L_{∞}	= final thickness of cake when θ_c or $\theta'_c = \infty$	[cm]
m	= ratio of wet to dry filter cake mass	[—]
p	= applied expression pressure	[g-force/cm ²]
p_i	= solid compressive pressure below which e is considered constant	[g-force/cm ²]
p_s	= solid compressive pressure	[g-force/cm ²]
s_i	= solid concentration of filter cake surface	[—]
T'_c	= consolidation time factor, defined by Eq. (5)	[—]
U_c	= average consolidation ratio	[—]
U'_c	= average consolidation ratio of hypothetical mixture, defined by Eq. (3)	[—]
$\alpha_{av \cdot F}$	= average specific resistance of filter cake	[cm/g]
ε	= local porosity	[—]
ε_i	= porosity of filter cake surface	[—]
η	= creep constant	[s ⁻¹]
θ_c	= consolidation time	[s]
θ'_c	= consolidation time for hypothetical expression	[s]
θ'_f	= imaginary filtering time	[s]
μ	= viscosity of filtrate	[g/cm·s]
ν	= consolidation behavior index concerning creep characteristic of materials	[—]
ρ	= density of filtrate	[g/cm ³]
ρ_s	= true density of solids	[g/cm ³]
ω_0	= total solid volume per unit sectional area	[cm]

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SHORT COMMUNICATIONS

TURBULENCE STRUCTURE OF SUBMERGED JETS OF DILUTE POLYMER SOLUTIONS

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Introduction

The reduction in wall shear stress caused by polymer additives of high molecular weight has been widely investigated because of its possible great importance in practical applications. Most investigations on drag reduction were done in a round tube or in a flat plate

boundary layer flow. Although the number of papers is relatively small, the study of turbulent jets has been another effort toward understanding the flow behavior of drag-reducing polymer solutions.

In an early experiment, Gadd²⁾ visualized the submerged jet of polymer solutions. In his observation, the presence of Polyox WSR-301 caused a significant suppression of small-scale turbulence. On the other hand, White¹⁾ and Goren³⁾ found no visible difference

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Fig. 1 Variation of jet center-line velocity along jet axis

Table 1 Integral scales on jet center-line

	a [cm]	$A_0/(x-a)$	$A_f/(x-a)$	A_f/A_0
water	3.0	0.0168	0.0324	1.93
50 ppm	7.0	0.0319	0.0877	2.75
200 ppm	13.5	0.0414	0.1187	2.87
Wignanski and Fiedler	3.0	0.0157	0.0385	2.43

between water and dilute polymer solutions. Recently, Barker¹⁾ employed a laser doppler technique to measure the velocity profile and the fluctuation velocity in an axisymmetric submerged jet of polymer solutions. His conclusion agreed with the observation of White¹¹⁾ and Goren³⁾. The gross structure of the submerged jet was not affected by the polymer additive. On the other hand, Shulman *et al.*⁹⁾ measured the fluctuating velocity of the submerged jet using a piezoceramic probe. In their experiments, a significant decrease of eddy diffusivity in a dilute polymer solution was observed, and these results corresponded to the observation of Gadd²⁾. The experimental results mentioned above are rather confusing, and more precise experimental work is required.

In this investigation, the authors attempted to explain the effect of the polymer additive on the structure of free turbulence. Turbulent motion of the submerged jet was recorded on high-speed cinefilm which provided the trajectory of tracer particles suspended in polymer solutions. Autocorrelation and space-correlation of the fluctuating velocity were calculated from the analyzed results of cinefilm.

1. Experimental Details

An axisymmetric jet submerged in a plexiglass tank (400×400×2500 mm) was used in this study. Dilute polymer solutions were run through once only using a big storage tank (1.5 m³) to minimize the effect of mechanical degradation of polymer additives. The jet was ejected from a convergent nozzle having an area ratio of 10:1 and an outlet section diameter of 1 cm. Ambient liquid in the test tank was the same polymer solution as contained in the storage tank.

In the preliminary experiments, the variation of the jet center-line velocity along the jet axis was measured

by a pitot tube. As already discussed by Smith *et al.*⁸⁾ the accuracy of pitot tube measurement in the viscoelastic fluid must be checked by an experiment. In this study, the read-out of the pitot tube was calibrated by the same polymer solution.

