AN EXTENDED POWER CORRELATION FOR ANCHOR AND HELICAL RIBBON IMPELLERS

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Introduction

On the basis of a physical model of the fluid motion around a blade, a power correlation for both anchor and helical ribbon impellers was previously proposed by the authors¹²⁾. This correlation assumed a constant blade width, w , although the theoretical term derived from the physical model included blade width as one of the geometrical variables.

In this work, with blade width and number of blades added to the experimental geometrical variables, power consumption measurements have been carried out in Newtonian liquids under laminar flow conditions. As a result, all the geometrical variables have been incorporated into the power correlation.

1. Experiment

The general configuration of a helical ribbon impeller is shown in **Fig. 1** and the geometrical variables of the impellers used are summarized in **Table 1**. In addition to these impellers, a special single helical ribbon impeller, SH6', was used to measure the power consumed around the arms which support the blades. This impeller had the same geometry as the SH6 with an additional six arms. The experimental procedure was as described in the previous paper¹²⁾.

2. Results and Discussion

The relation between power number, N_P , and Reynolds number, Re , can be represented by the following equation as suggested by many investigators^{2-4,9,10)}.

$$N_P \cdot Re = C \quad (1)$$

C is a function of the geometrical variables, and experimental values of C are also summarized in Table 1.

The power consumed around an arm was obtained from a comparison of the power consumption of SH6' impeller with that of SH6. The result is shown in **Table 2**. The product of power number for an arm and Reynolds number, $N_{Pa} \cdot Re$, was about 5. Therefore, in order for the effects of the geometrical

variables on power to be evaluated more precisely, it would be more satisfactory for the total power consumption to be given by the sum of the power consumption of the blades and that of the arms which support the blades.

From the drag flow analysis shown in the **appendix**, the power consumption of an arm can be described by the following equation.

$$N_{Pa} \cdot Re = 2.08\pi^3 \left(\frac{d_a}{d}\right)^{0.15} \left(\frac{r_{bi}}{d}\right)^{3.15} Re^{0.15} \quad (2)$$

Equation (2) indicates that the geometrical ratio of r_{bi}/d , which is equal to $1/2 - w/d$, has a large effect on the power consumption of an arm. With the substitution of $d_a/d = 3/115$ and $r_{bi}/d = 1/2$ into Eq. (2), a value of from 3.3 to 4.5 for $N_{Pa} \cdot Re$ was obtained for SH6' and was close to the measured value of 5. Since the difference between measured and calculated values is small and can be neglected compared with the value of C for SH6, Eq. (2) can be used to express the effect of the geometry of an arm on power consumption. The range of $Re^{0.15}$ was 0.7 to 1.4 within the limits of the experimental Re covered in this paper, i.e. $0.1 \leq Re \leq 10$. Therefore, we set $Re^{0.15} = 1$ for convenience.

Thus the total power for the impeller is given by

$$\begin{aligned} N_P \cdot Re = & \frac{16\pi^3}{2 \ln(4 + 8c/w) - 1} \left(\frac{L}{d}\right) f(c/D) \\ & \times (\sin \theta_B)^{\gamma} \left(\frac{n_p}{2}\right)^{\delta} (w/D)^{\epsilon} \\ & + 2.08\pi^3 n_a n_p \left(\frac{d_a}{d}\right)^{0.15} \left(\frac{r_{bi}}{d}\right)^{3.15} \end{aligned} \quad (3)$$

where

$$f(c/D) = 1 + \alpha(c/D)^{\beta}$$

The five constants in Eq. (3) were determined by a multiple non-linear regression using all the data shown in Table 1. The values of δ and ϵ were found to be 1.02 and -0.006 respectively. From the result, it can be concluded that power consumption increases nearly proportionally to the number of blades while the effect of blade width on power consumption is expressed by the theoretical term derived from the physical model previously proposed¹²⁾. Therefore,

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Table 1 Geometrical variables and measured power consumption for anchor and helical ribbon impellers

