DRYING CHARACTERISTICS OF PARTICLES IN A CONSTANT DRYING RATE PERIOD IN VIBRO-FLUIDIZED BED

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Particles were dried in a vibro-fluidized bed and the drying characteristics in a constant drying rate period are discussed. Ion exchange resin particles were used for sample because they were fluidized smoothly even at a somewhat high moisture content.

The effects of vibration on the uniformity of moisture content in the bed were remarkable when air velocities become lower than the minimum fluidization velocity \( u_{mf} \). The bed could be dried uniformly and it had no moisture content distributions under the appropriate vibrational conditions, even when air velocities were lower than \( u_{mf} \).

The uniformity of moisture content in the bed depended upon air velocity, vibrational intensity, and height of bed.

Introduction

Generally, air velocities more than several tens of times larger than the minimum fluidization velocity are used for the fluidized bed dryer\(^1\). This is required for good fluidization and for the uniformity of quality of the dried products. But these air conditions are undesirable from the standpoint of heat and other energy consumption and for the entrainment of undried particles. For the drying of oxygen-sensitive materials such as medicines and foods, the use of a large amount of air is undesirable for fear of oxidation and denaturation. Therefore, fluidized bed drying is expected to be operated at as low air velocities as possible. In this respect, the vibro-fluidized bed, which enhances the fluidity of the bed through addition of vibration to the bed, is considered as one of the more useful types of drying equipment and is in practical use in drying processes\(^7\).\(^8\).

On the effects of vibration, some research workers have already reported that it increases the circulation rate of the particles\(^2\) and the heat transfer coefficient\(^3,6,10,13\). Yet the investigations performed on the drying rate of particles in the vibro-fluidized bed are still few. Bukareva \( et \ al.\)\(^4\) and Arai \( et \ al.\)\(^1\) reported that the heat transfer rate and the drying rate were improved by the vibro-fluidized bed. But there is not yet a quantitative study available on the relation between the drying characteristic and the condition of vibration. We have thus tried to investigate the drying characteristics of particles in a bed using vibro-fluidized bed drying apparatus and to determine the usefulness of the vibro-fluidized bed as drying equipment.

In this study, we investigated the drying results of ion exchange resin particles that are fluidized smoothly even when their moisture contents are rather high, up to 0.5 kg-H\(_2\)O/kg-d.m. In the study of drying rates and moisture content distributions of the bed that depend upon the vibrational conditions, an analytical model that includes the particle circulation rate as a parameter was applied. The air velocities used in the experiments were mainly near or lower than the minimum fluidization velocity for the purpose of studying the effects of vibration in especially low air velocity regions.

1. Drying Model

In the vibro-fluidized bed, the particle circulation rate varied under the influence of vibrational conditions, and thus a fluidized bed drying model that assumes the complete mixing of particles could not be applied.

For this reason, a drying model which includes the particle circulation rate as a parameter has to be adopted to calculate drying rate and moisture content.
distribution in the bed in correspondence with all the states of fluidization.

This model is illustrated schematically in Fig. 1. It is similar to the fluidized bed solid circulation model which divides the bed into two sections. We extended the model in all sections, and further took into account the drying rate. It can be assumed that the rates of particles moving between one section and the other are identical, and that no moisture content deviations exist in any one of the sections. The moisture content distributions in axial direction within the bed are represented by the average moisture contents in each section. The material balances in each section can be expressed in the following equations:

\[ M_{d}W_{1}/d\theta = R_{1} - R_{1} \]
\[ M_{d}W_{2}/d\theta = R_{2} - R_{2} \]
\[ \vdots \]
\[ M_{d}W_{n-1}/d\theta = R_{n-1} - R_{n-1} \]
\[ M_{d}W_{n}/d\theta = R_{n} - R_{n} \]

where \( R_{n} \) is the drying rate in section \( n \). If the humidity in the air reaches saturation in any one section, the drying rates in the upper sections will be equal to zero. And the drying rates in the sections whose moisture contents are lower than the critical moisture content \( W_{c} \) are assumed to decrease in proportion to the free moisture content. Generally, most of the transport phenomena are achieved in the bottom region of the fluidized bed. Therefore, it is possible to determine the number of sections where the drying rates in the sections except in section 1 are nearly equal to zero.

