Fig. 2 Variation of wave inception

observed values for $\theta=9^\circ$ were excluded in the derivation of the correlation Eq. (1) because of the low accuracy of h_{c*} in that case, the agreement of the observed values of x_c with the calculated values is very good. It is concluded that the position of wave inception has been expressed successfully.

Nomenclature

h = liquid film thickness [cm]

h_* = $h(g \sin \theta)^{1/3}/\nu^{2/3}$ [—]
 Q = $\bar{u} \cdot h$ [cm²/sec]
 Re = $4Q/\nu$ [—]
 $\bar{u}$ = average fluid velocity [cm/sec]
 x = distance from the entrance [cm]
 x_* = $x(g \sin \theta)^{1/3}/\nu^{2/3}$ [—]
 z_* = $(h_c - h_\infty)(g \sin \theta)^{1/3}/\nu^{2/3}$ [—]

θ = inclination of plate to horizon [deg]
 ν = kinematic viscosity of liquid [cm²/sec]

<Subscripts>

c = at the position of wave inception
 ∞ = at infinity ($x=\infty$)

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COOLING OF HEAT-GENERATING FLUID BY A PARALLEL-PLATE HEAT EXCHANGER

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In the previous work²⁻⁴⁾, the conjugated transfer problem between fully-developed laminar flow and wall was studied for cases with and without heat generation in the fluid. The influence of two-dimensional thermal conduction in the wall and of heat generation on the heat transfer characteristics was clarified. In the present paper, the procedure used in the previous reports has been extended to analyze steady heat transfer in a parallel-plate heat exchanger, where Fluid 1 with uniform internal heat source is cooled by Fluid 2 flowing countercurrently.

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Theoretical

The heat transfer in both fluids was dealt with in a way similar to that for the classical Graetz problem, except for the heat source in Fluid 1. The problem is assumed to be symmetrical with respect to each flow-axis of both streams. Dimensionless variable y_i^* is the distance from axis of flow (i) towards the wall, and x_i^* is the dimensionless distance from the inlet of stream (i). Each dimensionless variable is so defined as to fit into the full range [0, 1]. Concerning the wall, y_s^* is used to indicate the dimensionless distance from the interface between Fluid 2 and the

wall. Fluid 1 flows into the heat exchanger at uniform temperature $\theta_{f1}=1$ and is accompanied by a uniform heat source.

Fluid 2 flows into the exchanger at uniform temperature $\theta_{f2}=0$. Both ends of the exchanger wall are thermally insulated. All the physical properties are assumed to be constant. Hereafter the interface between Fluid 1 and the wall is referred to as Interface I and that between Fluid 2 and the wall as Interface II.

If we set unknown temperatures at Interface I and Interface II as $f_1(x_1^*)$ and $f_2(x_2^*)$, respectively, we then have the temperature of Fluid 1 as an extension of the Graetz solution; that is⁽⁴⁾,

$$\theta_{f1} = \frac{Q^*}{2} (1 - y_1^{*2}) + \sum_{m=1}^{\infty} (C_m - Q^* C_c) Y_m(y_1^*) \times \exp\left(-\frac{8\lambda_m^2 x_1^*}{3Gz_1}\right) + \int_0^{x_1^*} f_1(\xi) \frac{\partial}{\partial x_1^*} \times \left[1 - \sum_{m=1}^{\infty} C_m Y_m(y_1^*) \exp\left\{-\frac{8\lambda_m^2 (x_1^* - \xi)}{3Gz_1}\right\}\right] d\xi \quad (1)$$

where

$$C_c = \frac{\int_0^1 (1 - y_1^{*2})^2 Y_m(y_1^*) dy_1^*}{2 \int_0^1 (1 - y_1^{*2}) Y_m^2(y_1^*) dy_1^*}$$

for Fluid 2

$$\theta_{f2} = \int_0^{x_2^*} f_2(\xi) \frac{\partial}{\partial x_2^*} \left[1 - \sum_{m=1}^{\infty} C_m Y_m(y_2^*) \times \exp\left\{-\frac{8\lambda_m^2 (x_2^* - \xi)}{3Gz_2}\right\}\right] d\xi \quad (2)$$

In the above relations, λ_m , C_m and $Y_m(\cdot)$ indicate the eigenvalues, expansion coefficients and eigen-functions of the Graetz problem, respectively.

The wall temperature is given as follows, using the Fourier cosine series.

$$\theta_s = y_s^* \int_0^1 f_1(\xi) d\xi + (1 - y_s^*) \int_0^1 f_2(1 - \xi) d\xi + 2 \sum_{m=1}^{\infty} \frac{\cos(m\pi x_1^*)}{\sinh(m\pi L^*)} \left[\sinh(m\pi L^* y_s^*) \int_0^1 f_1(\xi) \cos(m\pi \xi) d\xi + \sinh\{m\pi L^* (1 - y_s^*)\} \int_0^1 f_2(1 - \xi) \cos(m\pi \xi) d\xi \right] \quad (3)$$

The unknown interfacial temperatures are expanded as an infinite power series that includes unknown coefficients. Such coefficients are determined in a way similar to those in the previous work, using the following conjugated boundary conditions.

