used in Fig. 8. The Electric Power Research Institute provided support for analysis of experimental data related to liquefaction of coal.

Nomenclature

G_L	= dimensionless cake thickness	[—]
G_t	= dimensionless filtration time	[—]
G_v	= dimensionless filtrate volume	[—]
G_{edis}	= dimensionless displacement time	[—]
L	= cake thickness	[m]
p	= applied pressure	[Pa]
p_m	= pressure drop across medium	[Pa]
R_m	= medium resistance	[m ⁻¹]
t	= filtration time	[s]
t_{dis}	= displacement time	[s]
v	= filtrate volume per unit area	[m]
v_c	= cake volume per unit area	[m]
v_s	= slurry volume in tank per unit area	[m]
v_F	= feed slurry volume per unit area	[m]
w	= mass of dry solids per unit area	[kg/m ²]
α_{av}	= average specific cake resistance	[m/kg]

ϵ_{sav}	= average volume fraction of solids in cake	[—]
η	= ratio of volume fraction of solids in cake to feed	[—]
μ	= viscosity of liquid	[Pa·s]
ρ_s	= density of solids	[kg/m ³]
σ	= specific gravity of solids	[—]
ϕ_s	= volume fraction of solids in tank slurry	[—]
ϕ_{sF}	= volume fraction of solids in feed slurry	[—]
χ	= dimensionless medium resistance	[—]

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REMOVAL OF EMULSIFIED OIL PARTICLES BY DISPERSED AIR FLOTATION

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Dilute emulsified particles ($C \approx 300 \text{ g} \cdot \text{m}^{-3}$, $D_F \approx 8 \text{ } \mu\text{m}$) of heavy oil A were removed in a 65 mm-diameter, 0.32 m-deep column with a solution of pH 4 which minimized double-layer repulsion.

Firstly, flotation by bubbling through glass frits of three different pore sizes shows that the removal rate is proportional to the oil concentration to the first order at low gas velocity ($0.01 \leq U_g \leq 0.05 \text{ cm} \cdot \text{s}^{-1}$) and to the second order at high gas velocity ($0.1 \leq U_g \leq 3 \text{ cm} \cdot \text{s}^{-1}$). This fact implies that the predominant rate step is oil particle-bubble collision in the former region and particle-particle collision in the latter. Mechanical agitation by a paddle gives the same removal rate as bubbling in the high gas velocity region when the power consumption per unit volume is the same. This fact implies that the role of bubbling there is to give adequate turbulence to the liquid.

Secondly, the bubble diameter was changed from 40 μm to 2 mm under the mechanism of bubble-particle collision by electrolysis, dissolved and bubbling air flotation. With increasing bubble diameter, the collection efficiency decreases rapidly ($\approx$ to the -2 power) for small bubbles and much more gradually for large bubbles. This trend coincides with the existing and proposed trajectory theories.

Introduction

Air flotation has been applied to the separation of emulsified oil from water. The operation may be

classified into (1) dissolved air flotation and (2) dispersed air flotation with respect to the generation of fine bubbles. In the former, the dissolving pressure is a decisive operating variable. Its effect was discussed in the previous paper⁽¹⁾, along with the effect of the liquid properties. In the latter, the gas flow rate may be an important operating variable, and the

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(a) Bubbling air flotation (b) Electrolysis air flotation

Fig. 1 Schematic diagram of apparatus

Fig. 2 Variation of residual oil concentration after 30 minutes' bubbling with superficial gas velocity in bubbling flotation

present investigation deals primarily with its effect on the flotation efficiency and the flotation mechanism.

Bubble size varies widely, depending on the method of generation of bubbles, e. g. from the order of $10\text{ }\mu\text{m}$ in electrolysis to the order of 1 mm in bubbling through a porous frit. The second purpose of the present paper is to make clear its effect in terms of the collection factor.