Analytical solutions of the simultaneous differential equations (1) can be obtained only when \( n \) is less than 5 and \( R_{1} \) is equal to zero for \( f \geq 2 \). So we used a model in which the number of sections is four, and the drying rates in the sections higher than section 1 are zero in this study since this makes it easier to discuss the drying characteristics of the bed.

The dimensionless relations between the average moisture contents in each section and the drying time were derived from Eqs. (1) as follows (see Appendix):

\[ W_{1}/W_{0} = 1 - \frac{6}{SB} + \frac{l}{SB} \left\{ A_{1} + \frac{1}{V_{2}^{2}} A_{2} + \frac{1}{V_{3}^{2}} A_{3} \right\} \]
\[ W_{2}/W_{0} = 1 - \frac{6}{SB} + \frac{l}{SB} \left\{ A_{1} + \frac{1}{V_{2}^{2}} A_{2} + \frac{1}{V_{3}^{2}} A_{3} \right\} \]
\[ W_{3}/W_{0} = 1 - \frac{6}{SB} + \frac{l}{SB} \left\{ A_{1} + \frac{1}{V_{2}^{2}} A_{2} + \frac{1}{V_{3}^{2}} A_{3} \right\} \]
\[ W_{4}/W_{0} = 1 - \frac{6}{SB} + \frac{l}{SB} \left\{ A_{1} + \frac{1}{V_{2}^{2}} A_{2} + \frac{1}{V_{3}^{2}} A_{3} \right\} \]

where \( A_{1} = 1 - \exp \left( \frac{-8B}{SB} \right) \), \( A_{2} = 1 - \exp \left( \frac{-4(2 - \sqrt{2})B}{SB} \right) \), \( A_{3} = 1 - \exp \left( \frac{-4(2 + \sqrt{2})B}{SB} \right) \), \( \Theta = \theta / \theta_{0} \), \( \theta_{0} = W_{0} M_{0}/R_{c} \), \( B = R_{0}/M_{0} \), and \( R_{c} = (G_{c} C_{u} A / \lambda_{u}) (t_{w} - t) \left[ 1 - \exp \left( -haL/G_{c} C_{p} \right) \right] \). But the heat generated in the bed by the vibration was not taken into consideration for \( R_{c} \). The dimensionless particle circulation rate \( B \) may also be regarded as an index of the uniformity of moisture content in the bed.

When the change in moisture content in any section is measured, the optimum value of \( B \) or \( r \) can be estimated by Eqs. (2). Consequently, the particle circulation rate and the moisture contents in the other sections or the moisture content distributions can be evaluated by the value of \( B \).

2. Experimental Apparatus and Procedures

The schematic diagram of the experimental apparatus used in this study is shown in Fig. 2. The test cylinder was made of an acrylic resin tube, measuring 110 mm in inner diameter and 500 mm in height. The exterior wall of the cylinder was covered with a 20 mm-thick foaming polystyrol plate for heat insulation. A small part of the heat insulator was cut open and covered with a thin, transparent polyvinyl
chloride plate in order to measure the bed height. The distributor was of the perforated-plate type. It consisted of a 20 mm-thick acrylic resin plate covered at the upper surface with a 300-mesh stainless steel wire net. The hole diameter was 2.5 mm and the pitch was 4.5 mm. The system of the vibration generating device is shown in Fig. 2. The design error from the sinusoidal vibration was less than 3%.

The frequency of vibration was measured by a tachometer, and a calibrated wedge-shaped rule having an error less than 0.025 mm was used to measure the amplitude of vibration.

Ion exchange resin particles (polystyrol resin, $D_p=420-590 \mu\text{m}$, $\rho_p=1.2 \text{g/cm}^3$) were used for sample. The initial moisture contents prepared for the experiments were within a range from 0.3 to 0.5 kg-$\text{H}_2\text{O}$/kg-d.m. The measured value of critical moisture content was 0.16 kg-$\text{H}_2\text{O}$/kg-d.m. in the air conditions used.

