A SIMPLE THEORY OF THE MOTION OF SMALL PARTICLES SUSPENDED IN NONUNIFORM GASES

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The motion of small suspended particles (dust, smoke, aerosols, etc.) in nonuniform gases is a subject of importance in such fields as colloid chemistry, astrophysics and chemical engineering. Waldmann⁷⁾ has given a very complete theory of the thermal force and the diffusion force in a region of large Knudsen number by employing the rigorous kinetic model developed by Chapman and Enskog²⁾. Although his theory gives accurate results, the mathematical treatment is very complicated. It is thus useful to derive similar results from a much simplified kinetic model and to provide an easy understanding of these striking phenomena.

1. Theory

1.1 Background of simplified kinetic model

We first consider an infinitely expanding gas at a constant pressure but having a gradient of temperature in the z -direction and try to find an approximated form of the distribution function for the steady conduction of heat. Our starting point is to use the Boltzmann equation and the mass and heat flow equations:

$$c_z \frac{\partial}{\partial z} (nf) = \left[\frac{\partial (nf)}{\partial t} \right]_{\text{coll}} \quad (1)$$

$$\int n c_z f d\mathbf{c} = 0 \quad (2)$$

$$\int \frac{1}{2} m n c^2 c_z f d\mathbf{c} = -K \frac{dT}{dz} \quad (3)$$

If the deviation from uniform distribution is very slight, we can simply split f into a Maxwellian term f_0 and a first correction term f_s :

$$f = f_0 + f_s \quad (4)$$

where

$$f_0 = A \exp(-\beta^2 c^2), \quad A = \beta^3 / \pi^{3/2} \quad (5)$$

and $\beta = (m/2kT)^{1/2}$. Substituting Eq. (4) into Eq. (1) and considering that β is a function of z and that the term $\partial(nf_s)/\partial z$ is a small quantity which can be neglected, we have

$$\left[\frac{\partial (nf)}{\partial t} \right]_{\text{coll}} = -\frac{nA}{T} \frac{dT}{dz} c_z \left(\frac{5}{2} - \beta^2 c^2 \right) e^{-\beta^2 c^2} \quad (6)$$

We here adopt the concept of the linearized collision

model¹⁾ and put the collision term simply as

$$[\partial(nf)/\partial t]_{\text{coll}} = -n\Theta(f - f_0) \quad (7)$$

where Θ is the collision frequency. From Eqs. (4), (6), and (7), we can find the correction term

$$f_s = B c_z (5/2 - \beta^2 c^2) \exp(-\beta^2 c^2) \quad (8)$$

where

$$B = (A/T\Theta)(dT/dz) \quad (9)$$

At this stage B , as well as Θ , is arbitrary. Generally, Θ depends on the distribution of velocity. But, from the condition of Eq. (2), B , as well as Θ , is found to be a constant value independent of gas velocity. Substituting Eq. (4) with Eq. (8) into Eq. (3), we can express B in terms of more definite quantities:

$$B = (8/5\pi^{3/2})(\beta^7 K/mn)(dT/dz) \quad (10)$$

We now finish finding the distribution function. Although we do not aim at finding an expression of thermal conductivity, a brief discussion will be given here to check the validity of our theory. Equating Eq. (9) with Eq. (10), we have the expression of thermal conductivity

$$K = (5/4)(nk/\beta^2\Theta) \quad (11)$$

If Θ is selected suitably as $\Theta = (8/15)(\bar{c}/\lambda)$, Eq. (11) gives

$$K = (75\pi/128)nk\bar{c}\lambda \quad (=15 k\mu/4 m) \quad (12)$$

This is equivalent to Chapman's first approximation²⁾. It should be noted that Eq. (12) can be also derived without presuming the value of Θ if the rate of change in $c_z c^2$, $\Delta(c_z c^2)$ caused by collisions between molecules characterized by f_0 and those by f are known. The $\Delta(c_z c^2)$ has been given elsewhere^{4,5)}. Equating $\Delta(c_z c^2)$ with that obtained from Eq. (6), we can obtain another expression of B . Substitution of this into Eq. (10) gives the expression given by Eq. (12). Thus Eq. (14) with Eqs. (8) and (10) is thought to be an adequate expression of the distribution function to our problem.