1. Experimental

The model emulsion was prepared from a heavy oil A with particles of about $8\text{ }\mu\text{m}$ in arithmetic mean diameter, with a concentration of about $300\text{ g}\cdot\text{m}^{-3}$. Details of the preparation and properties of the emulsion and analysis of the oil concentration were described in the previous paper⁶⁾. The pH of liquid was adjusted to pH 4 by addition of HCl solution so that double-layer repulsion was minimized as confirmed in the previous paper⁶⁾ and so that the hydrodynamic effect may be observed in probably the most appropriate manner.

The separation of oil was carried out in two different systems of (1) bubbling and (2) electrolysis. Schematic diagrams are given in Figs. 1 (a) and (b), respectively. Each flotation column was a 0.5 m -long

acrylic cylinder of 65-mm diameter. Liquid depth was kept constant at 32 cm .

In a bubbling flotation, nitrogen rather than air was used for ready availability of a clean gas. Nitrogen was bubbled through porous glass frits of three different pore sizes and the bubble diameter varied over a range of $0.1\text{--}0.5\text{ cm}$ depending on frit pore size and gas flow rate. The residual oil concentration was analyzed after 30 minutes' bubbling.

In an electrolysis flotation, fine bubbles with an arithmetic mean diameter of $40\text{ }\mu\text{m}$ were generated from a platinum electrode consisting of a spiralled wire and a squared plate as shown in Fig. 1 (b). Direct current was applied intermittently to avoid column flooding due to excessive bubbles encountered in continuous bubble generation. The circuit was closed for ten seconds for bubble generation and then opened for three minutes for flotation under mild flow conditions. After this cycle was repeated twelve times, the oil concentration was analyzed. While the circuit was closed, large nitrogen bubbles ($\sim 5\text{ mm}$ in diameter) were introduced at the bottom with very low flow rate to facilitate the uniform dispersion of fine bubbles generated.

2. Effect of Gas Flow Rate

2.1 Bubbling flotation

Oil removal under various conditions is compared in terms of the oil concentration C after 30 minutes' bubbling through frits. The particular value of 30 minutes' bubbling is selected because sufficient change in oil concentration was observed in this period, and satisfactory accuracy of analysis was obtained over the corresponding range of oil concentration. The results are plotted against the superficial gas velocity U_g in Fig. 2. When the emulsion was left quiescent for 30 minutes, the oil concentration decreased to 80% of the initial value by spontaneous removal. The effect of frit size on residual concentration was unfortunately masked by much scattering in C/C_0 . However, the effect of gas flow rate can be seen clearly. The residual concentration C/C_0 decreases slightly with gas rate U_g at gas rate less than $0.05\text{ cm}\cdot\text{s}^{-1}$ (zone 1) and decreases more rapidly at gas flow rate greater than $0.1\text{ cm}\cdot\text{s}^{-1}$ (zone 2). At the transition between two zones the value of C/C_0 remains unchanged or even increases with increasing gas flow rate.

In some experimental runs, the variation of oil concentration with time was measured by sampling every 5 or 10 minutes. The average removal rates $\bar{r}$ per unit volume and unit time during the successive time intervals

$$\bar{r} = \Delta C / \Delta t \quad (1)$$

are plotted against time. The instantaneous removal

rates r were obtained by smoothing the $\bar{r}-t$ plot according to the method recommended by Churchill⁽²⁾.

The results are shown in Fig. 3 in the plot of r/C vs. C . The value of r/C is seen to be independent of C in zone 1 while it is proportional to C in zone 2, resulting in the rate equations

$$r=kC \quad \text{in zone 1} \quad (2)$$

$$r=kC^2 \quad \text{in zone 2} \quad (3)$$

Oil separation by bubbling has been investigated by Kondo *et al.*⁽³⁾, Saito *et al.*⁽⁵⁾, Angelldou *et al.*⁽¹⁾, and Takahashi *et al.*⁽⁹⁾. Their gas velocities were in the range of 0.1–15 cm·s⁻¹, which are within the zone 2 of the present investigation. Angelldou *et al.*⁽¹⁾ reported a linear rate equation at $U_g=0.13$ cm·s⁻¹ while the recent result of Takahashi *et al.*⁽⁹⁾ shows a rate equation of 2–3.5 order which increases with gas velocity in the range of 0.55–15 cm·s⁻¹. Thus, the variation of the order of rate equation with gas velocity agrees with available reports although the critical gas velocity is different due to the difference of operating conditions such as bubble diameter. Takahashi *et al.*⁽⁹⁾ have proposed a two-parameter rate equation based on linear rate process with distributed rate constant. A power law rate equation of a single parameter is used for greater simplicity in the present work.

