CSD ANALYSIS FROM A BATCH DILUTION CRYSTALLIZER

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The analysis of a batch dilution crystallizer is presented for different modes of operation using the Bransom crystal growth rate model and power law nucleation rate model. This analysis is used for potassium sulphate crystallization from aqueous solution using ethanol as a diluent. The process configuration is analysed over a wide range of variables in each mode of operation and suitable value of the operating variable is suggested for better and safer performance of a given batch dilution crystallizer.

Introduction

Prediction and analysis of crystal size distribution (CSD) with given process configuration, physical parameters and system kinetics is one of the important considerations in evaluating crystallizer performance. Most previous work on CSD analysis in batch crystallizers is related to cooling crystallizers. Recently the authors analysed batch evaporative crystallizers for potassium sulphate-water system and reported analogous results to those for cooling crystallizers. One of the other ways in which supersaturation can be effected is by the addition of a diluent which reduces the solubility of the solute in solvent. This process may be termed 'salting-out, precipitation or dilution crystallization'. The diluent (mostly liquid) used should be miscible with solvent of the original

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Literature Cited

solution and the solute should be insoluble in the diluent. The final solvent-diluent mixture should be capable of easy separation.

Although the dilution technique is widely employed industrially, relatively few published data are available regarding its use in crystallization. The watering-out technique may be employed for crystallizing organic substances from the water-miscible organic solvents. It may be useful in preparing pure inorganic salts from their aqueous solutions by organic solvents\(^5\). Higher throughputs by preparing very concentrated initial solutions and recovering most of the solute, lower working temperature for heat-labile substances and choice of suitable solvents and better and easier purifications as the mother liquor retains undesired impurities, are some of the advantages of the dilution technique. The need for the recovery unit to handle fairly large quantities of mother liquor in order to separate valuable solvent and/or diluent is the disadvantage of the technique. Timm and Larson\(^1\)\(^\text{11}\) used this technique for generating supersaturation in the kinetic study of crystallization of ammonium sulphate, sodium chloride and ammonium alum.

The object of the present communication is to analyse the CSD obtainable in a batch-operated dilution crystallizer under different modes of operation over a wide range of variables and to suggest suitable values of the controlled variable in each mode of operations. The system selected for this study was potassium sulphate-water-ethanol as the data regarding its use in crystallization. The watering-out technique may be employed for crystallizing industrially, relatively few published data are available out technique may be employed for crystallizing

The Bransom growth rate\(^2\) (Eq. (2)) and the power law nucleation rate model (Eq. (3)) are used in the present analysis. The crystallizer may contain two types of crystals: the seed and the newly generated nuclei. Therefore, it is convenient to distinguish between the former as S-crystals and the latter as N-crystals. It is assumed that the crystal nucleates at a finite size. Secondly, the size dependency function for newly generated nuclei is evaluated at the surface-volume mean crystal size and for seed crystals at actual mean size of the seed. In the Bransom growth rate model the growth rate tends to zero as the size of the crystals approaches zero for all supersaturation levels and gives discontinuity in growth rate function, which can be avoided by evaluating the size dependency at surface-volume mean crystal size for N-crystals. The power law nucleation model was used by many investigators in the past and the nucleation rate constant depends on number of parameters. A power law term of suspension density or crystal surface area and that of intensity of agitation in the nucleation rate model may also be used. In the present analysis the effects of these parameters were not considered. The agitation level can be kept at constant value in the present work for batch crystallizers. The variation in suspension density probably is not significant as the suspension mass and volume of the system varies with time.