A small portion of the bed was collected at every time interval to determine the moisture content. The sample collector is shown in Fig. 3. The inner diameter of the collecting tube was 4 mm, and a hand-operated net cap was set on top of the tube in order to minimize collecting error. The particles were collected at the center levels of sections 1 and 4, assuming that the moisture contents at those levels represented the average moisture contents in each section. In this study, the optimum values of $B$ were computed from the moisture content changes in both sections 1 and 4 by the trial-and-error method. The temperatures of the bed and the air were measured by copper-constantan thermocouples. The humidity in the air was determined by the dry- and wet-bulb temperatures measured by copper-constantan thermocouples, and those values had been pre-examined by a hygrometer (Ace Recording Hygrometer AR-33).

3. Bed Height per Section

If the heat generated in the bed by the vibration is not taken into account, the drying rate of the whole bed can be expressed by

$$-M_o dW/d\theta = (G_o C_H A/\lambda_w)(t_i - t_o)$$

Then the following relation between the volumetric coefficient of heat transfer and the height of the bed is derived:

$$h_a = -(G_o C_H / L) \ln (t_o - t_w)/(t_i - t_w)$$

Though the air temperature was measured nearly equal to the wet-bulb temperature at about 5 mm above the distributor, and so $t_w = t_o$ in the vibro-fluidized bed, the bed height per section used was determined in the following manner.

The empirical equation for the through-flow drying established by Wilke et al.\textsuperscript{11} was applied to calculate the value of the heat transfer coefficient $h$

$$h / G_o C_H = 2.407 (Re_p)^{-0.51} \text { for } Re_p < 250$$

The estimated value of $h_a$ is about 13,000 kcal/m$^3$·°C in the case of $u/u_{mf}=0.5$ ($Re_p=1.33$) as an example, hence the value of the specific surface area of particle $a$ is extrapolated to nearly one-fortieth of the theoretical value ($=6(1-s)/D_p$). Then the value of $l$ at which the humidity in the air reaches saturation was calculated to be 0.9 cm, when 95% saturation was postulated, and 1.5 cm for 99% saturation. Therefore, it is considered that 1 cm or somewhat more may be applied in the height of the bed for one section.

4. Experimental Results and Discussion

The vibro-fluidized bed exerted a remarkable effect on the uniformity of moisture content in the bed at air velocities lower than the minimum fluidization velocity $u_{mf}$. An example is shown in Fig. 4. In this figure, the superficial air velocity was 0.7 $u_{mf}$, and the bed height was 4 cm. On the relation between $W/W_o$ and $\theta/\theta_o$, the solid lines show the calculated values by the through-flow drying equation\textsuperscript{11}. The broken line indicates the computed values by assuming complete mixing and using the equation $R_i=(G_o C_H A/\lambda_w)(t_i - t_o)$. The temperature profiles are the measured values.
The drying characteristics under conditions of no vibration were the same as in fixed bed drying, and the change in bed temperatures showed that the humidity in the air probably reached saturation at less than 1 cm. But when the bed was vibrated under favorable conditions, the drying process was carried out with uniform moisture content and temperature in the bed. In our experiments, various profiles of moisture content and temperature, shown as examples in Fig. 4, were obtained in dependence with the conditions of vibration. Thus the effects of vibration on the uniformity of moisture content in the bed can be discussed quantitatively in terms of the particle circulation rate enhanced by the vibration.

4.1 Uniformity of moisture content depended on dimensionless particle circulation rate B

The difference in average moisture content between section 1 and section 4 is expressed by

\[ \frac{(W_4 - W_1)}{W_0} = \left( (3 + 2\sqrt{2})A_4 + (3 - 2\sqrt{2})A_1 \right) / AB \]

The dimensionless particle circulation rate B is defined as \( r_\theta \cdot M_0 / M_0 \) or \( rW_0 / Rc \), and it depends upon the initial moisture content \( W_0 \) and the drying rate \( Rc \). The relation between the calculated values of \((W_4 - W_1) / W_0\), and \( \Theta \) is shown in Fig. 5 as a function of \( B \). The uniformity of moisture content in the bed increases when the values of \( B \) increase. And when the values of \( B \) are over 150, it seems that the drying processes proceed with almost uniform moisture content, hence the values of \((W_4 - W_1)\) were evaluated and found to be less than 0.01 \( W_0 \). On the other hand, even if the particle circulation rates are equivalent, the uniformity of moisture content in the bed decreases according to the increase in value of \( Rc \), hence \( B \) is inversely proportional to \( Rc \).