The first-order rate equation implies that a possible mechanism of separation in zone 1 is the collision of oil particles with rising bubbles, as in deep-bed filtration. The 2nd-order rate equation in zone 2 implies that the collision of oil particles with each other is more important, as in flocculation sedimentation. When the emulsion was sampled during flotation, many agglomerates were observed through a microscope in the case of zone 2 but not in zone 1. This finding also supports the mechanism of oil particle collisions in zone 2.

2.2 Removal by mechanical agitation

The effect of mechanical agitation was examined by mixing the emulsion with double-stage paddles having four blades of 39-mm diameter and 19-mm width for thirty minutes without bubbling. The result of the residual oil concentration is plotted in Fig. 2 against the power consumption per unit liquid volume P_v which was estimated according to Nagata⁽⁴⁾. When the power for bubbling is taken to be

$$P_v = \rho g U_g \quad (4)$$

the corresponding power for bubbling can be scaled as shown in Fig. 2. The residual concentrations for bubbling flotation in zone 2 and for removal by agitation are shown to be the same at the same power consumption P_v . Thus, it can be concluded that the role of bubbling in zone 2 is to give suitable turbulence to the liquid and to give oil particles the kinetic energy

Fig. 3 Different dependence of removal rate on oil concentration between zone 1 and zone 2

Fig. 4 Result of electrolysis flotation

required to collide and coagulate with each other.

2.3 Electrolysis flotation

The result for electrolysis flotation is shown in Fig. 4. The volume of bubbles evolved during one run was measured in a separate experiment and was found to be about 60% of the maximum attainable volume which can be estimated from Faraday's law, as shown in Fig. 4. The evolved gas volume at the highest current corresponds to the superficial gas velocity U_g of 1.1×10^{-3} cm·s⁻¹ based on the net period of voltage application (2 minutes). Since this value of U_g is much smaller than that in zone 1 of the bubbling flotation, the oil particles would be removed by the particle-bubble collision as in zone 1. The residual oil concentration decreases here again with evolved gas volume or current.

3. Effect of Bubble Diameter

3.1 General trend

The electrolysis flotation, the bubbling flotation in zone 1 and the dissolved air flotation reported previ-

Fig. 5 Dependence of collection efficiency on bubble diameter

ously⁶⁾ all proceed in particle-bubble collision and the approximate bubble diameters in these systems are summarized as follows:

Bubbling flotation (zone 1)

Coarse frit	2 mm
Medium frit	1.5 mm
Fine frit	1 mm
Dissolved flotation	80 μ m (Ref. 6)
Electrolysis flotation	40 μ m

To see the general trend of the effect of bubble diameter, we considered three cases: (1) bubbling flotation at $U_g = 10^{-2} \text{ cm} \cdot \text{s}^{-1}$; (2) dissolved flotation at a pressure of $2 \times 10^5 \text{ Pa}$ and (3) electrolysis flotation at a current of 0.2 A. The residual oil concentrations are not much different among these cases, but the total bubble volumes consumed per unit liquid volume are quite different, i. e. 6×10^{-1} , 3×10^{-2} , 4×10^{-3} for cases (1), (2) and (3), respectively. This fact indicates that the smaller bubble is the more effective in the flotation of oil particles.