1. Population balance and moment equations

As the diluent is added to the crystallizer during operation the volume of the crystallizer contents varies with time. It is convenient to define the population density function on the basis of total working volume at any time as

\[ \bar{n} = \bar{n}_V \]

Other extensive quantities will also be defined on the basis of total working volume and will be represented by a bar on top of the corresponding symbol. For a batch-operated, perfectly mixed crystallizer with negligible crystal breakage and agglomeration, the population balance based on the total working volume at any time may be given as\(^1\)

\[ \frac{\partial \bar{n}}{\partial t} + \frac{\partial \bar{n} G}{\partial L} = - \bar{b}(L-L_n0); \quad L > 0 \]

\[ = 0 \quad ; \quad L_n(0) = 0 \]

As the crystallizer may contain N- and S-crystals the general solution of Eq. (5) should consist of the two parts representing the population density of S-crystals and that of N-crystals. Their respective number density functions are continuous functions of size and time.

The moment equations may be derived from the population balance equations (Eq. (5)) by moment transformation. The moment equations applicable in this process for N-crystals are

\[ d \bar{U}_n / dt = \bar{B}_n \]

\[ d \bar{U}_n / dt = \bar{U}_n G + B_n L_n0 \]
In above equations G represents the linear growth rate, (Eq. (2)) and can be represented as

\[ G = G_p G_s = (k_p P^2)(L^2) \]  

(10)

and the size dependency function, \( G_s \), for N-crystals should be evaluated at surface-volume mean crystal size defined as

\[ L = \frac{\Omega_{av}}{\Omega_{sv}} \]  

(11)

whereas for S-crystals it should be evaluated at actual mean size of the seed.

1.2 Supersaturation balance

In a batch dilution crystallizer if only S-crystals are present the rate of solid deposition is

\[ \frac{d W_s}{dt} = 3(W_{so}/L_{lo})(L_{n0}^2)G_s(L, t) \]  

(8)

and for N-crystals it is

\[ \frac{d W_n}{dt} = k_n \theta_n \left( dU_{av}/dt \right) \]  

(9)

The total rate of solid deposition for (N+S)-crystals is

\[ \frac{d W}{dt} = \frac{d W_s}{dt} + \frac{d W_n}{dt} \]  

(10)

The supersaturation balance for the batch dilution crystallizer based on total working volume at any time is given as

\[ \frac{d P}{dt} + \frac{d V C_s}{dt} + \frac{d W}{dt} = 0 \]  

(11)

In Eq. (15), the first term corresponds to accumulation, the second to generation of supersaturation and the third to disappearance of supersaturation from the solution to the solid deposition. The supersaturation balance along with moment equations may be used to find out the various operating modes.

2. Operating Modes

Batch dilution crystallizers may be operated under various modes, a few of which are considered here.

2.1 Constant rate of addition of a diluent

Under this mode of operation the rate of addition of a diluent is kept constant. The volume variation of the system will be

\[ V = V_0 + k_v \]  

(12)

and the concentration of a diluent with zero initial concentration will vary as

\[ C_s = k_s t/V \]  

(13)

The rate of supersaturation generation will be

\[ \frac{d V C_s}{dt} = k_s \theta_s \left( \frac{d U_{av}}{dt} \right) \]  

(14)

The moment equations for N-crystals will be described by Eqs. (6)-(9); the total rate of solid deposition by Eq. (14) and the supersaturation balance by Eq. (15). This set of equations along with other information like initial conditions and batch time will define the process configuration completely. Solution of supersaturation balance equation will define the variation of supersaturation with time with a given constant rate of addition of a diluent. A similar set of equations can also be derived for any other operating mode under which the diluent addition rate is a known function of time.

2.2 Constant rate of change of a diluent concentration

In this type of operation supersaturation in the crystallizer may be achieved by keeping the rate of change of a diluent concentration constant. Therefore,

\[ \frac{d C_s}{dt} = k_s \]  

(15)

Assuming initial concentration of a diluent to be zero the volume variation of the system is given as

\[ V = V_0 \left( 1 - k_v t \right) \]  

(16)

and the supersaturation generation term is

\[ \frac{d V C_s}{dt} = C_s \theta_s \left( \frac{d V}{V_0} \right) - k \]  

(17)

Using the above information and moment equations in supersaturation variation with time may be defined.

