KÁRMÁN VORTEX BEHIND A CIRCULAR CYLINDER IN DILUTE POLYMER SOLUTIONS

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In a previous paper³⁾, it was shown that the numerical solution of the cross flow around a circular cylinder submerged in viscoelastic fluids agreed with the experimental results obtained by James and Acosta¹⁾. The analysis was restricted to the moderate Reynolds number range ($1 \leq Re \leq 20$), because of vortex formation at higher Reynolds numbers. In the higher Reynolds number range, an individual fluid particle undergoes a more rapid change in deformation, and this rapid straining is expected to vary the flow field more significantly if the test fluid has viscoelasticity. James and Gupta²⁾ reported that the drag coefficient of a circular cylinder submerged in viscoelastic fluid flow showed a drastic increase in this Reynolds number range. For example, they observed that the drag coefficient in a 160-ppm polymer solution of Polyox Coagulant was almost 5 times the Newtonian drag coefficient at $Re=100$. This information should be combined with the stability and vortex formation problems in viscoelastic cross flow. The present work was thus planned to observe Kármán vortex formation in polymer solutions by a flow visualization technique. The results of Strouhal number measurements will be compared with the previous experimental results to discuss what kind of non-dimensional parameter should be suitable to correlate the observed phenomenon.

1. Experimental Apparatus

The water tunnel is in many respects similar to that described by Mizushima *et al.*⁴⁾. A detailed diagram of the test section is shown in Fig. 1. A one-pass system was adopted to minimize the mechanical degradation of polymer additives. Test fluids were forced to flow through the calming section and the convergent nozzle to obtain uniform laminar flow. A circular cylinder was set perpendicularly to the flow direction in the test section, which was made of Plexiglas with a cross-sectional area of $180 \times 80 \text{ mm}^2$. Three kinds of test cylinder made of stainless steel were used. Outer diameters of these cylinders were 1.6, 3 and 6 mm. The polymer solution, coloured by

India ink but otherwise the same as the test fluid, was injected from a fine glass tube (0.7 mm O.D.) at the same velocity as the main flow velocity. The height of the glass tube was adjusted carefully so that the stream line of coloured solution hit the front stagnation point of a circular cylinder.

The frequency of Kármán vortex formation was measured by stopwatch from observation of the fluctuation stream line just behind the circular cylinder. The approaching velocity was measured by the hydrogen bubble method or the particle tracer method. Both methods could be used in the case of a Newtonian fluid. But a viscoelastic fluid seemed to prevent clear streak-line formation in the hydrogen bubble method. The particle tracer method was thus adopted for velocity measurements of polymer solutions. A small amount of polyethylene particles (0.15 mm dia., $\rho=1.04 \text{ g/cm}^3$) was suspended and photographed in the light of a stroboscope. The velocity of a tracer particle was determined by a film motion analyser and was assumed to be equal to the fluid velocity. It was confirmed that the velocity in the test section was uniform except near the wall. The boundary-layer thickness was approximately 1 cm.

As the polymer additive, polyethylene oxide (PEO, grade ALCOX E-160, manufactured by Meisei Chemical Inc.) was used. The polymer additive was solved in tap water to give concentrations of 100, 200 and

Fig. 1 Diagram of experimental apparatus

Received May 21, 1979. Correspondence concerning this article should be addressed to H. Usui.

Fig. 2 Strouhal number of Newtonian fluid in terms of Reynolds number

Fig. 3 Strouhal number of polymer solutions in terms of Reynolds number

400 ppm. The viscosity of the polymer solution was measured by an Ostwald viscometer. Because of the diluteness of the solutions, the effect of shear-dependent viscosity was not taken into account in this study.

2. Results and Discussion

The results of Strouhal number measurements for Newtonian fluid are shown in Fig. 2. The broken line indicated in the same diagram is the experimental result of Roshko⁵⁾ given by

$$S = 0.212 - 4.5/Re \quad (1)$$

The results of this work are a little different from the result of Roshko. To verify the accuracy of the ex-

perimental results of this work, an electro-chemical technique was also adopted to measure Strouhal number. A hot-wire probe made of platinum was set behind the circular cylinder. The fluctuating velocity was recorded to calculate the frequency of the Kármán vortex formation. This result is also plotted in Fig. 2. The results by these two methods agreed with each other and resulted in an experimental correlation equation given by

$$S = 0.212 - 0.43/Re^{0.45} \quad (2)$$

The difference between Roshko and this work may be attributed to the experimental apparatus factors such as the blocking ratio or something similar. More precise discussion of this difference is not possible at the present stage. Thus a comparison of Strouhal number in polymer solutions with that of Newtonian fluid given by Eq. (2) will be made in the following section of this paper.

