APPLICATION OF ENERGY-DIRECTION FACTOR DIAGRAM FOR EXERGY ANALYSIS OF A DISTILLATION COLUMN

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An energy-direction factor diagram elucidating both the exergy required for distillation and the irreversible exergy loss in a distillation column is proposed. For this purpose the concept of a reversible distillation column with interstage heat exchangers is introduced and the energy exchange between the energy-donating processes and the energy-accepting ones is discussed. Although it is composed of a finite number of stages, this reversible column may theoretically perform separation without exergy loss.

By comparing a real distillation column with the corresponding reversible one with the same number of stages, the irreversible exergy loss in the distillation column can be obtained, facilitating a visual grasp of its thermodynamic characteristics.

1. Introduction

In general, separation processes in the process industry require a large input of energy. Therefore, many kinds of thermodynamic analyses have been performed, especially for distillation columns, from the viewpoint of energy saving.1,2,4,5,7-11 These analyses are based on both the first and second laws of thermodynamics.

Umeda et al.4,11 applied the $Q-(1 - T_f/T)$ diagram originally proposed for heat exchangers to distillation columns and tried to provide a visual representation of the irreversible exergy loss as the area on this diagram. However, the area they obtained was not the exergy destruction itself. Adopting a slightly different approach, Naka et al.9 discussed the effect of side-boilers and side-coolers on this diagram. However, their area includes various terms and does not correspond to the irreversible exergy loss either. Fitzmorris and Mah’s work11 may be the first in which the exergy loss in the nonisothermal distillation column is evaluated correctly. However, they could not represent it on a diagram.

One of the authors has discussed the degradation of energy in various unit processes and concluded that thermodynamic compass and the energy-direction factor diagram (or $AH-D$ diagram) are appropriate,3 because they allow not only thermal energy but also other kinds of energy to be treated. The direction factor $D$ is defined as $T_o \Delta S/\Delta H$, the slope of the process vector on the thermodynamic compass. As far as the thermal energy is concerned, it is equal to the ratio of the reference temperature $T_o$ (= 298.15 K) to the temperature $T$, and $\Delta H$ is equivalent to the exchanged heat $Q$. Hence the latter diagram is an upgraded version of the $Q-(1 - T_f/T)$ diagram.3

In this study, the concept of a reversible distillation column with interstage heat exchangers is introduced and this diagram is applied to correctly evaluate both the exergy required for the distillation process and the irreversible exergy loss in various types of distillation systems.

2. A Reversible Distillation Column with Interstage Heat Exchangers

Figure 1 outlines the distillation column with interstage heat exchangers. Although it has only a finite number of stages, it may theoretically perform reversible separation for a binary component system from the viewpoint of thermodynamics. The feed stream is supplied at the $f$-th stage from the top of the column. Since it is a reversible system, the temperature of the feed stream is assumed to be the same as...
that of the \( f \)-th stage.

First consider the interstage between the \((f-1)\)-th stage and the \( f \)-th stage in the rectifying section. Vapor rises from the \( f \)-th stage at a molar flow rate \( V \) and liquid falls from the \((f-1)\)-th stage at a molar flow rate \( L \).

To avoid exergy loss by mixing of streams of different compositions and temperatures, heat exchange between the rising and falling streams is carried out. Then the temperature of the falling stream at the entrance of the \( f \)-th stage may be set at the temperature of the \((f-1)\)-th stage, \( T_f \). When equilibrium between vapor and liquid phases is assumed, the falling stream which is now composed of vapor and liquid fractions may enter the \( f \)-th stage without exergy loss. Since \( V \) is generally greater than \( L \), the temperature of the rising stream at the entrance of the \((f-1)\)-th stage does not reach that of the \((f-1)\)-th stage. Hence an ideal auxiliary cooler is installed and reversible heat exchange with the rising stream is achieved. Then the exergy destruction in the rectifying section may theoretically be reduced to zero.

In the stripping section, on the other hand, the downward flow rate \( L' \) is greater than the upward flow rate \( V \). Hence reversible heaters are installed to completely cut down the exergy loss.