2.3 From a supersaturated solution

This is a special case of the above two modes. Supersaturation in the solution is achieved before crystallization. Crystal growth and nucleation can occur during operation without any generation of supersaturation. Normally such an operation proceeds via seeding and the rate of addition of a diluent is zero. Such an operation may find use in laboratory studies in evaluating the kinetic parameters.

2.4 Constant level of supersaturation

Mullin and his co-workers\(^6,7\) demonstrated that batch cooling crystallizers operated at a constant level of supersaturation gave better performance. In this mode, the level of supersaturation is kept constant by varying the amount of a diluent in the crystallizer. The nucleation rate (given by Eq. (3)) remains constant during operation. For a constant level of supersaturation,

\[ \frac{d P}{dt} = 0 \]  

(18)

As the variation in volume of the crystallizer is mainly due to the addition of a diluent it can be shown that

\[ \frac{d V}{dt} = d V_s/\theta \]  

(19)

The solubility variation with time is

\[ \frac{d C_s}{dt} = -K C_s \frac{d C_A}{dt} = -k C_s \frac{V_v}{V_0} \]  

(20)

The rate of addition of a diluent in this case may be defined from the supersaturation balance as

\[ \frac{d V_s}{dt} = -d W/\left( P + C_s - k C_s V_0 \right) \]  

(21)

Equation (25) defines the rate at which diluent must be added to the crystallizer so as to maintain the system at a constant level of supersaturation.

2.5 Constant level of supersaturation with ideal metastability

In this mode the situation may be simplified for
analysis from the previous mode by assuming ideal metastability at a constant level of supersaturation, i.e., the nucleation rate is negligible and there will be no N-crystals in the crystallizer. The growth occurs only on the added seeds, and the total rate of solid deposition may be modified to

$$dW/dt = (3W_{s}(G_{s}/L_{s})_{L}(t)^{g+a})$$

Using above solid deposition rate in Eq. (25) the rate of addition of a diluent with respect to time may be defined to maintain the crystallizer at a constant level of supersaturation.

3. Results and Discussion

The above treatment was used in an analysis of potassium sulphate crystallization from aqueous solution by adding ethanol as a diluent. The values of the physical parameters and system kinetics used in the present analysis are reported in Table 1. The value of the constant, \(k\), in the solubility relationship (Eq. (1)) reported here was estimated by the linear regression analysis of data available in ICT IV (p. 410). Other values of the physical parameters and system kinetics were used from the previous work^{5-8).

A crystallizer with constant rate of addition of a diluent is normally easy to operate. Moment equations for N-crystals and supersaturation balance equation for this case were solved numerically by the fourth-order Runge Kutta method on a dec 10 system digital computer. The variation of supersaturation, ethanol concentration, solubility of potassium sulphate in (ethanol+water) and increase in size of S-crystals with respect to time over a wide range of addition rates of a diluent is presented in Figs. 1-4 respectively. The addition rate was varied from 0 to 1.0 \(\times\) 10^{-4} kg of diluents. The effect of variation of this addition rate on sizes of N-, S- and (N+S)-crystals, percent coefficient of variation (CV) of N- and (N+S)-crystals and ratios \((W_{s}/W_{t})\) and \((W_{t}/W_{s})\) is shown in Fig. 5. Instead of keeping addition rate at a constant