	Geometry No.	n_p [—]	n_a [—]	d [mm]	c/D [—]	s/D [—]	L [mm]	w [mm]	$C(=N_p \cdot Re)$ [—]
Anchor impellers	AC1*	2	2	115.2	0.0500	∞	115.0	13.0	173.3
	AC2*	2	2	121.6	0.0250	∞	115.0	13.0	202.5
	AC3*	2	2	124.8	0.0125	∞	115.0	13.0	268.7
	AC4*	2	2	125.4	0.0100	∞	115.0	13.0	301.7
	AC5*	2	2	126.7	0.0050	∞	115.0	13.0	366.6
Helical ribbon impellers	SH1	1	3	120.0	0.0313	0.980	397.1	13.0	171.9
	SH2	1	3	114.0	0.0547	0.980	379.3	13.0	148.4
	SH3	1	3	103.9	0.0943	0.980	349.4	13.0	152.1
	SH4	1	4	113.2	0.0577	0.649	547.9	13.0	178.2
	SH5	1	5	114.6	0.0522	0.488	731.1	13.0	196.8
	SH6	1	3	116.4	0.0453	1.30	301.4	13.0	151.1
	SH7	1	3	115.7	0.0480	1.95	220.6	13.0	148.4
	DH1*	2	3	120.0	0.0313	0.980	397.2	13.0	351.1
	DH2*	2	3	112.4	0.0609	0.980	374.6	13.0	290.6
	DH3*	2	3	102.8	0.0984	0.980	346.3	13.0	275.3
	DH4*	2	4	112.9	0.0590	0.649	546.5	13.0	356.6
	DH5*	2	5	113.8	0.0555	0.488	725.9	13.0	425.1
	DH6	2	3	116.0	0.0469	1.30	300.6	13.0	291.0
	DH7	2	3	116.1	0.0465	1.95	221.1	13.0	272.5
	DH8	2	3	116.4	0.0454	0.980	386.5	9.75	294.1
	DH9	2	3	116.8	0.0437	0.980	387.6	16.3	332.7
	DH10	2	3	117.0	0.0428	0.980	388.2	19.5	339.4
	DH11	2	3	116.6	0.0444	0.980	387.1	26.0	372.4
	TH1	3	3	116.4	0.0453	1.30	301.4	13.0	448.1
	TH2	3	3	115.9	0.0473	1.95	220.8	13.0	416.3
	QH1	4	3	116.0	0.0469	1.95	221.0	13.0	491.5

$D=H=128.0$ [mm], $d_a=3.0$ [mm], $d_s=12.0$ [mm], * taken from previous work¹²⁾

Fig. 1 Sketch of helical ribbon agitator system

Table 2 Comparison of power consumption of SH6 and SH6' impellers

Geometry No.	Total number of arms	$C(=N_p \cdot Re)$ [—]
SH6	3	151.1
SH6'	9	181.6

$N_{Pa} \cdot Re = (181.6 - 151.1) / (9 - 3) \approx 5$
 N_{Pa} : power number per arm

the values of δ and ε were reset at 1 and 0 respectively and the other three constants α , β and γ were re-determined by non-linear regression.

$$N_p \cdot Re = \frac{16\pi^3}{2 \ln(4 + 8c/w) - 1} \left(\frac{L}{d} \right) f(c/D) (\sin \theta_B)^{0.555} \times \frac{n_p}{2} + 2.08\pi^3 n_a n_p \left(\frac{d_a}{d} \right)^{0.15} \left(\frac{r_{bt}}{d} \right)^{3.15} \quad (4)$$

where

$$f(c/D) = 1 + 0.00539(c/D)^{-0.976}$$

Equation (4) is compared with the experimental data obtained in this work and other published sources^{5-9, 13)} in Fig. 2. The agreement between measured and calculated values was satisfactory except in works by Nagata *et al.* and Zlokarnik. This correlation is valid for the following ranges of each geometrical variable: $n_p=1$ to 4, $0.019 \leq c/D \leq 0.13$, $0.49 \leq s/D$ and $0.076 \leq w/D \leq 0.20$ in the laminar flow region of $Re=0.1$ to 10.

