PRACTICAL SYNTHESIS METHOD FOR HEAT EXCHANGER NETWORK

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Synthesis of an optimal (minimum-cost) heat exchanger network is very important from the viewpoint of energy saving and cost reduction, but it is difficult to solve these synthesis problems because there exist an extremely large number of possible stream combinations to be considered.

A practical method composing three stages [pre-analysis, generation of initial networks, and evolution] was developed to synthesize an optimal network by hand calculation. The problem table is adopted to determine the minimum utility requirement precisely, and to search necessary conditions of maximum heat recovery in the pre-analysis stage. A simple algorithm of initial network generation is developed to generate nearly optimal networks considering maximum heat recovery and optimal locations of heaters and coolers. Initial networks are improved by simple evolutionary rules based on stream-splitting to increase the heat duties of units and to vary the exchanger sizes in the evolution stage. The effectiveness of this method is demonstrated by solving the synthesis problems reported in previous works.

Introduction

Heat exchanger networks are widely used to reduce energy consumption in the petroleum and petrochemical industries, and the utility and capital cost to accomplish the required heat exchange can often account for the major portion of the plant cost. With the recent energy cost increase and quantitative limits on supply of energy, it is very important to solve the problem of synthesizing an optimal (minimum-cost) network of exchangers, heaters, and/or coolers in view of energy saving and cost reduction.

Many different methods have been proposed to synthesize an optimal network, and recent reviews can be found in Nishida *et al.*⁽¹⁾ and Hlavacek⁽²⁾. The difficulties of synthesizing an optimal heat exchanger network arise from the extremely large number of possible stream combinations. Algorithmic methods, such as branch-and-bound method⁽³⁾, tree search method⁽⁷⁾, depth first tree search method⁽⁸⁾, and implicit enumeration method⁽²⁾, have been developed to reduce the combinatorial problem, but application of these methods requires some special mathematical background and computational skill, and the optimality of the produced network is not guaranteed. Alternative approaches, such as the heuristic method⁽⁹⁾ and systematic methods^(1,5,6), have been developed to avoid the combinatorial problem. The heuristic method, always matching the hot stream of highest

supply temperature with the cold stream of highest target temperature, was developed by Ponton and Donaldson⁽⁹⁾. This method, in many cases, generates surprisingly good networks, but it sometimes generates networks which are quite far from optimal. This is mainly caused by insufficient consideration of maximum heat recovery (maximum amount of heat exchange among hot and cold streams) and optimal locations of heaters and coolers. Rathore and Powers⁽⁹⁾ pointed out that the utility cost associated with heating and cooling is normally more important than the capital cost, so recent works have concentrated on finding networks which achieve maximum heat recovery while showing low capital cost. Systematic methods have been developed by Nishida *et al.*⁽⁸⁾, Linnhoff and Flower⁽⁵⁾, and Flower and Linnhoff⁽¹⁾, and these methods generate the networks achieving maximum heat recovery. Nishida *et al.*⁽⁸⁾ and Linnhoff and Flower⁽⁵⁾ generated the initial networks, and improved them using evolutionary rules. The former generated the initial networks using the minimum total area algorithm, and the latter generated them using the problem table. The initial networks generated by these methods consist of a large number of units. The calculation load in the evolution stage is in proportion to the number of units in the initial network. It was mentioned that these methods can be implemented by hand calculation without special mathematical background and computational skill, but it is difficult to apply them to large problems because the number of units in the initial network

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increases rapidly as the number of process streams of the problem increases. Flower and Linnhoff¹⁾ developed a thermodynamic-combinatorial method using the minimum number of units, but this method could not produce networks for problems having difficult heat recovery situations.