| Table 1 Parameter values used in analysis for potassium sulphate-water-ethanol system |
|-------------------------------------|------|
| 1. Batch time                       | s    | 10800 |
| 2. Operating temperature            | K    | 298   |
| 3. Solubility of potassium sulphate in water at 298 K, \(C_{s}\) | kg/kg | 0.12035 |
| 4. Value of constant, \(k\), in Eq. (1) |     | 9.5513 |
| 5. Density of potassium sulphate crystals, \(\rho_{s}\) | kg/m³ | 2662 |
| 6. Volume shape factor, \(k_{a}\)    |      | 0.525 |
| 7. Initial supersaturation for constant-rate operations, \(P_{0}\) | kg K₂SO₄/kg H₂O | 0.00732 |
| 8. Initial seed charge, \(W_{t0}\)  | kg/kg | 0.00236 |
| 9. Growth rate order, \(g\)         |      | 2.0   |
| 10. Nucleation rate order, \(n\)    |      | 7.63  |
| 11. Exponent of crystal size in growth rate |  | 0.5   |
| 12. Growth rate constant, \(k_{g}\) | kg m⁻¹ (kg/kg)⁻¹ (m)⁰ | 0.0317 |
| 13. Nucleation rate constant, \(k_{n}\) [number/(kg·s)·(kg/kg)⁷¹] |  | \(6.67 \times 10^{17}\) |
| 14. Critical nucleus size, \(L_{n0}\) [m] |  | \(6 \times 10^{-9}\) |
| 15. Initial seed size, \(L_{s0}\)    [m] |  | \(5.5 \times 10^{-4}\) |
| 16. Initial solvent capacity, \(V_{0}\) [kg] |  | 1.0   |
Fig. 3 Variation of the solubility of potassium sulphate in ethanol-water with time using (a) rate of addition of the diluent (in kg/s) and (b) rate of change of diluent concentration (in kg/kg·s) as parameters.

Fig. 4 Variation of the increase in size of the seed with times at different values of rate (in kg/s) of addition of the diluent in constant rate operation.

Fig. 5 Plots of various variables versus the rate of addition of the diluent.

Fig. 6 Plots of supersaturation versus time with the rate of change of the diluent concentration (in kg/kg·s) as a parameter in a constant rate of change of a diluent concentration mode.

Batch time = 10800 s; temperature = 298 K; \( W_{o} = 0.00236 \text{ kg/kg}; \quad P_{0} = 0.00732 \text{ kg/kg}, \quad L_{m} = 550 \mu\text{m}, \) Variables: (1) Size of N-crystals (●); (2) Size of S-crystals (○), (3) Size of (N+S)-crystals (▲); (4) CV of N-crystals (●); (5) CV of (N+S)-crystals (●); (6) ratio of weight on N-crystals to that of S-crystals in product (■); (7) ratio of weight of S-crystals in product to that of S-crystals in feed (○).

Fig. 7 Variation of ethanol concentration and solubility of potassium sulphate in (ethanol+water) can be evaluated and are shown in Figs. 9-10 respectively. Variation of increase in size of S-crystals for various levels of supersaturation. With these rates the variation of the diluent concentration and solubility of potassium sulphate in (ethanol+water) can be evaluated and are shown in Figs. 9-10 respectively. Variation of increase in size of S-crystals.
3. 1 Constant-rate operations

Normally batch operations are flexible. It is rather difficult to assign all the operating conditions rigidly for the desired product specification. In an operation with a constant rate of addition of a diluent the supersaturation passes through a maximum. The peak height of supersaturation goes on increasing and peak width decreases with an increase in rate of addition of a diluent (Fig. 1). Supersaturation is generated because of the addition of a diluent. The diluent concentration at lower rates varies linearly; at higher rates its rate of change lessens with time and the solubility follows an exponentially decaying relationship (Figs. 2–3). For a particular diluent addition rate the supersaturation generation term is initially high but goes on diminishing with time. Supersaturation will start increasing and along with it growth and nucleation will increase. The rate of disappearance of supersaturation will also increase. The overall result

--- curves for the operation with constant level of supersaturation
----- curves for the operation with constant level of supersaturation under ideal metastability

---, --- curves for the operation with constant level of supersaturation

---, ---, --- curves for the operation with constant level of supersaturation under ideal metastability