1.2 Motion in a temperature gradient

Let us now consider a spherical particle suspended in nonuniform gas expressed by Eq. (4). The coordinate system is shown in Fig. 1. In the region of $\lambda/a \gg 1$, it can be assumed that the distribution function of gas molecules towards the particle suffers no change caused by existence of the particle. Then the normal com-

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ponent of momentum transferred by the molecules striking the particle can be calculated as

$$\begin{aligned}\tau_{rr} &= mn \int_{c_r < 0} c_r^2 [A + B(c_\theta \cos \theta - c_r \sin \theta) \\ &\quad \times (5/2 - \beta^2 c^2) e^{-\beta^2 c^2}] dc \\ &= mn/4\beta^2 + (3\mu/2\pi\beta^2 \bar{c} T)(dT/dz) \cos \theta\end{aligned}\quad (13)$$

The shear component of impulse is similarly

$$\tau_{r\theta} = -(3\mu/4\pi\beta^2 \bar{c} T)(dT/dz) \sin \theta \quad (14)$$

The impulse in the z -direction is

$$\begin{aligned}F &= 2\pi a^2 \int_0^\pi \sin \theta d\theta [-\tau_{rr}(\cos \theta) - \tau_{r\theta}(-\sin \theta)] \\ &= -(8a^2/15)(2\pi m/kT)^{1/2} K(dT/dz)\end{aligned}\quad (15)$$

This shows that the thermal force should act on the particle to move from hotter to colder region.

The drift velocity of the particle can be easily estimated by utilizing Einstein's equation⁹⁾

$$v_1 = D_{12} F/kT \quad (16)$$

where D_{12} is the diffusivity of the particle through gas. In our case, the particle diameter is much larger than that of a molecule and the Maxwell-Chapman-Enskog expression for D_{12} can be expressed as

$$D_{12}^{-1} = (8n/3)(2m/\pi kT)^{1/2} \pi a^2 \Omega_{12} \quad (17)$$

where Ω_{12} is the collision integral for diffusion; $\Omega_{12} = 1$ for elastic sphere of specular reflection and $\Omega_{12} = 13/9$ for that of perfect diffuse reflection⁶⁾. Substituting Eqs. (15) and (17) into Eq. (16), we have

$$v_1 = -(1/5)(K/\Omega_{12} p)(dT/dz) \quad (18)$$

When the former case is selected for Ω_{12} , Eq. (18) as well as Eq. (15) agrees with Waldmann's formula⁷⁾ and also with the Mason-Chapman equation⁶⁾ derived from the dusty gas model.

If the particle motion caused by a gradient of temperature is regarded as a process similar to thermal diffusion, our problem can be expressed by the following phenomenological equation:

$$v_1 = -D_{12} \alpha_T d(\ln T)/dz \quad (19)$$

where α_T is the thermal diffusion factor. Comparison between Eq. (18) and Eq. (19) gives

$$\alpha_T = (1/5)(K/nkD_{12}) \quad (20)$$

This is precisely the thermal diffusion factor for a quasi-Lorentzian gas⁶⁾.

1.3 Motion in a concentration gradient

We here consider a binary gas mixture diffusing at a constant pressure and a constant temperature. The method of simplified kinetic model similar to that used for conduction of heat in section 1.1 can be employed to find the distribution function for each kind of diffusing molecule. We resolve the distribution function into two terms, the Maxwellian and correction terms

$$f_I = f_{I0} + f_{Is} \quad (I=2 \text{ or } 3) \quad (21)$$

Fig. 1 A spherical particle suspended in gases

where

$$f_{I0} = A_I \exp(-\beta_I^2 c_I^2), \quad A_I = \beta_I^3/\pi^{3/2}$$

Then the Boltzmann equation Eq. (1) with the aid of constant pressure and temperature gives

$$f_{Is} = C_I c_z \exp(-\beta_I^2 c_I^2) \quad (22)$$

After an algebraic manipulation similar to that used to determine factor B in Eq. (8), we find

$$C_I = -(4\beta_I^4 D_{IJ}/\pi^2 n_I \bar{c}_I)(dn_I/dz) \quad (23)$$

Now imagine a small particle suspended in a gas mixture whose nonuniformity is expressed by the split distribution functions Eq. (21). The normal component of momentum transferred to the particle by molecules I can be obtained by use of Eq. (21) with Eqs. (22) and (23),

$$\tau_{rr,I} = m_I n_I/4\beta_I^2 + (m_I D_{IJ}/\pi^{1/2} \beta_I)(dn_I/dz) \cos \theta \quad (24)$$

The shear component is

$$\tau_{r\theta,I} = -(m_I D_{IJ}/2\pi^{1/2} \beta_I)(dn_I/dz) \sin \theta \quad (25)$$

The total impulse of molecules 2 and 3 in the z -direction is

$$F = -(8\pi a^2/3)(2kT/\pi)^{1/2} (\sqrt{m_2} - \sqrt{m_3}) D_{23} (dn_2/dz) \quad (26)$$

This agrees precisely with Waldmann's formula⁷⁾. Equation (26) shows that the dragging force of 2 and 3 causes the particle to move in the direction of diffusion of heavier molecules.