The purpose of this study is to develop a practical method of synthesizing an optimal network by hand calculation, so it is necessary to minimize the total annual cost (utility and capital) of the network and also to reduce the calculation load. The method developed consists of three stages (pre-analysis, generation of initial networks, and evolution). In the pre-analysis stage the thermodynamical restrictions of the synthesis problem are investigated in order to achieve maximum heat recovery, since the utility cost heavily dominates the total cost. The problem table, used in the temperature interval method⁶⁾, was adopted to determine the minimum utility requirement and also to search the necessary conditions for maximum heat recovery. In the generation stage the initial network under the condition of maximum heat recovery is synthesized by a simple algorithm, intended to decrease the number of units as much as possible and to locate them optimally for reduction of capital cost and calculation load in the evolution stage. In the evolution stage the initial network is improved by evolutionary rules based on stream-splitting, which make possible a reduction in capital cost by increasing the heat duties of units and varying the exchanger sizes. The total annual cost and calculation load in the evolution stage can be greatly reduced by this method. The effectiveness of this method is demonstrated by solving problems which have been reported in previous works.

1. Formulation of the Problem

The problem is to synthesize a minimum-cost network of exchangers, heaters, and/or coolers and is stated as follows:

There are n_H hot streams with heat capacity flowrates F_{Hi} at the inlet temperatures T_{Hi} and the target temperatures T_{H2i} ($i=1, 2, \dots, n_H$).

There are n_C cold streams with heat capacity flowrates F_{Cj} at the inlet temperatures T_{C1j} and the target temperatures T_{C2j} ($j=1, 2, \dots, n_C$).

Heating and cooling utilities are available at specified conditions, and single-pass heat exchangers of countercurrent type are used. Heat transfer coefficients U_{ij} and heat capacity flowrates F_{Hi} , F_{Cj} are assumed to be independent of temperature.

The objective function is the total annual cost, which consists of capital and utility costs. The capital cost for the i^{th} exchanger, heater, and cooler denoted by C_{Ei} , C_{Hi} , and C_{Ci} , respectively can be

Table 1 Process streams specification and design data

	F_H	T_{H1}	T_{H2}		F_C	T_{C1}	T_{C2}
Problem 4SP1							
Sh1*	16,666.8	320	200	Sc1*	14,450.1	140	320
Sh2	20,000.0	480	280	Sc2	11,530.0	320	200
Problem 4SP2							
Sh1	20,000	500	110	Sc1	70,000	25	420
Sh2	50,000	430	230				
Sh3	30,000	400	110				
Problem 5SP1							
Sh1	31,500	480	250	Sc1	24,700	200	400
Sh2	25,200	400	150	Sc2	21,600	100	400
				Sc3	24,500	150	360
Problem 6SP1							
Sh1	23,800	520	300	Sc1	16,000	100	430
Sh2	28,000	440	150	Sc2	26,350	200	400
Sh3	33,600	390	150	Sc3	32,760	180	350
Problem 7SP1							
Sh1	23,800	520	300	Sc1	16,000	100	430
Sh2	28,000	440	150	Sc2	19,840	350	410
Sh3	33,600	390	150	Sc3	26,350	200	400
				Sc4	32,760	180	350
Problem 7SP3							
Sh1	153,018	675	150	Sc1	478,351	60	710
Sh2	111,220	592	452				
Sh3	45,507	543	115				
Sh4	598,500	428	344				
Sh5	116,482	398	100				
Sh6	1,277,434	301	231				
Problem 8SP1							
Sh1	22,400	470	320	Sh1	16,800	200	420
Sh2	17,500	450	240	Sh2	23,200	150	400
Sh3	28,500	370	150	Sh3	35,100	185	330
Sh4	20,100	310	200	Sh4	17,250	140	300
Problem 10SP1							
Sh1	23,800	520	300	Sc1	11,530	240	431
Sh2	20,000	480	280	Sc2	16,000	100	430
Sh3	38,000	440	150	Sc3	26,350	200	400
Sh4	33,600	390	200	Sc4	32,760	180	350
Sh5	16,670	320	200	Sc5	14,450	140	320

* Sh (Sc) represents a hot (cold) stream having flow rate F_H (F_C), inlet temperature T_{H1} (T_{C1}), and target temperature T_{H2} (T_{C2}).