For the infinitesimal volume between temperatures \( T \) and \( T + dT \) shown in Fig. 2, the energy balance gives

\[
c_p V_f (T + dT - T_f) + V_{v(T+dT)} \lambda + c_p L_{j-1}(T - T_f) \\
+ L_{v(T+dT)} \lambda = c_p V_f (T - T_f) + V_{v(T)} \lambda + c_p L_{j-1}(T + dT - T_f) \\
+ L_{v(T+dT)} \lambda + dQ_{R(T)}
\]

where \( V_v \) and \( L_v \) are the molar flow rates of the vapor in the rising and falling streams, respectively. Then we have

\[
dQ_R = c_p dT + [(V_v - L_v)(T + dT) - (V_v - L_v)(T)] \lambda \tag{1}
\]

When the equilibrium mole fractions of the light component in vapor and liquid phases are denoted by \( y \) and \( x \), respectively, the material balance gives

\[
V_{v(T)} = V_f (y_f - x_f)/(y - x) \\
L_{v(T)} = L_{j-1}(x_{j-1} - x_f)/(y - x)
\]

Besides, the overall material balance yields

\[
V_f y_f = L_{j-1} x_{j-1} + t_{z_f}
\]

Therefore,

\[
V_{v(T)} - L_{v(T)} = t(z_f - x)/(y - x)
\]

Equation (1) is reduced to

\[
dQ_{R(T)} = t \left[ c_p dT + \lambda d \left( \frac{z_f - x_f}{y - x} \right) \right] \tag{2}
\]

Generally, \( dQ_R \) is positive. Hence, in the rectifying section, the thermal process to remove heat, i.e., the process to heat the coolant, becomes the energy-accepting process and the separation process becomes the energy-donating process.

\[
dH_{ha} = dH_{sep} = -dQ_R \tag{3}
\]

\[
dH_{ha} = dH_R = dQ_R \tag{4}
\]

where subscript \( R \) denotes the thermal process in the rectifying section.

Since the temperature in the infinitesimal volume is \( T \), the entropy change is given by

\[
dS_{ha} = dS_{sep} = -dQ_R/T \tag{5}
\]

\[
dS_{ha} = dS_R = dQ_R/T \tag{6}
\]

Therefore, the direction factor \( D \) is given as

\[
D_{sep} = D_{hd} = D_R = T_o/T \tag{7}
\]

Integration of Eq. (2) from the 1st stage to the \( f \)-th feed stage gives the following equation.

\[
Q_R = t c_p (T_f - T_i) + t L \lambda + L_{min,f} \lambda \tag{8}
\]

where the minimum internal reflux rate at stage \( f \) \( L_{min,f} \) is determined for a column with an infinite number of stages.

\[
V_{min,f} = L_{min,f} + t \tag{9}
\]

\[
V_{min,f} y_f = L_{min,f} x_f + t z_f \tag{10}
\]

Similarly, for the stripping section we have

\[
dQ_{S(T)} = b \left[ c_p dT + \lambda d \left( \frac{x_f - x_s}{y - x} \right) \right] \tag{11}
\]

Hence, in the stripping section, the thermal process to supply heat becomes the energy-donating process and the separation process becomes the energy-accepting process.

\[
dH_{ha} = dH_{sep} = dQ_S \tag{12}
\]

\[
dH_{ha} = dH_S = -dQ_S \tag{13}
\]

\[
dS_{ha} = dS_{sep} = -dQ_S/T \tag{14}
\]

\[
dS_{ha} = dS_S = -dQ_S \tag{15}
\]

\[
D_S = T_h = D_{sep} = D_{ha} = T_o/T \tag{16}
\]
Integration of Eq. (11) from the feed stage to the bottom stage yields

$$Q_s = b c_p (T_b - T_f) + V_{\text{min}, f}$$

(17)

where the minimum internal reflux rate at the feed stage $V_{\text{min}, f}$ is determined by

$$L_{\text{min}, f} = V_{\text{min}, f} + b$$

(18)

$$L_{\text{min}, f} x_f = V_{\text{min}, f} y_f + b x_b$$

(19)

3. Minimum Exergy Requirement for Separation

The minimum exergy requirement in the given reversible distillation process is obtained as

$$\Delta e_{\text{sep}} = \Delta H_{\text{sep}} - T_o \Delta S_{\text{sep}}$$

$$= \int (1 - D_{\text{sep}}) dH_{\text{sep}}$$

$$= \int (1 - D_s) dQ_s - \int (1 - D_R) dQ_R$$

$$= e_s - e_R$$

(20)

This result is in accord with that obtained by Fitzmorris and Mah.1)

When $D_{\text{sep}}$ is plotted against $\Delta H_{\text{har}}$, we have Fig. 3 for benzene and toluene mixture with saturated liquid feed and saturated vapor distillate. The equilibrium data are based on the literature.6) On this diagram, $\varepsilon_s$ and $\varepsilon_R$ are obtained as the area bounded by $D_{\text{sep}}$ and the line $D = 1$ (i.e., $D$ for heat sources or sinks at the reference temperature $T_o$). Therefore, the minimum exergy requirement $\Delta e_{\text{sep}}$ is obtained as the difference between these two areas.