We next calculate the drift velocity of the particle in a binary mixture having a concentration gradient. The effective diffusivity of the particle is

$$D_{\text{eff},1}^{-1} = x_2 D_{12}^{-1} + x_3 D_{13}^{-1} \quad (27)$$

where D_{IJ} ($J=2, 3$) is given by an expression similar to Eq. (17). Then Einstein's equation gives

$$v_1 = -\frac{(\sqrt{m_2} - \sqrt{m_3}) D_{23}}{(\sqrt{m_2} x_2 \Omega_{12} + \sqrt{m_3} x_3 \Omega_{13})} \frac{dx_2}{dz} \quad (28)$$

Equation (28) is equivalent to the expression given by Waldmann⁷⁾ and also to that given by Mason and Chapman⁶⁾ for the elastic sphere of specular reflection, but our method to derive the expression is much simpler when compared with those employed by Waldmann and Mason *et al.*

Conclusion

In this note is demonstrated that the method of distribution function starting from a much-simplified

kinetic model leads us directly to as accurate an expression of the motion of particles in nonuniform gases as is obtained from the very rigorous kinetic model. The expressions given by Eqs. (18) and (28) will be useful in treating the thermophoresis and diffusio-phoresis of small particles suspended in nonuniform gases inasmuch as the processes are in the free molecule region.

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Nomenclature

A_I	= constant = $\beta_I^2/\pi^{3/2}$	$[m^{-3} \cdot s^3]$
a	= radius of particle	$[m]$
B	= constant given by Eq. (10)	$[m^{-4} \cdot s^4]$
C_I	= constant given by Eq. (23)	$[m^{-4} \cdot s^4]$
c_I	= velocity of molecule I	$[m \cdot s^{-1}]$
$\bar{c}_I$	= averaged velocity of molecule I = $\sqrt{8kT/\pi m_I}$	$[m \cdot s^{-1}]$
D_{IJ}	= diffusivity	$[m^2 \cdot s^{-1}]$
$D_{eff, I}$	= effective diffusivity defined by Eq. (27)	$[m^2 \cdot s^{-1}]$
F	= thermal or diffusional force	$[N]$
f_i	= velocity distribution function	$[m^{-3} \cdot s^3]$
f_{10}	= Maxwell's distribution function	$[m^{-3} \cdot s^3]$
f_{1s}	= small deviation from f_{10}	$[m^{-3} \cdot s^3]$
K	= thermal conductivity	$[J \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}]$
k	= Boltzmann's constant	$[J \cdot K^{-1}]$
m_I	= mass of molecule I	$[kg]$
n_I	= number density of molecule I	$[m^{-3}]$
p	= pressure of gas = $(\sum n_i)kT$	$[Pa]$
r	= distance in radial direction	$[m]$

T	= absolute temperature	$[K]$
t	= time	$[s]$
v_1	= drift velocity of particle	$[m \cdot s^{-1}]$
x_I	= fraction of constituent gas	$[-]$
z	= distance in direction of temperature or concentration gradient	$[m]$

α_T	= thermal diffusion factor	$[-]$
β_I	= inverse of most probable velocity of molecule I = $\sqrt{m_I/2kT}$	$[m^{-1} \cdot s]$
Θ	= collision frequency	$[s^{-1}]$
θ	= polar angle	$[-]$
λ	= mean free path	$[m]$
μ	= viscosity	$[kg \cdot m^{-1} \cdot s^{-1}]$
τ_I	= stress tensor	$[Pa]$
Ω_{IJ}	= collision integral for diffusional process	$[-]$

<Subscripts>

1	= particle
2, 3	= gas molecules
I, J	= dropped for a pure gas

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COMPARISON BETWEEN ELECTROOSMOTIC DEWATERING EFFICIENCIES UNDER CONDITIONS OF CONSTANT ELECTRIC CURRENT AND CONSTANT VOLTAGE

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In previous papers^{1,2)}, it was found that electroosmotic dewatering was effective for gelatinous sludge which was difficult to dewater by the mechanical method. The electroosmotically dewatered volume and the electric power consumption per unit dewatered volume were analysed theoretically based on a model of electroosmotic flow through a compressible-particle

packed bed under conditions of constant electric current and constant voltage respectively. These equations, obtained theoretically, were confirmed to be appropriate for the design of electroosmotic dewatering equipment.

In this paper, a comparison of electroosmotic dewatering efficiency under constant electric current with that under constant voltage is made, involving dewatering rate, water content in dewatered sludge

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