Fig. 7  Plots of increase in seed size versus time at different values of rates (in kg/kg · s) in an operation with constant rate of change of a diluent concentration

Fig. 8  Variation of rate of addition of the diluent with time at various levels of supersaturation (kg/kg)

Fig. 9  Plots of ethanol concentration versus time at various levels of supersaturation (kg/kg)

Fig. 10  Plots of the solubility versus time at various levels of supersaturation (in kg/kg)

with time using supersaturation level as a parameter is presented in Fig. 11. The analysis was also carried out by assuming ideal metastable conditions (no N-crystals). The results of the variation of diluent addition rate, diluent concentration, solubility and increase in size of S-crystals with time are shown in Figs. 8–11 respectively. The statistics for the CSD at various levels of supersaturation are reported in Table 2.
Table 2 Statistics obtained from CSD analysis in an operation at a constant level of supersaturation with and without metastability

<table>
<thead>
<tr>
<th>P x 10^2 [kg/kg]</th>
<th>Without metastability</th>
<th>With metastability</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_s [m]</td>
<td>L_n [m]</td>
<td>L_p [m]</td>
</tr>
<tr>
<td>CV_n [kg]</td>
<td>CV_p [kg]</td>
<td>CV [kg]</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1098 1913 18.0</td>
<td>770 1098 16.5 6.0</td>
<td>1098 0.0</td>
</tr>
</tbody>
</table>

will be that supersaturation level will pass through a maximum. With increase in the addition rate the maximum of supersaturation will occur at lower period as the supersaturation generation term increases with it initially and then decreases at a faster rate. At the level of zero rate, supersaturation goes on diminishing continuously. Increase in size of S-crystals increases with time. With an increase in the addition rate the increase in size of S-crystals increases in the initial period and in the latter period initially it increases, passes through a maximum and then starts decreasing. This behaviour is due to the supersaturation variation of the system. Growth and nucleation processes are competitive, just like parallel reactions. Initially, as supersaturation increases, growth and nucleation start. In a system with positive relative kinetic order at a relatively higher supersaturation level nucleation becomes predominant. At higher addition rates the maximum of supersaturation level is achieved at a higher value in the initial period. This supersaturation is used in generating large number of nuclei rather than in growing crystals. The final size of S-crystals increases with a diluent addition rate at a lower value, passes through the maximum and then decreases (Fig. 5). The final weight ratio of N- to S-crystals and the final CV of (N+S)-crystals start increasing rapidly and the final size of the product, i.e. (N+S)-crystals, starts decreasing rapidly after an addition rate of about 1 x 10^-6 kg/s. It is rather difficult to decide the objectives in assigning the addition rate of a diluent in an operation. Normally the larger size of seed crystals with minimum CV of product crystals will be desirable. With these conflicting objectives, for the present process configuration an addition rate of a diluent around 5 x 10^-6 kg/s may be used (Fig. 5). At relatively lower addition rates lower CV with lower size can be realised. The size and the CV of N-crystals remain more or less constant over the range of rates considered. However, at higher rates, nucleation effects are predominant and the average weight mean size of (N+S)-crystals decreases considerably. It is interesting to note that the final size of S-crystals at the highest rate (1 x 10^-4 kg/s) is less than that at the limit of zero addition rate of a diluent. The analysis in the operation with constant rate of change of a diluent concentration is more or less similar to that of the previous mode and a rate around the same value may be used for this mode.