All capacity heat flowrates are given in Btu/hr·°F and temperatures in °F.

(continued)

correlated to their corresponding heat transfer areas A_{Ei} , A_{Hi} , and A_{Ci} by the empirical expressions $C_{Ei} = aA_{Ei}^b$, $C_{Hi} = aA_{Hi}^b$, and $C_{Ci} = aA_{Ci}^b$. The total annual cost of the network can be expressed as

$$J = \delta(\sum C_{Ei} + \sum C_{Hi} + \sum C_{Ci}) + \sum r_k Q_k \theta$$

where δ is annual rate of return, r is unit cost of utility, Q is amount of utility per hour, and θ is operating hours per year.

The illustrative examples in this paper are all taken from previous works, and their specification of process streams and design data are summarized in **Table 1**. The minimum allowable approach temperatures are very important design variables, but no attempt has been made to optimize them in this study, as in most previous works, since their optimization should be left to final design with detailed design equations.

Table 1 (continued)							
Heat transfer coefficients		[Btu/hr·ft ² ·°F]					
7SP3		1	2	3	4	5	6
Hot stream							
Cold stream		100	70	60	70	60	55
Coolers		80	70	60	70	40	20
All other problems							
Heat exchanger		150					
Water cooler		150					
Steam heater		200					
Problems		4SP1, 4SP2, 5SP1, 7SP1, 8SP1, 10SP1			7SP3		
Cooling water							
Heat capacity [Btu/lb·°F]			1.0			1.0	
Temperature [°F]			100			80	
Maximum water output temperature [°F]			180			140	
Cost [\$ / lb]			5 × 10 ⁻⁵			5 × 10 ⁻⁵	
Fuel							
Caloric power [Btu/lb]						2.0 × 10 ⁴	
Cost [\$ / lb]						1.0 × 10 ⁻²	
Steam							
Pressure [lb/in ² ·abs]			450.0 (962.5 for 4SP1)				
Latent heat [Btu/lb]			767.5 (656.6 for 4SP1, 768.0 for 5SP1)				
Temperature [°F]			456 (540 for 4SP1)				
Minimum allowable approach temperature							
Heat exchanger [°F]			20			20	
Water cooler [°F]			20			20	
Steam heater [°F]			25				
Operating hours [hr/yr]			8380 for 4SP1, 5SP1 and 6SP1 8500 for all other problems				
Annual rate of return [l/yr]			0.10			0.33	
Investment cost							
Heat exchanger [\$]			350A ^{0.6}			350A ^{0.6}	
Water cooler [\$]			350A ^{0.6}			350A ^{0.6}	
Steam heater [\$]			350A ^{0.6}				
Furnace [\$]						1.2Q ^{0.7}	

2. Pre-analysis

The utility cost associated with heating and cooling is normally more important than the capital cost, so recent work has concentrated on finding networks which achieve maximum heat recovery while showing low capital cost. This study also synthesizes an optimal network under the condition of maximum heat recovery, so it is necessary to calculate minimum utility requirement precisely, and to search the conditions necessary to achieve maximum heat recovery.

The problem table, which was used in the temperature interval method⁵⁾, was adopted in this pre-analysis, and the problem table of 7SP3, which is specified in Table 1, is shown in Table 2. Linnhoff and Flower⁶⁾ described the physical significance of the columns of the problem table. The figures in columns 4 and 5 represent the heat flows into and out of the sub-networks for the case where the minimum