3.2 Collection efficiency E_c

The effect of bubble size may be twofold. One is the geometrical effect that small bubbles have a large projected area, facilitating collision with oil particles. The other is the hydrodynamic effect that the oil particle is more accessible to the smaller bubble. The latter effect can be evaluated in terms of the collection efficiency E_c which is often used to characterize filtration performance. It is defined by the ratio of the number of oil particles collected by a bubble to that existing in the volume swept by the bubble.

The collection efficiency E_c is related to the residual concentration at a particular point by a simple material balance, i. e.

$$\left(\frac{C}{C_0}\right)_{\text{local}} = \exp\left\{-\frac{3}{2} \frac{E_c}{D_B} v\right\} \quad (5)$$

in which v is the cumulative volume of bubbles which passes the column section of unit area. In bubbling flotation the value of v is uniform with column height and equal to $U_g t$. Thus,

$$\frac{C}{C_0} = \exp\left\{-\frac{3}{2} \frac{E_c U_g t}{D_B}\right\} \quad (6)$$

In electrolysis and dissolved flotations, bubbles are initially distributed uniformly and the quantity v is proportional to the distance x above the bottom according to $v = \alpha x$, where α is the volume of bubbles evolved per unit liquid volume. The observed concentration is the average along the height and the collection efficiency is given by the following equation:

$$\frac{C}{C_0} = \frac{1}{h} \int_0^h \left(\frac{C}{C_0}\right)_{\text{local}} dx = \frac{2}{3} \frac{D_B}{E_c \alpha h} \left[1 - \exp\left\{-\frac{3}{2} \frac{E_c \alpha h}{D_B}\right\}\right] \quad (7)$$

The experimental values of E_c were calculated from Eqs. (6) and (7) and are plotted against bubble diameter in Fig. 5. In the case of electrolysis and bubbling flotations, the initial concentration C_0 was taken to be 80% of actual value, assuming that the spontaneous removal of 20% in 30 minutes took place independently of flotation.

3.3 Comparison with theories

When the oil particles colliding with the bubble are assumed to be all collected and removed, the collection efficiency E_c may be predicted by a simple trajectory theory based on the interception:

$$E_c = \text{maximum of } \left\{ \frac{2\phi}{U_\infty (D_B/2)^2} \right\}_{y=D_p/2} \quad (8)$$

where ϕ is the stream function of particle free flow around a bubble and y is the distance from the bubble surface. Since the bubble behaves like a solid sphere, E_c for creeping flow is given as⁸⁾

$$E_c = \frac{3}{2} \left(\frac{D_p}{D_B}\right)^2 \left(1 + \frac{2}{3} \frac{D_p}{D_B}\right) / \left(1 + \frac{D_p}{D_B}\right) \quad (9)$$

Similarly, E_c for boundary layer flow is given as

$$E_c = 0.995 (D_p/D_B)^2 \sqrt{Re} \quad (10)$$

of which the derivation is given in Appendix. Equations (9) and (10) are also shown in Fig. 5 for comparison with the experimental values.

The general trend of dependence of experimental collection efficiency E_c on the bubble diameter D_B can be found in Fig. 5 in spite of much scattering. With increasing diameter the factor E_c decreases rapidly for small bubbles and much more gradually for large bubbles as the theories of Eqs. (9) and (10) predict, respectively. However, the measured E_c for small bubbles falls below the E_c predicted by Eq. (9). This difference may be in part explained by the modernized trajectory theory⁸⁾, which takes into account many relevant forces as well as interception. The parameter N_{ad} :

$$N_{ad} = \frac{Q(D_B/2)^2}{9\pi\mu(D_P/2)^4 U_\infty} \quad (11)$$

is important in the present case with negligibly small gravity force and double-layer repulsion. The value of N_{ad} with the most probable value of $Q=3 \times 10^{-3}$ J is 1.9×10^{-3} and the theory of Spielman and Fitzpatrick⁸⁾ predicts the value of E_c to be 40% of that given by Eq. (9), as shown by the broken line in Fig. 5.

