be well explained through the change of weight fraction of fine particles near the bed surface.

Nomenclature

d_p	= particle diameter	[mm]
$\bar{d}_p$	= mean particle diameter	[mm]
E	= elutriation rate	[kg/m ² s]
h	= height from gas distributor	[m]
H_o	= static bed height	[m]
U	= superficial gas velocity	[m/s]
U_{mf}	= minimum fluidisation velocity of monocomponent particle	[m/s]
$\bar{U}_{mf}$	= minimum fluidisation velocity of binary particle mixture	[m/s]
U_t	= terminal velocity	[m/s]
X	= weight fraction of fine particles	[—]
$\bar{X}$	= bulk weight fraction of fine particles	[—]
Z	= dimensionless bed height	[—]
ϵ_{mf}	= voidage of incipiently fluidised bed	[—]
ρ_{mf}	= particle density of incipiently fluidised bed	[kg/m ³]
ρ_p	= particle density	[kg/m ³]

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(Presented at the 47th Annual Meeting of The Society of Chemical Engineers, Japan, at Tokyo, March 29, 1982.)

NUMERICAL METHOD FOR SOLVING STEADY THREE-DIMENSIONAL HEAT CONDUCTION PROBLEM IN SECTOR COLUMN

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Generally, the partial differential equation, which describes the three-dimensional steady heat conduction problem by use of the cylindrical coordinate system, is given by the following equation.

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (1)$$

where the thermal conductivity is assumed to be constant. Using the relationship $y \equiv \ln r$, Eq.(1) can be transformed into the following equation.

$$e^{2y} \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (2)$$

Then, the following 7-point finite difference equations (F.D.E. hereafter) for Eqs. (1) and (2) by means of the computational molecules, i.e.,

$$\left[\begin{array}{c} a_2 \\ a_5 \quad a_0 \quad a_6 \\ a_3 \quad a_1 \end{array} \right] T_{l,m,n} + O(\bar{k}^2) = 0 \quad (3)$$

Received April 19, 1982. Correspondence concerning this article should be addressed to I. Yamada. H. Nakamura is at Dept. of Mech. Eng., Daido Inst. of Tech. Nagoya 457.

Table 1. Elements in computational molecules

$$a_0 = -2 \left(1 + \bar{p}^2 + \frac{s^2}{r^2} \right) \quad b_0 = -2(e^{2y} + p^2 + q^2)$$

$$a_1 = a_2 = 1 \quad b_1 = b_2 = e^{2y}$$

$$a_3 = a_4 = \frac{s^2}{r^2} \quad b_3 = b_4 = q^2$$

$$a_5 = \bar{p}^2 - \frac{h\bar{p}}{2r} \quad b_5 = b_6 = p^2$$

$$a_6 = \bar{p}^2 + \frac{h\bar{p}}{2r}$$

$$c_0 = -\frac{4}{3} \left(1 + \bar{p}^2 + \frac{q^2}{r^2} \right) + \frac{\bar{k}^2}{6r^2} + \frac{h^2}{3r^2} - \frac{\bar{k}^2 q^2}{2r^4}$$

$$c_1 = c_2 = \frac{2}{3} - \frac{1}{6} \left(\bar{p}^2 + \frac{q^2}{r^2} \right) - \frac{\bar{k}^2}{12r^2}$$

$$c_3 = c_4 = \frac{2q^2}{3r^2} - \frac{1}{6} (1 + \bar{p}^2) + \frac{q^2 \bar{k}^2}{4r^4}$$

$$c_5 = \frac{2}{3} \bar{p}^2 - \frac{1}{6} \left(1 + \frac{q^2}{r^2} + \frac{h^2}{r^2} \right) - \frac{\bar{p}h}{2r} + \frac{1}{12r} \left(2\bar{p}h + \bar{k} - \frac{3q^2 \bar{k}}{r^2} \right)$$

$$c_6 = \frac{2}{3} \bar{p}^2 - \frac{1}{6} \left(1 + \frac{q^2}{r^2} + \frac{h^2}{r^2} \right) + \frac{\bar{p}h}{2r} - \frac{1}{12r} \left(2\bar{p}h + \bar{k} - \frac{3q^2 \bar{k}}{r^2} \right)$$

$$c_7 = c_8 = \frac{1}{12} \left(\bar{p}^2 + \frac{q^2}{r^2} \right) - \frac{1}{24r} \left(\bar{p}h - \frac{3q^2 \bar{k}}{r^2} \right)$$

$$c_9 = c_{10} = \frac{1}{12} \left(\bar{p}^2 + \frac{q^2}{r^2} \right) - \frac{1}{24r} \left(\bar{p}h - \frac{3q^2 \bar{k}}{r^2} \right)$$