3.2 Operations with constant level of supersaturation

In this work operation at a constant level of supersaturation with and without metastable conditions was considered over a range of supersaturation from 0.005 to 0.015 kg/kg. Assumption of ideal metastability is normally used for the simplification of analysis, and in the present work it can be used to see its effect in the analysis. It is interesting to note that at supersaturation levels of 0.015 and 0.010 kg/kg the system reaches its physical limits in between the batch time. The solubility of potassium sulphate tends to zero and supersaturation cannot be generated by adding the diluent to the system. Thus it is impossible to maintain the system at a constant level of supersaturation. Such a high level cannot be used for this process configuration. Therefore, it would be better to operate the crystallizer at the supersaturation level around 0.0075 kg/kg. Operation with ideal metastable condition can be carried out at relatively higher (N 0.01 kg/kg) supersaturation level. Addition rates are initially lower and increase with time, and the concentration of diluent and its rate increases with time. Variation of diluent concentration and solubility in this case is quite different from that of the constant rate of addition operation. Addition rates of a diluent at a lower supersaturation level are the same in operations with and without metastable conditions. However, a substantial difference at higher supersaturation level is observed because of the nucleation effects in the system. Increase in size of S-crystals will be the same for both operations. However, operation with met-
Astable condition reaches its physical limits at a much later time (Fig. 11). From the statistical parameters (Table 2) operation at a constant level of supersaturation with metastable condition gives the best performance. At supersaturation level of 0.010 kg/kg the product size obtained in this mode is 1650 μm with minimum CV. However, such an operation cannot be realised in the crystallizer. The crystallizer operated at a constant level of supersaturation gives relatively better performance. A crystallizer operated at the highest safe supersaturation level (0.0075 kg/kg) gives product crystals of 913 μm weight average size with 40% CV whereas at supersaturation level of 0.005 kg/kg the product is of 767 μm weight average size with 6% CV.

Mullin and Jones6) and Jones5) analysed and reported the results for similar modes in cooling crystallizers and the authors8) also analysed the same system under more or less similar conditions in an evaporative crystallizer and obtained an analogous trend in the results for both types. However, their analyses were restricted to limited values of the controlled variables. In the present analysis, the batch dilution crystallizer was analysed over a wide range of controlled variables and suitable values of the controlled variables are suggested for better performance in each mode of operation. Temperature used in the present work is lower than that used in previous work and relatively lower size of S-crystals is expected under similar operating conditions.

Conclusions

Analytical description of a batch dilution crystallizer is developed and used in the analysis of crystallization of potassium sulphate crystals from aqueous solution by adding ethanol as a diluent. Operations with a constant rate of addition of a diluent and a constant rate of change of the diluent concentration are analysed over a wide range of rates and a rate around 5x10⁻⁶ kg/s for a large size of S-crystal with lower CV of (N+S)-crystals is desirable for this system. Performance of a crystallizer operated at a constant level of supersaturation was evaluated over a range of supersaturation from 0.005 to 0.015 kg/kg. Operation at higher supersaturation reaches its physical limits. A supersaturation level of around 0.0075 kg/kg for operation at a constant level of supersaturation and around 0.010 kg/kg for operation at a constant level of supersaturation with ideal metastable condition can be used. Crystallizers operated at a constant level of supersaturation give better performance.

Appendix: Derivations of Eqs. (18), (21), (24), (25) and solubility data

1. Derivation of Eq. (18)
   Differentiating the solubility relationship, Eq. (1), with respect to t,
   \[ \frac{dC_E}{dt} = -C_E \exp(-kC_A) \frac{dC_A}{dt} = -kC_E \frac{dC_A}{dt} \]  \[ \text{A-1} \]
   The time rate of change of diluent concentration can be obtained from Eqs. (15) and (17) as
   \[ \frac{dC_A}{dt} = \frac{d}{dt} \left( \frac{V-V_0}{V} \right) = \frac{dV}{dt} = \frac{V}{V_0} k_s \]  \[ \text{A-2} \]
   Substituting Eq. (A-2) in Eq. (A-1) the time rate of change of solubility is given by
   \[ \frac{dC_E}{dt} = -kC_E k_s \left( V/V_0 \right)^2 \]  \[ \text{A-3} \]
   The rate of supersaturation generation will be
   \[ \frac{dV_C}{dt} = V \frac{dC_E}{dt} = \frac{V}{V_0} k_s \]  \[ \text{A-4} \]
   Using Eqs. (A-3) and (16) in Eq. (A-4) it can be modified to
   \[ \frac{dV_C}{dt} = k_s C_A [1 - (V_0/V)] \]  \[ \text{A-5} \]

