed solid flux, \( c_fQ_f/A \), exceeds the maximum capacity of the semi-continuous thickener, partial solids of the feed will leave with the overflow. Such an operation may be termed “overloading operation” and the solid particle balance must be as follows:

\[
c_fQ_f/A > c_f(c_f) \tag{10}
\]

Since the maximum solid flux moving downwards in the settling zone is equal to \( c_f(c_f) \), the excess solid flux, which amounts to \( c_fQ_f/A - c_f(c_f) \), will be equal to the overflow solid flux, \( c_0Q_0/A \). In such a case, the concentrations of both the settling zone and the overflow zone are evidently the same as the feed concentration, \( c_f \). The concentration of the overflow, \( c_0 \), is determined from the following equation:

\[
c_0Q_0/A = c_fQ_f/A / c_f(c_f) \tag{11}
\]

As shown in Fig. 6(a), if the feed solid flux, \( c_fQ_f/A \), conveniently corresponds to a distance, \( H_fR \), the left-hand side of Eq. (11) can be graphically shown by a distance, \( H_fX \). The concentration of the overflow can, therefore, be graphically drawn directly from the following proportional expression as Fig. 6(b).

\[
c_0/c_f = H_fX/H_fR \tag{12}
\]

Consequently the concentration distribution in the semi-continuous thickener operated at overloading will be obtained as illustrated in Fig. 6(b). Although actual thickeners for clarifying have not been operated at overloading, the overloading operation is discussed here to recognize the analysis of semi-continuous thickeners.

### 3. Cyclic Discharge

The change of concentration in a settling tank, a semi-continuous thickener operating with the cyclic discharge of sludge, will be shown below according to an example of an optimum operation cycle proposed by Robins.

Assume that the thickener operates under the critical-loading conditions with a settling zone concentration, \( c_x = c_f \), and no bottom discharge. Let the discharge be periodic (period \( T \)) with an underflow velocity, \( Q_f/A \), for a fraction, \( f \), of the period and zero discharge for the rest of the cycle, as shown in Fig. 7.

With zero discharge, the discontinuity between \( c_f \) and \( c_x \), which is termed by Kynch the first-order discontinuity, rises with an upward velocity, \( U(c_x) \), from the bottom of the thickener to the height of the feed inlet, \( H_f = (1 - f)TU(c_x) \), after a time, \( t_x = (1 - f)T \). Immediately the discontinuity reaches the height, \( H_f \), at the time, \( t_x \), the bottom discharge starts with an underflow velocity, \( Q_u/A \), equal to \( tan \beta \) in Fig. 8. With underflow velocity, \( Q_u/A \), the concentration of the settling zone may change from \( c_f \) to \( c_{1w} \), and instead of the discontinuity between \( c_f \) and \( c_x \), a discontinuity between \( c_{1w} \) and \( c_{ex} \) will be formed and propagated downwards at a velocity

\[
Q_u/A - U(c_{ex}) = tan \beta - tan \gamma \tag{13}
\]

where \( U(c_{ex}) \) is the upward velocity of the discon-
continuity given by the slope of the line joining the points R and X. The fraction, $f$, and the period, $T$, can be therefore calculated from the following equation:

$$\frac{(1-f)TU(c_0)}{TQ_A-U(c_{ax})}=H_f \quad (14)$$

If the slope corresponding to $Q/A$ is decreased to that of the line joining the points R and S, $Q/A$ is equal to the upward velocity, $U(c_{ax})$, of a discontinuity between $c_{ax}$ and $c_{ax}$. Since the fraction, $f$, of the period, $T$, is unity in the constant discharge, the semi-continuous thickener operated under these conditions is the same as a continuous thickener operated at the maximum capacity described in the preceding paper. The concentrations, $c_{ax}$ and $c_{as}$, of the settling zone are here estimated on the assumption that the settling velocity, $v(c)$, of the concentration, $c(c < c_s)$, will be similar to that of the feed concentration, $c_f$.

The height of practical semi-continuous thickeners makes no contribution to increasing the capacity for thickening, as mentioned above, but does serve to operate the thickener steadily. If the height of the thickener were doubled, the period, $T$, is also doubled while other controlling factors, $c_s$, $c_{as}$, and $U(c_0)$, are naturally unchanged.

4. Experimental Results and Discussion

A suspension of precipitated calcium carbonate in water was used in this experiment. The dimensions of the experimental apparatus are shown in Fig. 9(b) and sampling and operations are the same as those in the preceding paper.

In Figs. 9(a) and 10(a), two height-time curves are shown for the initial concentrations 0.047 and 0.031 g/cm$^3$ with a glass measuring cylinder 30 cm high of 1000 ml. The critical settling points expressed in $Y$ were distinctly observed in each case, marked by a sudden decrease in settling velocity.

Since the critical-loading operation is unstable with a small deviation in the rate or concentration of the feed, experiments under only two different conditions, underloading operation and overloading operation, will be discussed below.

4.1 Underloading operation

With a feed concentration of 0.047 g/cm$^3$, a feed rate, $Q_f$, of 191 cm$^3$/min and zero discharge, the operating point could be obtained as P shown in Fig. 9(a). Since the settling velocity at the feed concentration was faster than the feed flow velocity, $Q/A$, this was an underloading operation of the semi-continuous thickener as previously mentioned.