4.2 Relation between particle circulation rate and vibrational intensity

Here, the effect of vibration on particle circulation rate \( r \) was studied, hence the value of \( B \) cannot be used to illustrate the particle circulation rate itself for its dependency on \( W_0 \) and \( Rc \). The value of \( r \) is expressed as the bone-dry material basis, but the net value of particle circulation rate expressed by the wet material basis depends on \( W_0 \). The influence of \( W_0 \) on the experimental values of \( r \) was calculated and found to be less than 15% in this study.

The experimental values of \( \log r \) are plotted in Fig. 6 against \( \log u \) as a function of the acceleration of vibration \( a_\omega \), where the bed height is 4 cm. The values of \( B \) were roughly estimated to be 60 \( r \) at \( u=3 \text{ cm/sec} \), and to be 20 \( r \) at 8 cm/sec in this figure. So, when the values of \( r \) were larger than 1, the differences between \( W_4 \) and \( W_1 \) became less than 0.1 \( W_0 \).

When the superficial air velocities were less than \( u_\text{mf} \), the relation between \( \log r \) and \( \log u \) were linear. And the gradients \( C_1 \) and the intercepts \( C_2 \) of the lines calculated by the least-squares method are shown in Fig. 7. The values of \( C_1 \) depended upon the acceleration of vibration, and were approximated to be 6 \( a_\omega \). The values of \( C_2 \) were nearly the same, and the average value was \(-1.4 (C_2=0.040) \). Figure 8 shows the values of \( ru^{-a_\omega \omega / 2} / g \) as a function of \( a_\omega \).
Though the average value of $C_2$ was calculated to be 0.040, the optimum value of $C_2$ fitted to Fig. 8 was 0.037. Consequently, the relationship among $r$, $u$, and $a_0\omega^3/g$ was expressed approximately by

$$r = 0.037 \ u^{0.042}$$

for $3 \text{ cm} \leq u \leq 8 \text{ cm}$

$$0.17 \leq a_0\omega^3/g \leq 0.45$$

This equation was obtained only when the bed height $L$ was 4 cm, and it expressed the experimental values within $\pm 48\%$ error.

Figure 6 also shows clearly that the effect of vibration on the particle circulation rate is remarkable when the air velocities are less than $u_{mf}$, and it decreases promptly at air velocities over $u_{mf}$.

### 4.3 Influence of bed height

Figure 9 shows that the effect of vibration on the particle circulation rate decreased swiftly as the height of bed increased. Therefore, appropriate vibrational intensities have to be determined experimentally to obtain the desired particle circulation rate or uniformity of moisture content in the bed according to the materials used and the bed height.

### Conclusion

The drying characteristics of particles in the vibro-fluidized bed were discussed in terms of the particle circulation rate. The conclusions obtained were:

1) The effect of vibration on the uniformity of moisture content in the bed was remarkable when the air velocities were less than $u_{mf}$.

2) The bed could be dried with uniform moisture content in the bed from the beginning of drying under the appropriate conditions of vibration, even if the air velocities were lower than $u_{mf}$.

3) The uniformity of moisture content in the bed depended upon air velocity, vibrational intensity, and the bed height.