The experimental results for polymer solutions are shown in Fig. 3. A striking suppression of vortex formation is observed. Increase of polymer concentration and decrease of cylinder diameter seem to have a more significant effect in the decrease of Strouhal number in polymer solutions. A decrease in the frequency of Kármán vortex means an increase in the drag coefficient of a circular cylinder in cross flow. This observation agrees qualitatively with measurements of the drag coefficient made by James and Gupta²⁾.

To correlate the experimental results, a characteristic time scale for dilute polymer solutions was calculated from the following equation.

$$\lambda = \frac{2}{5} \frac{\eta_s [\eta]^2 Mc}{RT} \quad (3)$$

This relaxation time was given by James¹⁾ for dilute polymer solutions. In this experiment M and $[\eta]$ were determined from measurement of the intrinsic viscosity as 4.3×10^6 [g/g-mole] and 1550 [cm³/g] respectively. Weissenberg number, $We (= \lambda U/D)$, was employed as the non-dimensional parameter to express the phenomenon of vortex formation in viscoelastic fluids. The correlation of Strouhal number with the Weissenberg number is shown in Fig. 4. In this diagram, S_N and S_P indicate the Strouhal number of Newtonian fluid and viscoelastic fluid at the same Reynolds number. All the results seem to be well correlated by

$$S_P = 0.212 - (0.43/Re^{0.45}) - 0.203 We^{0.32} \quad (4)$$

In the case of PEO=100 ppm with $D=3$ mm, the difference between S_N and S_P is very small and the experimental error seems to become relatively large. Although considerable scatter of experimental results is observed it appears that we can confirm the effectiveness of the Weissenberg number in correlating

vortex formation of viscoelastic fluids.

James *et al.*^{1,2)} successfully used the non-dimensional parameter $D/\sqrt{\lambda\nu}$ ($=\sqrt{Re/We}$) to explain the anomalous effects of polymer additives on the heat transfer and drag of a circular cylinder in cross flow. Ultman and Denn⁶⁾ also recommended use of the same parameter to describe the flow behavior of polymer solutions in cross flow, although their analysis was restricted to very low Reynolds numbers. Thus the parameter $D/\sqrt{\lambda\nu}$ was employed to correlate the experimental results of this work as shown in Fig. 5. This diagram indicates that the parameter $D/\sqrt{\lambda\nu}$ is also a good parameter to explain the effect of polymer additive on the vortex formation. If Reynolds number becomes sufficiently high, the parameter $D/\sqrt{\lambda\nu}$, which contains the effect of the inertial term, should be a more preferable parameter than the Weissenberg number. But the fact that the Weissenberg number can correlate Kármán vortex formation, as shown in Fig. 4, means that the inertial term may not so effective for this phenomenon. This is because the Weissenberg number represents the ratio of elastic force to viscous force. Thus it is hard to say which parameter, We or $D/\sqrt{\lambda\nu}$, better explains the phenomena of this work.

Conclusion

Measurements of Strouhal number in cross flow showed that Kármán vortex formation was strongly suppressed by the polymer additive. The deviation of Strouhal number in polymer solutions from the Newtonian case was well correlated by either Weissenberg number or $D/\sqrt{\lambda\nu}$.

Nomenclature

c	= polymer concentration	[ppm or kg/m ³]
D	= cylinder diameter	[m]
M	= molecular weight of polymer	[kg/kg-mole]
n	= frequency of vortex formation	[1/sec]
R	= gas constant	[kg ² ·m/(kg-mole·°K·sec ²)]
Re	= Reynolds number ($=DU/\lambda$)	[—]
S	= Strouhal number ($=nD/U$)	[—]
T	= temperature	[°K]
U	= approaching velocity	[m/sec]
We	= Weissenberg number ($=\lambda U/D$)	[—]
η_s	= solvent viscosity	[kg/m·sec]

Fig. 4 Correlation of Strouhal number of polymer solutions with Weissenberg number

Fig. 5 Correlation of Strouhal number of polymer solutions with the parameter $D/\sqrt{\lambda\nu}$

$[\eta]$	= intrinsic viscosity	[m ³ /kg]
λ	= relaxation time	[sec]
ν	= kinematic viscosity of polymer solutions	[m ² /sec]

<Subscript>

N	= Newtonian fluids
P	= polymer solutions

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