Fig. 2 Power correlation for both anchor and helical ribbon impellers

Appendix: Power consumption of the arm

The drag coefficient at low Reynolds number for a cylinder of diameter d_a and infinite length can be represented by the following equation¹⁾.

$$C_D = \frac{8\pi}{Re' \ln(7.4/Re')} \quad (\text{A-1})$$

where

$$Re' (= \rho U d_a / \mu) < 1$$

The logarithmic plot of $\ln(7.4/Re')$ against Re' is an almost straight line with slope -0.15 in the investigated region of $0.001 < Re' < 0.1$. Therefore, Eq. (A-1) was rewritten as follows.

$$C_D = \frac{8\pi}{Re'(3.2Re')^{-0.15}} = \frac{8\pi}{3.2Re'^{0.85}} \quad (\text{A-1})'$$

The drag D_a exerted by the liquid can be represented as follows.

$$D_a = \frac{1}{2} \rho U^2 d_a C_D = \frac{4\pi}{3.2} \mu U \left(\frac{d_a U \rho}{\mu} \right)^{0.15} \quad (\text{A-2})$$

where

$$U = 2\pi r N$$

The arm can be regarded as a cylinder of length $(r_{bi} - r_s)$. The torque T_a produced by rotation of the cylinder as shown in **Fig. A-1** is represented as follows:

$$\begin{aligned} T_a &= \int_{r_s}^{r_{bi}} D_a r dr \\ &= 1.04\pi^2 \mu N \left(\frac{\rho d_a N}{\mu} \right)^{0.15} (r_{bi}^{3.15} - r_s^{3.15}) \quad (\text{A-3}) \end{aligned}$$

Consequently, the following equation is obtained:

$$\begin{aligned} N_{Pa} \cdot Re &= 2.08\pi^2 \left(\frac{d_a}{d} \right)^{0.15} \left(\frac{r_{bi}^{3.15} - r_s^{3.15}}{d^{3.15}} \right) Re^{0.15} \\ &\doteq 2.08\pi^2 \left(\frac{d_a}{d} \right)^{0.15} \left(\frac{r_{bi}}{d} \right)^{3.15} Re^{0.15} \quad (2) \end{aligned}$$

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Nomenclature

C	= geometrical constant in Eq. (1)	[—]
C_D	= drag coefficient	[—]
c	= clearance between impeller and vessel wall	[m]
D	= vessel diameter	[m]
D_a	= drag for cylinder per unit length	[Nm ⁻¹]
d	= impeller diameter	[m]
d_a	= arm diameter	[m]
d_s	= shaft diameter	[m]
H	= height of vessel	[m]

Fig. A-1 Schematic diagram of arm motion

h	= height of blade	[m]
L	= length of blade ($= h/\sin \theta_B$)	[m]
N	= rotational speed of the impeller	[s ⁻¹]
N_P	= power number of impellers ($= P/\rho d^5 N^3$)	[—]
N_{Pa}	= power number of an arm ($= P_a/\rho d_a^5 N^3$)	[—]
n_a	= number of arms on each blade	[—]
n_p	= number of blades	[—]
P	= power consumption of impeller	[W]
P_a	= power consumption of an arm	[W]
Re	= Reynolds number for mixing systems ($= d^2 N \rho / \mu$)	[—]
Re'	= Reynolds number for a cylinder ($= \rho U d_a / \mu$)	[—]
r	= radial position	[m]
r_s	= radius of shaft	[m]
s	= impeller pitch	[m]
T_a	= torque of a cylinder	[N·m]
U	= velocity of uniform flow	[m·s ⁻¹]
w	= blade width	[m]
$\alpha, \beta, \gamma, \delta, \varepsilon$	= constants in Eq. (3)	[—]
θ_B	= blade angle	[—]
μ	= viscosity	[Pa·s]
ρ	= density	[kg·m ⁻³]

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