Table 2 Problem table for 7SP3						
SN	Column					
	1	2	3	4	5	
	Temp. [°F]	Deficit [Btu/hr]	Accumulated [Btu/hr]	Maximum permissible [Btu/hr]		
	hot cold	× 10 ⁷	input × 10 ⁷ output × 10 ⁷	input × 10 ⁷	output × 10 ⁷	
	730 710					
SN(1)		2.63	0.00	-2.63	8.59	5.96
	675 655					
SN(2)		2.70	-2.63	-5.33	5.96	3.26
	592 572					
SN(3)		1.05	-5.33	-6.38	3.26	2.21
	543 523					
SN(4)		1.53	-6.38	-7.91	2.21	0.67
	452 432					
SN(5)		0.67	-7.91	-8.59	0.67	0.00
	428 408					
SN(6)		-0.96	-8.59	-7.63	0.00	0.96
	398 378					
SN(7)		-2.35	-7.63	-5.28	0.96	3.31
	344 324					
SN(8)		0.70	-5.28	-5.98	3.31	2.60
	301 281					
SN(9)		-7.80	-5.98	1.82	2.60	10.40
	231 211					
SN(10)		1.32	1.82	0.49	10.40	9.08
	150 130					
SN(11)		1.11	0.49	-0.61	9.08	7.97
	115 95					
SN(12)		0.54	-0.61	-1.16	7.97	7.43
	100 80					
SN(13)		0.96	-1.16	-2.11	7.43	6.47
	80 60					

utility heating requirement is received at the highest possible temperature. Thus the top figure in column 4 is the minimum utility heating requirement and the bottom figure in column 5 is the minimum utility cooling requirement. For the case of 7SP3, the minimum utility heating and cooling requirements are 8.58×10^7 and 6.47×10^7 Btu/hr respectively. This problem table is further used to search the necessary conditions to achieve maximum heat recovery. For the example of 7SP3, it is clearly necessary to separate 7SP3 into two sub-problems, one consisting of SN(1)–SN(5) and the other consisting of SN(6)–SN(13), because the figure of SN(5) in column 5 (the figure of SN(6) in column 4) is 0. In other words, it indicates that heat flow between these two sub-problems does not exist when maximum heat recovery is achieved. If heat is exchanged between these two sub-problems, it is impossible to achieve maximum heat recovery. It is also clear from table 2 that Scl is necessary to match Sh1 and Sh3 simultaneously in SN(5), because there exists in SN(5) a minimum allowable approach temperature constraint produced by decomposition, and it indicates that stream-splitting of Scl is necessary in SN(5). This pre-analysis is necessary and effective to achieving maximum heat recovery before synthesis of networks.

Fig. 1 Algorithm of initial network generation

3. Generation of Initial Networks

The purpose of this stage is to generate networks as close to the optimal network as possible, so that the calculation load in the evolution stage can be greatly decreased. Nishida *et al.*⁶⁾ generated initial networks which have minimum total heat transfer area. Linnhoff and Flower⁵⁾ generated them from the problem tables mentioned above, and improved them by evolutionary rules. They could generate good networks, but it is necessary for them to calculate the total cost of a large number of neighboring networks, or to consider a large number of possible matches, in the evolution stage before producing the optimal network. The heuristic method⁸⁾, always matching the hot stream of highest supply temperature with the cold stream of highest target temperature, can generate surprisingly good networks in many cases, but sometimes it generates networks which are quite far from optimal. This is due to insufficient consideration of maximum heat recovery and the optimal locations of heaters and coolers in this method.

Considering these results, a simple algorithm is developed to generate nearly optimal networks, which achieve maximum heat recovery while showing low capital costs. Capital costs depend on the combinations of streams and the number of units, and these two factors strongly interact with each other. The algorithm is shown in Fig. 1. The results of the pre-analysis stage are adopted to achieve maximum heat recovery, and available utilities are treated as process streams to determine the optimal locations of heaters and coolers. The matching rule of the minimum area algorithm⁶⁾ results in an increase of calculation load in the evolutionary stage because the network generated consists of many small units, and the