The pictures of the jets were taken by a high-speed cinecamera (Hitachi-16H) with 100 feet of colour film. Tracer particles made from polyethylene and coloured with orange fluorescent pigment were suspended in the test liquids. The particle diameter, density and concentration were 200 μ , 1.03 g/cm³ and 0.2 wt% respectively. The tracer particles were illuminated by the parallel light of a mercury lamp which came in the lateral direction of the jet through a slit of 3 mm width. The measuring section was selected to be 3 mm in the radial direction and 6 mm in the axial direction of the jet. A film motion analyzer (Bell & Howell model 7399) was used to measure the coordinate differences of a tracer particle between two successive frames. The instantaneous velocity of a particle was determined by axial and radial displacement and the time lapse per frame. The velocity of liquid was assumed to be equal to that of the tracer particle.

The velocity data was obtained from the cinefilm of about 3000 frames, which corresponds to a time of 5 sec. As the individual frame data involved considerable reading error of coordinates on the film motion analyser, the data of 5 successive frames were averaged to give the instantaneous velocity at the middle frame time.

Polyethylene oxide (PEO: grade ALCOX, E160 supplied by Meisei Chemical Corp. was used as the polymer additive. The intrinsic viscosity of ALCOX E160 in water was 1200 cm³/g. A stabilizer, SANDEX-C (Meisei Chemical Corp.), was added to prevent the chemical degradation of the polymer additive. Tap water was used to dissolve the polymer to concentrations of 50 and 200 ppm. It was certified preliminarily that SANDEX-C had no effect on drag reduction in a turbulent pipe flow.

2. Results and Discussion

The variation of the mean velocity along the jet axis is shown in Fig. 1. All the results of this work were obtained at $Re=20,000$. The length of potential core in the polymer solutions increases according to the increase of polymer concentration. The hypothetical origin of the jet, i. e. the "origin of similarity", was determined from Fig. 1. The distance between the origin of similarity and the jet outlet, indicated by a , is given in Table 1. The value of a in a Newtonian fluid coincides with that of Wignanski and Fiedler¹²⁾, whose data were taken at $x/D < 50$. In the following discussion, the length scale of the jet would be referred to $x-a$, not x , to emphasize the similarity relation-

ship of the jet.

As mentioned in the previous section, the fluctuating velocity in the longitudinal direction was measured simultaneously at some different positions in the radial direction. Only the results of auto- and space-correlation of the fluctuating velocity are presented in this paper because of space limitation. More detailed information is available in reference 10). The auto-correlation of the u' signal along the center-line of the jet was calculated according to the following definition.

$$R_E(T) = \overline{u'(t)u'(t-T)} / \overline{u'^2} \quad (1)$$

The lateral space-correlation was obtained from the following definition.

$$R_{uu}(\Delta y) = \overline{u'(y)u'(y+\Delta y)} / \{\sqrt{\overline{u'^2(y)}}\sqrt{\overline{u'^2(y+\Delta y)}}\} \quad (2)$$

In these experiments, the value of y in Eq. (2) was set to equal zero. This means that in the calculation of the space correlation one point was fixed on the jet center-line and the other point was moved in the radial direction. Δy indicates the distance between the two positions. The associated integral scales A_E and A_g were calculated from the correlation curves. It was pretty difficult to obtain the correct values of integral scales because of the scatter of the experimental results. Thus the following conventional approximations were adopted.

$$A_E = \int_0^{T_c} R_E(T) dT \quad (3)$$

$$A_g = \int_0^{\Delta y_c} R_{uu}(\Delta y) d(\Delta y) \quad (4)$$

T_c and Δy_c were defined to be equal to the position where the correlation curves crossed the zero level for the first time. This calculation was intended to give a rough estimation of the effect of the polymer additive on the structure of turbulence in the submerged jet. In the analysis of the autocorrelation, Taylor's hypothesis was adopted to calculate the value of the integral scale of longitudinal space correlation. For this calculation, the following equation was used.

$$A_f = U A_E \quad (5)$$

It had already been noted that Taylor's hypothesis was basically invalid for the turbulent free jet¹²⁾. In particular, this hypothesis fails near the edges of a submerged jet. The measuring points of this study were restricted to the central region of the submerged jet where the intermittency factor was close to unity. Thus the integral scale A_f determined by Taylor's hypothesis was expected to give only qualitative information about the effect of the polymer additive.