$$c_{11} = c_{12} = \frac{1}{12} (1 + \bar{p}^2) - \frac{1}{24r} (\bar{p}h + \bar{k})$$

$$c_{13} = c_{14} = \frac{1}{12} (1 + \bar{p}^2) + \frac{1}{24r} (\bar{p}h + \bar{k})$$

$$c_{15} = c_{16} = c_{17} = c_{18} = \frac{1}{12} \left(1 + \frac{q^2}{r^2} \right)$$

$$d_0 = -\frac{1}{3} \left\{ e^{2y} (4 + 2k^2) + \left(4 - \frac{k^2}{3} \right) (p^2 + q^2) \right\}$$

$$d_1 = d_2 = \frac{1}{3} \left\{ e^{2y} (2 + k^2) - \frac{1}{2} \left(1 + \frac{k^2}{3} \right) (p^2 + q^2) \right\}$$

$$d_3 = d_4 = \left(\frac{2}{3} - \frac{k^2}{18} \right) q^2 - \frac{1}{6} (e^{2y} + p^2)$$

$$d_5 = \frac{e^{2y}}{6} (2k - 1) + \left(\frac{2}{3} - \frac{k^2}{18} \right) p^2 - \frac{1}{6} q^2$$

$$d_6 = \frac{e^{2y}}{6} (-2k - 1) + \left(\frac{2}{3} - \frac{k^2}{18} \right) p^2 - \frac{1}{6} q^2$$

$$d_7 = d_8 = d_9 = d_{10} = \frac{1}{12} (p^2 + q^2)$$

$$d_{11} = d_{12} = \frac{1}{12} \left\{ e^{2y} (1 - 2k) + \left(1 + \frac{k^2}{3} \right) p^2 \right\}$$

$$d_{13} = d_{14} = \frac{1}{12} \left\{ e^{2y} (1 + 2k) + \left(1 + \frac{k^2}{3} \right) p^2 \right\}$$

$$d_{15} = d_{16} = d_{17} = d_{18} = \frac{1}{12} \left\{ e^{2y} + \left(1 + \frac{k^2}{3} \right) q^2 \right\}$$

where $\bar{p} \equiv \frac{h}{k}$, $p \equiv \frac{h}{k}$, and $q \equiv \frac{h}{s}$

$$\left[\begin{array}{c} b_2 \\ b_5 \quad b_6 \quad b_4 \\ b_3 \quad b_1 \end{array} \right] T_{l,m,n} + O(h^2) = 0 \quad (4)$$

have frequently been used to obtain the approximated temperature profile in a sector column under various boundary conditions, where $a_0 - a_6$ and $b_0 - b_6$ in the molecules are listed in **Table 1** and the relationship between the direction of coordinates and the position of molecules is presented in **Fig. 1**.

The purpose of this paper is to compare 19-point F.D.E. with 7-point F.D.E. The 19-point F.D.E. are

Fig. 1. Relationship between direction of coordinates and position of molecules.

derived by a derivation process^{1,2)} similar to that for Greenspan's 9-point finite difference equation for the two-dimensional orthogonal coordinate system.

1. Derivation of 19-point F.D.E.

Using the central difference operator δ , Eqs. (1) and (2) can be written by the following equations.

For Eq. (1)

$$\left[\left(\delta_z^2 - \frac{1}{12} \delta_z^4 + \frac{1}{90} \delta_z^6 \cdots \right) + \bar{p}^2 \left(\delta_r^2 - \frac{1}{12} \delta_r^4 + \frac{1}{90} \delta_r^6 \cdots \right) + \frac{\bar{p}h}{r} \mu \left(\delta_r - \frac{1}{6} \delta_r^3 + \frac{1}{60} \delta_r^5 \cdots \right) + \frac{q^2}{r^2} \left(\delta_\theta^2 - \frac{1}{12} \delta_\theta^4 + \frac{1}{90} \delta_\theta^6 \cdots \right) \right] T_{l,m,n} = 0 \quad (5)$$

For Eq. (2)

$$\left[e^{2y} \left(\delta_z^2 - \frac{1}{12} \delta_z^4 + \frac{1}{90} \delta_z^6 \cdots \right) + p^2 \left(\delta_y^2 - \frac{1}{12} \delta_y^4 + \frac{1}{90} \delta_y^6 \cdots \right) + q^2 \left(\delta_\theta^2 - \frac{1}{12} \delta_\theta^4 + \frac{1}{90} \delta_\theta^6 \cdots \right) \right] T_{l,m,n} = 0 \quad (6)$$

When only the first terms in parentheses in Eqs. (5) and (6) are retained, Eqs. (5) and (6) are reduced to Eqs. (3) and (4) respectively. To obtain more accurate finite difference equations for Eqs. (1) and (2), the second terms in parentheses, $\mu\delta_r^3$, δ_z^4 , δ_r^4 and δ_z^4 for Eq. (5) and δ_z^4 , δ_θ^4 and δ_r^4 for Eq. (6), should be considered. However, computations of higher-order differences at points adjacent to the boundary are very difficult. To avoid this computational difficulty the following procedure is adopted here.