Fig. 2 Evolutionary rules of increasing heat duty

minimum-area network is not the minimum-cost network because the power factor of costing equations for heat transfer area is less than 1.0. So a new matching rule is developed to generate nearly optimal networks. This rule is that the highest target temperature cold stream is matched with the hot stream having the highest value of Q/aA^b . The initial networks of this study correspond to those which are improved by evolutionary rules^{5,6)} because the matches are determined in consideration of cost performance. In the majority of cases this rule generates the same matches that the rule of the hottest hot with the hottest cold streams⁸⁾ generates. When an infeasible match is encountered during the generation of the initial network, firstly the heat duties of the units are modified in the reverse order of generation without increase of number of units. If this correction is not effective, the matches determined before rearranged in the reverse order of generation so as to solve this infeasible match.

4. Evolution

The purpose of this stage is to reduce the capital cost of the network under the condition achieving maximum heat recovery. Simple evolutionary rules are adopted in this study because the network generated in the previous stage is nearly optimal. If the power factor of the costing equation for heat transfer area is less than 1.0, two ways are effective for capital cost reduction. One is to reduce the number of units, and the other is to vary exchanger sizes, as pointed out by Nishida *et al.*⁶⁾, and Linnhoff and Flower⁵⁾.

It is essential for reduction of the number of units to increase heat duties of the units, and stream-splitting is effective for this. There are three cases where stream-splitting can increase heat duties of units, as shown in Fig. 2.

Stream-splitting is sometimes effective in varying exchanger sizes without influencing the other

Fig. 3 Evolutionary rules of varying exchanger sizes

Fig. 4 Progress of initial network generation for 5SP1 problem

Fig. 5 Initial network for 5SP1 problem

Fig. 6 Final network for 5SP1 problem

matches⁵⁾, but a comparison of costs is necessary to decide whether this evolutionary rule should be adopted or not. Figure 3 shows this rule.

The simple evolutionary rules mentioned above are applied to improve the initial network according to their order of generation. Not only the capital cost but also the calculation load in the evolution stage can be decreased greatly.

5. Illustrative Example

This method is applied to the synthesis problems specified in Table 1 to demonstrate its effectiveness. The calculation loads can be decreased greatly in comparison with the systematic methods^{1,5,6)} mentioned above, but the effectiveness in decreasing the

Table 3 Problem table for 5SP1

SN	Temp. [°F]		Deficit [Btu/hr]		Accumulated [Btu/hr]		Maximum premissible [Btu/hr]	
	hot	cold	$\times 10^6$	$\times 10^6$	input $\times 10^6$	output $\times 10^6$	input $\times 10^6$	output $\times 10^6$
SN(1)	480	460	-1.89	0.00	1.89	3.02	4.91	
SN(2)	420	400	0.30	1.89	1.59	4.91	4.61	
SN(3)	400	380	-0.21	1.59	1.80	4.61	4.82	
SN(4)	380	360	1.83	1.80	-0.03	4.82	2.99	
SN(5)	250	230	1.37	-0.03	-1.40	2.99	1.62	
SN(6)	220	200	1.05	-1.40	-2.44	1.62	0.58	
SN(7)	170	150	-0.07	-2.44	-2.37	0.58	0.65	
SN(8)	150	130	0.65	-2.37	-3.02	0.65	0.00	
	120	100						

calculation loads cannot be evaluated quantitatively. It is proved that this method is very effective for problems which must be separated into two sub-problems, such as 7SP3 and 8SP1. Problems 5SP1 and 7SP3 are solved to illustrate this method.