Figure 4(a) shows the case when the feed is introduced in the saturated vapor, while the other conditions are kept the same as those in Fig. 3. As may be anticipated intuitively, the vapor feed reduces $\Delta e_{\text{sep}}$. In this example, it becomes negative, indicating that the excess exergy may be recovered from the distillation column. However, some exergy has been consumed outside the column to evaporate the feed stream.10)

Figure 4(b) shows the case when the distillate is withdrawn in the saturated liquid. The minimum exergy requirement is reduced because of the increase in amount of exergy recovered in the rectifying section.

4. Exergy Destruction

When energy is exchanged between an energy-donating process and an energy-accepting process, the exergy destruction caused by this exchange may be obtained as the area bounded by the direction factor curves for these processes on the energy-direction factor diagram.3)

4.1 Reversible distillation columns

In the reversible distillation column with interstage heat exchangers, the direction factor of the energy-donating process $D_{\text{ha}}$ is just equal to that of the energy-accepting process $D_{\text{ha}}$, resulting in no exergy destruction.

4.2 Columns with a reboiler and a condenser

Let us consider first a distillation column with an infinite number of stages, driven by the minimum reflux. Then we may obtain equations similar to Eqs. (8) and (17), but $Q_R$ and $Q_S$ are substituted for the quantity of heat removed in the condenser stage $Q_c$ and supplied in the reboiler stage $Q_r$, respectively. Hence the process to remove heat by the coolant in the condenser stage is the energy-accepting process and the process to supply heat by the heating medium in the reboiler stage is the energy-donating process.

When the temperature of the coolant and the heating medium are assumed to be equal to the
temperatures of the stages \( T_t \) and \( T_b \) respectively, the exergy destruction may be obtained as the shaded area in Fig. 5.

The exergy destruction caused by the heat exchange may be obtained as the dotted area in Fig. 5, by plotting the direction factor curves \( T_jT_{j'} \) and \( T_jT_r \). In many cases, this term is quite large.

It is to be noted that there is an intersection point, i.e., a pinch point, between the curves of \( D_{ha} \) and \( D_{hd} \) at the feed stage, indicating the requirement of an infinite number of stages immediately above and immediately below the feed stage.

When the internal reflux rate becomes greater than the minimum rate, the condensation of excess reflux appears as another energy-donating process in the rectifying section, while the evaporation of excess reflux is another energy-accepting process in the stripping section, as is schematically shown in Fig. 6(a). Since these processes apparently take place at the temperature of the stage under consideration (the feed stage in this example), their direction factor \( D \) is given as \( T_d/T_r \). When the excess reflux \( L - L_{\min,d} \) is denoted by \( E_f \), the enthalpy changes of these processes are given as \( -E_f \lambda \) and \( E_f \lambda \). Then the exergy destruction in the rectifying section is represented as the upper part of the shaded area in Fig. 6(b), while the exergy destruction in the stripping section is the lower part.

### 4.3 Columns with side-boilers and side-coolers

The thermodynamic characteristics of the column with side-boilers and/or side-coolers in the stages may be examined for each block between the two stages with a cooler, a boiler, or a feed or product stream as follows.

The energy-accepting process in the first block between the top and second coolers is the process to remove heat of the quantity \( Q_1 \) from the top stage at temperature \( T_{c1} \), as shown in Fig. 7(a). On the other hand, there are two energy-donating processes: the condensation process with minimum internal flow rates \( V_{\min,c2} \) and \( L_{\min,c2} \), which contributes to the separation; and the condensation process of excess reflux \( E_{c2} \) which takes place apparently at temperature \( T_{c2} \). \( V_{\min,c2} \) and \( L_{\min,c2} \) at the \( c2 \)-nd stage in the rectifying section may be obtained by replacing the subscript \( f \) in Eqs. (9) and (10) by \( c2 \). The direction factor curve for the former process corresponds to \( D_{sep} \) discussed in the previous section and is represented by the oblique curve in Fig. 7(b), while that for the latter is given by the horizontal line with \( D = T_{c2}/T_{c2} \). Hence the exergy destruction in the first block is obtained as the area bounded by these direction factor curves. This figure indicates quite clearly that a pinch point may appear at the \( c2 \)-nd stage when excess reflux \( E_{c2} \) becomes zero.

In the second block between the second and third side-coolers, there are two energy-accepting processes: the evaporation process of excess reflux \( E_{c2} \) at temperature \( T_{c3} \) and the process to remove heat of quantity \( Q_{c2} \) from the stage at \( T_{c2} \). On the other hand, there are also two energy-donating processes: the condensation process with minimum internal flow rates \( V_{\min,c3} \) and \( L_{\min,c3} \), contributing to the separation, and the condensation of excess reflux \( E_{c3} \) at stage temperature \( T_{c3} \).