For Eq. (1): Partially differentiating Eq. (1) once with respect to r and twice with respect to z , r and θ respectively, one can finally obtain the following relationships.

$$\mu\delta_r^3 = -\frac{1}{r} \bar{p}^{-2} k \delta_z^2 - \frac{2k}{r} \delta_r^2 + \frac{1}{r^3} q^2 \bar{p}^{-2} k \delta_\theta^2 - \frac{1}{r^2} \bar{p}^{-2} q^2 \mu \delta_r \delta_\theta^2 - \bar{p}^{-2} \mu \delta_r \delta_z^2 \quad (7)$$

$$\delta_z^4 = -\bar{p}^2 \delta_z^2 \delta_r^2 - \frac{1}{r} \bar{p} h \delta_z^2 \mu \delta_r - \frac{1}{r^2} q^2 \delta_z^2 \delta_\theta^2 \quad (8)$$

$$\delta_r^4 = \frac{3}{r^2} \bar{p}^{-2} k^2 \delta_z^2 + \frac{6}{r^2} k^2 \delta_r^2 - \frac{5}{r^4} p^{-2} q^2 k^2 \delta_\theta^2 + \frac{5}{r^3} \bar{p}^{-2} q^2 k \mu \delta_r \delta_\theta^2 - \frac{1}{r^2} \bar{p}^{-2} q^2 \delta_r^2 \delta_\theta^2 - \bar{p}^{-2} \delta_r^2 \delta_z^2 + \frac{1}{r} p^{-2} k \mu \delta_r \delta_z^2 \quad (9)$$

$$\delta_\theta^4 = -r^2 q^{-2} \delta_z^2 \delta_\theta^2 - r^2 p^2 q^{-2} \delta_r^2 \delta_\theta^2 - r p q^{-1} s \delta_\theta^2 \mu \delta_r \quad (10)$$

Substitution of Eqs. (7)–(10) into Eq. (5) gives the following 19-point finite difference equation, where

the values of c_0 – c_{18} are listed in Table 1.

$$\left[\begin{array}{c} c_{12} \quad c_2 \quad c_{18} \quad c_{14} \\ c_8 \quad c_{16} \quad c_4 \quad c_{10} \\ c_5 \quad c_6 \quad c_9 \quad c_7 \\ c_{11} \quad c_3 \quad c_{17} \quad c_{13} \\ c_{15} \quad c_1 \quad c_{19} \end{array} \right] T_{l,m,n} + O(\bar{k}^3) = 0 \quad (11)$$

For Eq. (2): Partially differentiating Eq. (2) twice with respect to z , y and θ respectively, the following relationships can be obtained.

$$\delta_z^4 = -e^{-2y} (p^2 \delta_z^2 \delta_y^2 + q^2 \delta_z^2 \delta_\theta^2) \quad (12)$$

$$\delta_y^4 = -4e^{2y} p^{-2} k^2 \delta_z^2 - 4e^{2y} p^{-2} k \delta_z^2 \mu \delta_y - e^{2y} p^{-2} \delta_z^2 \delta_y^2 - p^{-2} q^2 \delta_y^2 \delta_\theta^2 - \frac{1}{3} (k^2 \delta_z^2 \delta_y^2 + k^2 p^{-2} q^2 \delta_z^2 \delta_\theta^2) \quad (13)$$

$$\delta_\theta^4 = -e^{2y} q^{-2} \delta_z^2 \delta_\theta^2 - p^2 q^{-2} \delta_y^2 \delta_\theta^2 \quad (14)$$

Substitution of Eqs. (12)–(14) into Eq. (6) gives the following 19-point finite difference equation, where the values of d_0 – d_{18} are also listed in Table 1.

$$\left[\begin{array}{c} d_{12} \quad d_2 \quad d_{18} \quad d_{14} \\ d_8 \quad d_{16} \quad d_4 \quad d_{10} \\ d_5 \quad d_6 \quad d_9 \quad d_7 \\ d_{11} \quad d_3 \quad d_{17} \quad d_{13} \\ d_{15} \quad d_1 \quad d_{19} \end{array} \right] T_{l,m,n} + O(h^4) = 0 \quad (15)$$

2. Discussion and Conclusion

To demonstrate the superiority of Eqs. (11) and (15), sample calculations are illustrated for the steady heat conduction problem in a sector column with the shape and boundary conditions shown in Fig. 2. Fortunately, in this problem the analytical solution is given by the following equation.