There are still some differences between the experimental and theoretical collection efficiency, probably because of partial collection of colliding oil particles, rather wide size distribution of bubbles and oil particles and so on. However, the available and proposed trajectory theories are useful since they predict the collection efficiency to within one order of magnitude at worst.

Conclusions

The following conclusions are derived for the flotation of oil particles in the absence of repulsion due to the electric double layer.

Firstly, the flotation rate in bubbling flotation through glass frit is proportional to the oil concentration to the first order at low gas velocity (zone 1, $0.01 \leq U_g \leq 0.05 \text{ cm} \cdot \text{s}^{-1}$) and to the second order (zone 2, $0.1 \leq U_g \leq 3 \text{ cm} \cdot \text{s}^{-1}$). The predominant rate step may be oil particle-bubble collision in zone 1 and particle-particle collision in zone 2. Mechanical agitation and bubbling flotation in zone 2 give the same removal rate with the same power consumption. The role of bubbling in zone 2 is to give adequate turbulence to the liquid.

Secondly, with increasing bubble diameter, the collection efficiency for particle-bubble collision decreases rapidly ($\approx$ to the -2 power) for small bubbles and much more gradually for large bubbles. This trend coincides with the existing and proposed trajectory theories.

Appendix: Derivation of Eq. (10)

When the diameter of the oil particle is much smaller than the boundary layer thickness, the distribution can be approximated by the asymptotic one at the bubble surface, given by Schlichting⁷⁾ as

$$\frac{u}{U_\infty} = 1.392 \sqrt{\frac{3}{2}} (1 - 0.3925\theta^2 + 0.0421\theta^4 + \dots) \sqrt{Re} \frac{y}{(D_B/2)} \quad (\text{A-1})$$

By definition of ϕ it follows that

$$\begin{aligned} \frac{2\phi}{U_\infty(D_B/2)^2} &= \frac{2 \sin \theta}{(D_B/2)} \int_0^{D_P/2} \frac{u}{U_\infty} dy \\ &= \left[1.392 \sqrt{\frac{3}{2}} (1 - 0.3925\theta^2 + 0.0421\theta^4 + \dots) \sin \theta \right] \left(\frac{D_P}{D_B} \right) \sqrt{Re} \end{aligned} \quad (\text{A-2})$$

The function in brackets has the maximum of 0.995 at $\theta=68^\circ$, and Eq. (10) is readily derived.

Nomenclature

C, C_0	= residual and initial oil concentration, respectively	$[\text{g} \cdot \text{m}^{-3}]$
D_B, D_P	= diameter of bubble and oil particle, respectively	$[\text{m}]$
E_c	= collection efficiency	
g	= acceleration due to gravity	$[\text{m} \cdot \text{s}^{-2}]$
h	= height of liquid	$[\text{m}]$
N_{ad}	= parameter defined by Eq. (11)	
P_v	= power consumption per unit volume	$[\text{W} \cdot \text{m}^{-3}]$
Q	= Hamaker constant	$[\text{J}]$
Re	= Reynolds number, $D_B U_\infty \rho / \mu$	
$r, \bar{r}$	= instantaneous and average rates of oil removal per unit volume and unit time, respectively	$[\text{g} \cdot \text{m}^{-3} \cdot \text{s}^{-1}]$
t	= flotation time	$[\text{s}]$
U_g, U_∞, u	= superficial gas velocity, bubble rising velocity, local fluid velocity, respectively	$[\text{m} \cdot \text{s}^{-1}]$
v	= cumulative volume of bubbles passing the column of unit area	$[\text{m}^3 \cdot \text{m}^{-2}]$
x	= distance from the bottom of flotation column	$[\text{m}]$
y	= distance from the bubble surface	$[\text{m}]$
α	= bubble volume evolved per unit liquid volume	
θ	= angle from the front stagnation point	
μ, ρ	= viscosity and density of liquid, respectively	$[\text{Pa} \cdot \text{s}], [\text{kg} \cdot \text{m}^{-3}]$
ϕ	= stream function	$[\text{m}^2 \cdot \text{s}^{-1}]$

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