Problem 5SP1 The problem table representing the 5SP1 problem is shown in Table 3, and it is determined that minimum utility heating requirement is 3.02×10^6 Btu/hr, and that the portion less than 170°F of Sh2 must be matched with Sc1. Both Sc1 and Sc3 are selected as the cold streams of highest target temperature, and the values of Q/aA^b are respectively calculated for feasible matches, Sc1-Sh1, Sc1-Steam, Sc3-Sh1, and Sc3-Steam. Sc1-Sh1 is selected and represented as E1. Sc3-Steam and Sc2-Sh2 are selected and represented as H and E2. It is impossible to match the portion less than 277.81°F of Sc3 with hot streams, as shown in Fig. 4. According to the algorithm of initial network generation, heat duty of E2 cannot be modified without increasing the number of units, but E1 is possible, so E1 is modified to be able to match the portion of less than 277.81°F of Sc3. Figure 5 shows the initial network of the 5SP1 problem generated by this method with a total annual cost of \$39,013/yr. Next, the application of evolutionary rules gives an improved network with a total annual cost of \$38,713/yr, as shown in Fig. 6. The same network was generated by Nishida *et al.*⁶⁾, and Linnhoff and Flower⁵⁾ further optimized the stream-splitting ratio, but this study leaves this optimization to final design.

Problem 7SP3 This problem corresponds to a crude oil heating train in which a furnace is used instead of a steam heater and the heat transfer coefficients vary with the combinations of streams. The problem

table is shown in Table 2. As mentioned above, it is necessary to separate this problem into two sub-problems, one consisting of SN(1)–SN(5) and the other consisting of SN(6)–SN(13), and the minimum utility heating and cooling requirements are 8.58×10^7 and 6.47×10^7 Btu/hr respectively. The initial networks, generated separately for the two sub-problems, are shown in Fig. 7. Next, evolutionary rules are applied separately and Fig. 8 shows the improved network with a total annual cost of \$1,181,295/yr. Grossmann and Sargent²⁾ generated the networks using the implicit enumeration method, but the total annual cost of their network was \$1,199,158/yr, because maximum heat recovery is not achieved in their network. Table 4 shows the calculation results in comparison with recent work. It is clear that this method can generate networks equal to those generated by other methods or better.

Conclusion

A practical synthesis method of heat exchanger network, which can be implemented by hand calculation, is developed and applied to problems reported in previous works. This method consists of pre-analysis, initial network generation, and evolution stages. In the pre-analysis stage, the problem table is used to determine the minimum utility requirement and necessary conditions. Separation into sub-problems is effective in achieving maximum heat recovery. In the initial network generation stage, nearly optimal networks are generated, considering maximum heat recovery and cost performance of units. In the evolution stage, the initial networks are improved by simple evolutionary rules, increasing heat duties of units and varying exchanger sizes.

The calculation loads can be decreased greatly in comparison with systematic methods^{1,5,6)}, but the effectiveness in decreasing the calculation loads cannot be evaluated quantitatively. It is clear that this method can generate networks equal to those generated by other methods or better.

Nomenclature

A	= heat transfer area	[ft ²]
a	= parameter of cost function	[\$]
b	= parameter of cost function	[—]
C	= capital cost of unit	[\$]
F	= heat capacity flowrate	[Btu/hr·°F]
J	= total annual cost	[\$/y]
n	= number of streams	[—]
Q	= heat to be transferred	[Btu/hr]
r	= unit cost of utility	[\$/Btu]
T	= temperature	[°F]
U	= heat transfer coefficient	[Btu/hr·°F·ft ²]
δ	= annual rate of return	[1/yr]

Fig. 7 Initial network for 7SP3 problem

Fig. 8 Final network for 7SP3 problem

Table 4 Calculation results

	This work	[\$/yr] Nishida <i>et al.</i> ⁶⁾	Grossmann ²⁾
4SP1	13,590	13,590	13,590
4SP2	19,655	20,353	21,663
5SP1	38,713	38,713	38,288
6SP1	35,010	35,010	35,010
7SP1	30,172	—	30,172
7SP3	1,181,295	—	1,199,158
8SP1	38,303	—	41,228
10SP1	43,884	43,984	44,160

θ = operating hours per year [hr/yr]

<Subscripts>

C	= cold stream, cooler
E	= heat exchanger
H	= hot stream, heater
i	= hot stream number
j	= cold stream number
k	= kind of utility
1	= inlet
2	= target

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