$$T = \sum_{m'=1}^{\infty} \sum_{n'=1}^{\infty} \frac{4\{1-(-1)^{m'}\}\{1-(-1)^{n'}\}}{m'n'\pi^2} \frac{F(m', n', r)}{F(m', n', r_i)} \times \sin n'\pi z \sin m'\theta \quad (16)$$

where

$$F(m', n', r_j) = I_{m'}(n'\pi r) K_{m'}'(n'\pi r_0) - K_{m'}(n'\pi r) I_{m'}'(n'\pi r_i) \quad (j=i, 0)$$

The numerical calculation was carried out by using the Gauss-Seidel algorithm for the various number of divisions n which was taken to be $n \times n \times n$ in the z , r and θ -directions, where the initial values used were $T_{l,m,n} = 0$ for all lattice points, and the convergence

criterion was $|\bar{T}_{l,m,n}^v - \bar{T}_{l,m,n}^{v-1}|_{\max} \leq 10^{-6}$. Moreover, the temperature T_B at the boundaries of $(z, 0.5, 0)$ and $(z, 0.5, \pi)$, which provides the minimum deviation

Table 2. Radial distribution of dimensionless temperature at $\theta=\pi/5$, $z=0.4$ in semisector column shown in Fig. 2

r	Eq. (3)			Eq. (11)		Eq. (16)
	$n=10$	$n=20$	$n=30$	$n=10$	$n=20$	
0.700	0.4707	0.4764	0.4778	0.4799	0.4789	0.4788
0.900	0.2156	0.2169	0.2172	0.2177	0.2175	0.2175
1.100	0.0961	0.0960	0.0960	0.0960	0.0960	0.0960
1.300	0.0363	0.0361	0.0360	0.0360	0.0360	0.0360
Dev. (%)	0.87	0.25	0.11	0.11	0.02	—
Time (s)	5	96	682	8	134	—

r	Eq. (4)			Eq. (15)		Eq. (16)
	$n=10$	$n=20$	$n=30$	$n=10$	$n=20$	
0.623	0.6320	0.6404	0.6420	0.6450	0.6435	0.6434
0.776	0.3746	0.3535	0.3547	0.3559	0.3557	0.3557
0.967	0.1634	0.1657	0.1662	0.1664	0.1665	0.1665
1.204	0.0593	0.0599	0.0600	0.0600	0.0601	0.0601
Dev. (%)	1.80	0.47	0.21	0.12	0.01	—
Time (s)	6	104	728	8	139	—

Fig. 2. Shape of sector column for numerical example where $T=1$ for the surface of $r_i=0.5$ and $T=0$ for the other surfaces.

from the analytical solution obtained from Eq. (16) was expressed by the following equation.

$$T_B = \frac{1}{1 + \rho^2} \quad (18)$$

where ρ denotes any of the ratios of grid spacing k/h , $\bar{k}/h$ or s/h .

The values of dimensionless temperature for r -direction at $z=0.4$, $\theta=\pi/5$ obtained by Eqs. (3), (4), (11), (15) and (16) are shown in **Table 2**. Additionally, mean value of deviation from Eq. (16) and C.P.U. time with M-170 (HITACHI) are also shown in this table. It is found that the results of Eqs. (11) and (15) for $n=20$ are very accurate. On the other hand, though the deviation of $n=30$ in Eq. (3) is equal to that of $n=10$ in Eq. (11), the C.P.U. time required for the former is observed to be about 85 times as long as for the latter.

Therefore, the superiority of the present 19-point F.D.E. to the 7-point F.D.E. is clear.

Nomenclature

a_j, b_j	= elements of computational molecule in Eqs. (3) and (11)	[—]
h	= grid spacing for z -direction	[—]
$I_m(x)$	= modified Bessel function of 1st kind of order m	[—]
$K_m(x)$	= modified Bessel function of 2nd kind of order m	[—]
k	= grid spacing for y -direction	[—]
$\bar{k}$	= grid spacing for r -direction	[—]
n	= number of divisions in z , r and θ -directions	[—]
$O(\)$	= order of truncation error in F.D.E.	[—]
p	= defined by h/k	[—]
$\bar{p}$	= defined by $h/\bar{k}$	[—]
q	= defined by h/s	[—]
r	= dimensionless distance for r -direction	[—]
s	= grid spacing for θ -direction	[—]
T	= dimensionless temperature	[—]
y	= defined by $\ln r$	[—]
z	= dimensionless distance for z -direction	[—]
δ	= central-difference operator	[—]
θ	= dimensionless distance for angle	[—]
μ	= average operator	[—]

<Subscripts>

B	= boundary intersected by the planes at $T=0$ and $T=1$
i	= inner radius
l, m, n	= space subscript of grid in z , y (or r) and θ direction
o	= outer radius

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