The experimental semi-continuous thickener was filled with the feed slurry at the beginning of experiments. The height of an observable discontinuity between $c_s$ and $c_{ax}$ reached a height of 12 cm 86 minutes after the feed slurry was supplied to the thickener, and then the concentration distribution in the thickener was measured. As the discharge pump was started and adjusted to a discharge rate of 155 cm$^3$/min after 87 minutes, the operating point shifted from P to P' in Fig. 9(a). After 102 minutes a new observable discontinuity between $c_s$ and $c_{as}$ reached the bottom of the thickener, and then the concentration distribution was also measured. With or without bottom discharge, these discontinuities in concentration will differ respectively: with zero discharge the theoretical concentration, $c_s$, of the settling zone corresponds to the intercept, $H_s$, and the critical concentration, $c_{ax}$, to $H_{ax}$, while with discharge rate of 155 cm$^3$/min, $c_s$ corresponds to $H_s'$ and $c_{as}$ to $H_{as}'$.

A predicted height, $H_s$, of the discontinuity which would be reached 86 minutes after the feed slurry was supplied to the thickener could be calculated as follows:

$$H_s=U(c_f)t=0.136\times86=11.7 \text{ cm}$$

Supposing that the required period of time before the concentration of the settling zone varied from $c_s$ to $c_{ax}$ was negligible, a theoretical discharge duration could be given by

$$H_{as}=[Q/A-U(c_s)]=12/(0.939-0.060)=13.7 \text{ minutes, while the actual discharge duration was 15 minutes.}$$

The period, $T$, of one cycle (cf. Fig. 7) and the fraction, $f$, of the period mentioned above could be given by Eq. (14). That is

$$f=U(c_s)[Q/A-U(c_s)+U(c_0)]$$

$$=0.136/(191/165.1-0.060)+0.136=0.110 \quad [-]$$

and

$$T=H_s/(1-f)U(c_s)=22/(1-0.11)\times0.136=182 \text{ minutes.}$$

Therefore, a mean theoretical concentration, $\bar{c}_s$, of

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the underflow would be
\[ c_u = \frac{c_f Q_f}{f Q_u} = (0.047 \times 191)/(0.11 \times 155) \]
\[ = 0.527 \text{ g/cm}^3 \]
while the measured average concentration of bottom discharge was 0.48 g/cm³.

The predicted and measured concentration distributions in the thickener 86 and 102 minutes after the feed pump was started are also shown in Fig. 9(b).

4.2 Overloading operation

With a feed concentration of 0.031 g/cm³, a feed rate of 336 cm³/min and zero discharge, the operating point, P, as well as with a discharge rate of 110 cm³/min, the operating point, P', were determined as shown in Fig. 10(a). Since a part of the feed solids was contained in the overflow operated under these conditions, this was called "overloading operation".

After 65 minutes from the fine feed slurry was supplied to the thickener without discharge, an observable discontinuity between \( c_1(=c_f) \) and \( c_c \) reached a height 12 cm from the bottom of the thickener, while a predicted height, \( H_d \), of the discontinuity could be calculated as follows:

\[ H_d = U(c_c) t = 0.165 \times 65 = 10.7 \text{ cm} \]
The concentration distribution in the thickener was then measured and is shown in Fig. 10(b). Both the measured concentration, \( c_1 \), of the settling zone and \( c_s \) of the overflow zone were exactly equal to the feed concentration as expected. The theoretical concentration, \( c_u \), of the overflow could be given by Eq. (12), that is

\[ c_u = c_f (H_f X/H_f R) = 0.031 \times (7.6/30) \]
\[ = 0.0079 \text{ g/cm}^3 \]
while a mean measured value was 0.007 g/cm³.

The discharge pump was started 66 minutes after the feed pump was started, and after 96 minutes a new observable discontinuity between \( c'_1 \) and \( c'_c \) reached the bottom of the thickener. With a discharge rate of 110 cm³/min, the overflow was, of course, transparent.

As can be seen from Figs. 9 and 10, the theoretical values were in good agreement with the measured values. Although there were small differences in the upward and downward velocities of the discontinuity between theory and experiment, the presented analysis might be in practice a useful method for the semi-continuous thickener.

Conclusion

The analysis of semi-continuous thickeners using the batch settling curve of a suspension at the feed concentration can be derived. This method, based on the Kynch theory, is quite adequate for flocculated suspensions like precipitated calcium carbonate slurry in water.

The effectiveness of the analysis has been demonstrated by comparing theoretical concentration distributions with corresponding experimental data, showing excellent agreement.

Nomenclature

\[ A = \text{cross-sectional area of thickener} \quad [\text{cm}^2] \]
\[ c = \text{solid concentration} \quad [\text{g/cm}^3] \]
\[ f = \text{fraction of period during which discharge takes place in cyclic discharge} \quad [-] \]
\[ H = \text{height from the bottom} \quad [\text{cm}] \]
\[ Q = \text{volumetric flow rate} \quad [\text{cm}^3/\text{min}] \]
\[ T = \text{period of one pumping cycle} \quad [\text{min}] \]
\[ t = \text{time} \quad [\text{min}] \]
\[ v = \text{settling velocity of solid particles} \quad [\text{cm/min}] \]
\[ U = \text{upward or downward velocity of a discontinuity in concentration} \quad [\text{cm/min}] \]

(Subscripts)

\[ c = \text{critical} \]
\[ d = \text{discontinuity} \]
\[ f = \text{feed} \]
\[ i = \text{interface} \]
\[ o = \text{overflow} \]
\[ s = \text{sludge} \]
\[ u = \text{underflow} \]
\[ 1 = \text{below the feed inlet of thickener} \]
\[ 2 = \text{above the feed inlet of thickener} \]

Literature Cited