### Appendix

In the case of $n=4$ and $R_i=0$ for $i \geq 2$, Laplace transformation of Eqs. (1) is

$$sL(W_1)-W_1(0)=kL(W_2)-kL(W_3)-(K/s)$$

$$sL(W_2)-W_2(0)=kL(W_3)-2kL(W_4)+kL(W_1)$$

$$sL(W_3)-W_3(0)=kL(W_4)-2kL(W_5)+kL(W_1)$$

$$sL(W_4)-W_4(0)=kL(W_5)-kL(W_6)$$

The initial conditions for Eq. (A-1) are given by

$$\theta=0, W_1=W_2=W_3=W_4=W_5=W_6=0$$

Then, these simultaneous equations are solved. For example:

$$W_i = W_o \ s^{-1} \left( s^2 + 5k^2s + 6k^2s + 10k^2s + 14k^2s \right) - K$$

where $k=r/M$ and $K=R_f/M$.

The solution is obtained by the inverse Laplace transformation of Eq. (A-3) resolved into factors:

$$W_i = W_o \left( 1 - e^{-t/4} \right) - k/8k \left[ (1-e^{-t/2}) + (3+2\sqrt{2}) \right]$$

$$\times \left( 1-e^{-t/8\sqrt{2}} \right) + (3-2\sqrt{2}) \left[ 1-e^{-t/(2+\sqrt{2})} \right]$$

The relations between the moisture contents in other sections and the drying time are derived in the same manner.

### Nomenclature

- $A =$ cross-sectional area of bed [m$^2$]
- $a =$ specific surface area [m$^2$/m$^3$-bed]
- $a_0 =$ amplitude of vibration [cm], [mm]
- $B =$ dimensionless particle circulation rate [-]
- $C_1 =$ gradient of lines in Fig. 6 [-]
- $C_2 =$ log $C_2$ is intercept of lines in Fig. 6 [1/h]
- $C_{st} =$ specific heat of air [kcal/kg·°C]
- $D_p =$ particle diameter [m], [µm]
- $d.m.$ = bone-dry material
- $f =$ frequency of vibration [1/sec]
- $G_o =$ mass velocity of air [kg/m$^2$·h]
- $g =$ acceleration of gravity [cm/sec$^2$]
- $H =$ humidity [kg-H$_2$O/kg-dry air]
- $h =$ heat coefficient [kcal/m$^2$·h·°C]
- $K =$ $R_f/M$ [kg-H$_2$O/h·kg-d.m.]
- $k =$ $r/M$ [1/h]
- $L =$ height of bed [cm]
- $l =$ distance from distributor [mm]
- $M =$ mass of dry particles in one section [kg]
- $M_o =$ mass of dry particles in bed [kg]
- $n =$ number of sections [-]
- $R_e =$ constant drying rate [kg-H$_2$O/h]
- $R_o =$ drying rate in section $n$ [kg-H$_2$O/h]
- $R_{H} =$ relative humidity [%]
- $R_{f} =$ Reynolds number [-]
- $r =$ particle circulation rate [kg-d.m./h]
- $t_i =$ air temperature at inlet of bed [°C]
- $t_o =$ air temperature at outlet of bed [°C]
- $u =$ superficial air velocity [cm/sec]
- $u_{mf} =$ minimum fluidization velocity [cm/sec]
- $W_c =$ critical moisture content [kg-H$_2$O/kg-d.m.]
\[ W_e = \text{equilibrium moisture content} \quad \text{[kg-H}_2\text{O/kg-d.m.]} \]
\[ W_0 = \text{initial moisture content} \quad \text{[kg-H}_2\text{O/kg-d.m.]} \]
\[ W_n = \text{average moisture content in section } n \quad \text{[kg-H}_2\text{O/kg-d.m.]} \]
\[ w_o = \text{initial weight of sample} \quad \text{[kg]} \]
\[ \varepsilon = \text{bed voidage} \quad \text{[—]} \]
\[ \Theta = \frac{\theta}{\theta_o} \quad \text{[—]} \]
\[ \theta = \text{drying time} \quad \text{[h]} \]
\[ \theta_o = \frac{W_o M_o}{R_o} \quad \text{[h]} \]
\[ \lambda_o = \text{latent heat of vaporization} \quad \text{[kcal/kg]} \]
\[ \rho_o = \text{particle density} \quad \text{[g/cm}^3\text{]} \]
\[ \omega = \text{angular frequency} \quad \text{[1/sec]} \]

**Literature Cited**


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