$$-\frac{\partial \theta_{f1}}{\partial y_1^*} \bigg|_{y_1^*=1} = R_{w1} \frac{\partial \theta_s}{\partial y_s^*} \bigg|_{y_s^*=1} \quad (4)$$

$$-\frac{\partial \theta_{f2}}{\partial y_2^*} \bigg|_{y_2^*=1} = -R_{w2} \frac{\partial \theta_s}{\partial y_s^*} \bigg|_{y_s^*=0} \quad (5)$$

To carry out theoretical calculations, the first twenty-five eigenvalues were used. Parameters were arbi-

Table 1 Conditions for theoretical calculations

Case	Stream	Heat source	R_w	Problem
1	multiple	$Q^*=20$ in Fluid 1	10	heat exchanger
2	multiple	$Q^*=20$ in Fluid 1	1000	heat exchanger
3	multiple	—	10	heat exchanger
4	multiple	—	1000	heat exchanger
5	single	$Q^*=20$	10	conjugated between flow and wall
6	single	$Q^*=20$	1000	
7	single	$Q^*=20$	—	const. temp. at interface
8	single	—	—	const. temp. at interface (Graetz)

trarily fixed as $Gz_1=Gz_2=10$, $L^*=0.01$ and $R_{w1}=R_{w2}=R_w$. Representative cases where theoretical calculations were made are listed in **Table 1**.

Temperature profiles in the heat exchanger for Case 1 are given in **Fig. 1**. In Case 1 accompanied by the uniform heat source, the exchanger wall has the lesser thermal conductance. The slope of the wall temperature profile along y_s^* varies with x_1^* markedly. Therefore, the thermal conduction process in this case should not be approximated as a one-dimensional problem. The temperatures at Interface I and Interface II depend on x_1^* and have certain maximum values. The temperature profile at $x_1^*=0.02$ shows that Fluid 1 is not cooled but rather is heated by Fluid 2.

Figure 2 shows temperature profiles in the exchanger for Case 2. In this case the thermal conductance of the wall is as high as $R_w=1000$, in comparison with that for Case 1. There is no appreciable variation in wall temperature with y_s^* . This is the most important difference from Case 1. However, the temperatures for various values of x_1^* are clearly different from each other. It is known in this case that the thermal conduction in the wall can be treated as a one-dimensional problem only in the x_1^* direction. Kroeger studied theoretically the influence of longitudinal conduction in the wall on the effectiveness of the exchanger⁽¹⁾. He approximated the heat conduction in the wall as a one-dimensional case, just as in the above case. However, he did not take into account the mutual influence between the two fluid streams and the wall. Consequently, the present results cannot be compared with his. In contrast with that in Case 1, Fluid 2 in Case 2 is constantly heated throughout the full range of x_1^* . On the other hand, in the small x_1^* region Fluid 1 is heated by the wall. Accordingly, it is found that a part of the heat transferred from Fluid 1 to the wall in the large x_1^* region is consumed to heat Fluid 1 itself in the small x_1^* region.

Nunge *et al.*⁽⁵⁾ made a theoretical analysis of the counterflow, parallel-plate heat exchanger without heat source. They assumed that heat fluxes at Inter-

Fig. 1 Temperature profiles for Case 1

Fig. 2 Temperature profiles for Case 2

face I and Interface II at any specified x_1^* are equal to each other, and neglected longitudinal conduction in the exchanger wall. In the present cases with an internal heat source, the heat fluxes at the interfaces are not identical with each other. As a result, that assumption is known to be inadequate in cases with internal heat generation.

Concluding Remarks

A counterflow heat exchanger in which a heat-

generating fluid is cooled shows thermal behavior different from that of the usual exchanger without heat source. According to its operating conditions, self-heating of the heat-generating fluid can occur through the exchanger wall, and heating of the heat-generating fluid by the cooling fluid also occurs. In addition, the influence of longitudinal conduction in the exchanger wall is significant, compared to that in the usual heat exchanger.

Nomenclature

C_c	= expansion coefficients defined in Eq. (1)	[—]
C_m	= expansion coefficients of Graetz problem	[—]
C_p	= heat capacity of fluid	[J/(kg·K)]
Gz	= Graetz number, $4w^2V\rho C_p/(k_f L)$	[—]
k	= thermal conductivity	[W/(m·K)]
L	= length of heat exchanger	[m]
L^*	= δ/L	[—]
Q	= volumetric heat source	[W/m ³]
Q^*	= $Qw_1^2/\{k_{f1}(t_{e1}-t_{e2})\}$	[—]
R_w	= $k_s w/(k_f \delta)$	[—]
t	= temperature	[°C]
V	= mean velocity of fluid	[m/s]
w	= half-height of duct	[m]
x	= longitudinal coordinate	[m]
x^*	= x/L	[—]
$Y_m(\cdot)$	= eigenfunctions of Graetz problem	[—]
y	= coordinate perpendicular to flow direction	[m]
y^*	= y/w	[—]
y_s	= distance from Fluid 2-wall interface	[m]
y_s^*	= y_s/δ	[—]
δ	= wall thickness	[m]
Θ	= $(t-t_{e2})/(t_{e1}-t_{e2})$	[—]
λ_m	= eigenvalues of Graetz problem	[—]
ξ	= dummy variable	[—]
ρ	= fluid density	[kg/m ³]

<Subscripts>

e	= entrance of heat exchanger
f	= fluid
i	= both Fluid 1 and Fluid 2
s	= wall
1	= Fluid 1
2	= Fluid 2

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