2. Derivation of Eq. (21)
   Using Eq. (19) in Eq. (A-1) the rate of change of the solubility may be written as
   \[ \frac{dC_E}{dt} = -kC_E V \]  \[ \text{A-6} \]
   The rate of change of column with time from Eq. (20) can be obtained as
   \[ \frac{dV}{dt} = \frac{V_kk_s}{1 - k_c} \]  \[ \text{A-7} \]
   Using above values in Eq. (A-4) the supersaturation generation term will be
   \[ \frac{dV_C}{dt} = k_s C_A [V/V_0 - k] \]  \[ \text{A-8} \]

3. Derivation of Eqs. (24) and (25)
   The rate of change of solubility with time obtained by differentiating Eq. (1) is represented by Eq. (A-1) and the rate of change of diluent concentration may be obtained by using Eq. (23) as
   \[ \frac{dC_A}{dt} = \frac{d(V/A)}{dt} = \frac{V_kdV_A}{V^2} \]  \[ \text{A-9} \]
   Equation (1) can be rewritten as
   \[ \frac{dC_E}{dt} = \frac{dV}{dt} \left( \frac{V}{V_0 \frac{dC_A}{dt}} \right) \]  \[ \text{A-10} \]
   The supersaturation balance (Eq. (15)) can be expressed as
   \[ \frac{dV}{dt} + \frac{d^2V}{dt} + \frac{C_E dV}{dt} + \frac{dC_E}{dt} = 0 \]  \[ \text{A-11} \]
   Using Eqs. (22), (23) and (24) in Eq. (A-8) the addition rate of diluent may be defined as
   \[ \frac{dV_A}{dt} = \frac{dW}{dt} \left( \frac{P + C_E - kC_E V}{V_0} \right) \]  \[ \text{A-12} \]

4. Solubility of potassium sulphate in ethanol-water at 298 K

<table>
<thead>
<tr>
<th>Solvent</th>
<th>Potassium sulphate kg/kg of (EtOH+H₂O)</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
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<tr>
<td>0.014</td>
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<td>0.002</td>
</tr>
<tr>
<td>0.692</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Nomenclature

$a$ = exponent of size in growth rate model

$(Eq. \ (2))$

$B$ = nucleation rate [number/s \cdot kg]

$b$ = birth rate function [number/s \cdot kg]

$C$ = solution concentration [kg/kg]

$G, G_P, G_{L} = \text{linear growth rate, all, supersaturation, size}$ [m/s, m^{-1}/s, m^{-2}]

$g$ = order of growth $(Eq. \ (2))$

$k, k_v, k_c$ = constants

$k_v$ = volume shape factor

$k_g$ = growth rate constant [m/s \cdot (kg/kg)^2 \cdot (m)^3]

$k_n$ = nucleation rate constant [number/(kg/kg) \cdot (m/s)]

$L$ = crystal size [m]

$n$ = population (number) density function [number/m \cdot kg]

$n$ = order of nucleation $(Eq. \ (3))$

$P$ = supersaturation

$t$ = time [s]

$U_{n,t}$ = $n$th moment of N-crystals

$V$ = solvent capacity [kg]

$W$ = weight of crystals [kg/kg]

$\delta$ = Dirac delta function

$\rho$ = density [kg/m^3]

$\langle \text{Subscripts}\rangle$

$A$ = diluent

$C$ = crystal

$E$ = equilibrium

$f$ = final

$c$ = at time $t=0$, initial

$p$ = product, supersaturation

$s$ = seed

$0$ = nucleic

$\langle \text{Superscript}\rangle$

$- = \text{total volume}$

Literature Cited