The autocorrelation of the u' signal on the jet center-line is plotted in Fig. 2. The lateral space correlation is shown in Fig. 3. These diagrams show that the correlation in the polymer solutions is stronger in a wide range of $TU_m/(x-a)$ or $\Delta y/(x-a)$ than in

Fig. 2 Autocorrelation of u' signal along jet center-line

Fig. 3 Variation of lateral correlation across jet

the case of Newtonian fluid (water). The integral scales calculated by Eqs. (3), (4) and (5) are shown in Table 1. This Table also contains the experimental results obtained by Wignanski and Fiedler¹²⁾. The value of Reynolds number in their experiments was 10^5 , and the data were taken at $x/D=30-90$. Assuming that the flow was self-preserving, the values of these experiments at $x/D=35$ and 50 were averaged to give this table. It is obvious that the integral scale in the polymer solutions is greater than in the case of Newtonian fluid. It has been shown for isotropic turbulence that the ratio, A_f/A_g , is equal to the value of 2⁴⁾. The results for Newtonian fluid, both in this study and in the study of Wignanski and Fiedler, show that the value of A_f/A_g is almost equal to the value of 2. This means that the turbulence near the jet center-line is similar to isotropic turbulence. In the case of polymer solutions, the values of A_f/A_g are bigger than that for Newtonian fluid flow. This means that the representative eddy is stretched into the longitudinal direction.

The variation of A_f in the radial direction is plotted in Fig. 4. The solid line in this diagram is the experimental result of Wignanski and Fiedler¹²⁾. The integral scale A_f of Newtonian fluid becomes greater according to the increase of distance from the jet center-line. This tendency is just the same as the re-

Fig. 4 Distribution of longitudinal integral scale across jet

sults obtained by Wignanski and Fiedler. The difference of the absolute values of A_f between the experimental results of this study and Wignanski and Fiedler seems to be caused by the Taylor's assumption in this study. Wignanski and Fiedler measured the longitudinal space correlation and calculated the value of A_f from the space correlation.

The behavior of the polymer solutions in the variation of A_f is quite different from that of Newtonian fluid. The maximum value of A_f was observed on the center-line of the submerged jet, and A_f seems to decrease according to the increase of y . This means that the effect of the polymer additive on the structure of turbulence appears most significantly in the central portion of the submerged jet, where the velocity gradient becomes zero. Most observations of turbulent pipe flow of the drag-reducing fluid confirmed that the effect of polymer additive appeared in the wall region where the turbulent energy production showed a maximum value⁵⁻⁷). The interaction between polymer additives and turbulence has been thought to occur most significantly at the position where energy production had a maximum value. The experimental results of Wignanski and Fiedler for the submerged jet showed that the turbulence energy production had a maximum value at $y/x=0.052$ where the distribution of Reynolds stress showed a maximum value. They also showed that the dissipation had a maximum on the jet center-line. The results shown in Fig. 4 mean that the suppression of turbulence caused by polymer additives is not related to turbulence production, but is related more closely to its dissipation. Summing up the results of this work, the effect of polymer additives on the structure of a turbulent round jet gave qualitative support to the results obtained by Gadd²) and Shulman *et al.*⁹). This means that polymer additives cause a significant interaction with turbulent eddy, even in a free turbulent shear flow.

Conclusion

The following information was obtained.

- 1) The increase of the integral scale in the submerged jet of the polymer solutions is not isotropic. The eddy is stretched more significantly in the longitudinal direction.
- 2) The most significant effect of the polymer additive on the enlargement of the integral scale appears at the jet center-line, where the turbulent dissipation, not the turbulent production, has a maximum value.

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Nomenclature

a	= position of the origin of similarity	[m]
c	= concentration of polymer additive	[ppm]
D	= nozzle diameter at the outlet section	[m]
Re	= Reynolds number ($= U_0 D / \nu$)	[—]
$R_E(T)$	= autocorrelation coefficient	[—]
$R_{uu}(\Delta y)$	= lateral space correlation coefficient	[—]
t	= time	[s]
T	= delay time of autocorrelation	[s]
U	= time-smoothed velocity in the x direction	[m/s]
U_0	= velocity at the nozzle outlet	[m/s]
U_m	= velocity on the jet center-line	[m/s]
u'	= fluctuating velocity in the x direction	[m/s]
x	= longitudinal coordinate in flow direction ($x=0$ corresponds to the nozzle outlet)	[m]
y	= lateral coordinate ($y=0$ corresponds to the jet center-line)	[m]
Δy	= distance between the two points where the space-correlation is taken	[m]
A_E	= integral time scale defined by Eq. (3)	[s]
A_f	= longitudinal integral scale defined by Eq. (5)	[m]
A_g	= lateral integral scale defined by Eq. (4)	[m]
ν	= kinematic viscosity	[m ² /s]

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