It is found in Fig. 7(b) that by increasing \( Q_{c2} \), the excess reflux \( E_{c2} \) in the \( c2 \)-nd stage is reduced, giving rise to a pinch point as mentioned previously. In this manner, the maximum quantity of heat removed in the stage and the corresponding exergy destruction in the block can be represented quite clearly in this diagram.

On the other hand, the exergy destruction relating to the heat exchange between the stage and the coolant may be obtained as the dotted area shown in Fig. 7(b), by plotting \( T_d/T_{c3} \) against \( Q_{c3} \).

**Figure 8** shows the whole result for the case when two side-boilers and two side-coolers are installed and the values of \( E_f/L_{\min,f} \) or \( E_f/V_{\min,f} \) are set at 0.5 and 0.23, respectively. The dashed line shows the boundary for the column with only a condenser and a reboiler. The difference in the area of exergy destruction shows the amount of the exergy saving achieved by the installation of the side-boilers and side-coolers. It should be noted, however, that more stages are required when such side-boilers and side-coolers are attached. Hence the optimal excess reflux may be obtained by the following equation.

\[
E_f = E_f (L_{\min,c} / L_{\min,f})^\alpha = E_f (V_{\min,c} / V_{\min,f})^\beta
\]  

where \( \alpha \) and \( \beta \) are chosen as appropriate values.
between zero and unity.

5. Mixture Showing Strong Deviation from Ideality

Up to this point, only the case giving the pinch point at the feed stage has been discussed. However, it is well known that there are systems such as ethanol-acetic acid mixture which show an equilibrium relation deviating far from the ideal one, as shown in Fig. 9(a).5,9

Then Eq. (11) gives a sigmoid curve for \( D_{sep} \) as shown in Fig. 9(b). The portions between \( b \) and \( m \) and between \( n \) and \( f \) show a regular inclination, showing that the separation process is an energy-accepting evaporation. On the other hand, the portion between \( m \) and \( n \) shows the reverse inclination, indicating that the separation process is an energy-donating condensation, even in the stripping section.

The exergy loss for minimum reflux is obtained
from the energy–direction factor diagram as shown in Fig. 10(a), giving rise to a pinch point at stage m. The examination of each block in a manner similar to that used in 4.3 shows that the excess reflux changes quite complicately. It is to be noted that the location of the pinch point m may readily be found in this diagram, too.

However, we may set side-boilers and side-coolers to satisfy the criterion for excess reflux rate, Eq. (21). Then the amount of heat removal or heat supply in each stage can easily be determined in the diagram, as shown in Fig. 10(b).

These results together with those obtained in the previous paper3) indicate that the energy–direction factor diagram is effective in considering not only energy saving but also the thermodynamic characteristics of the process system.

Since the concept $D_{sep}$ may be applied also to multicomponent systems, the best way to analyze or synthesize a distillation system using multicolumns may be conducted in the following manner. First, make an energy–direction factor diagram for each column. Then, check the effect of various parameters in the diagrams. Finally, select proper values for the parameters so that the total exergy loss may be reduced.

6. Conclusion

1) By introducing the concept of the ideal distillation column with interstage heat exchangers, a real column with any number of stages and any internal reflux rate may be compared with a reversible column with the same number of stages and the same reflux rate.

2) By applying the energy–direction factor diagram, the thermodynamic characteristics of various types of distillation columns may be represented clearly.

3) The exergy required for the separation is obtained as the difference of the two areas $\varepsilon_S$ and $\varepsilon_R$ in this diagram.

4) The exergy destruction for various types of columns is represented as the area bounded by the direction factor curves in this diagram. Furthermore,
the location of the pinch point may readily be found.

### Nomenclature

- $b$ = bottoms flow rate
- $c_p$ = heat capacity
- $D$ = direction factor ($= T_o \Delta H / \Delta S$)
- $E$ = excess reflux rate
- $f$ = feed flow rate
- $\Delta H$ = enthalpy change
- $L$ = falling rate in rectifying section
- $L'$ = falling rate in stripping section
- $Q$ = heat flow rate
- $\Delta S$ = entropy change
- $T'$ = temperature
- $T_o$ = temperature of coolant or heating medium
- $\dot{x}$ = top distillate rate
- $y$ = mole fraction in liquid phase
- $z$ = mole fraction in total stream
- $V$ = rising flow rate in rectifying section
- $V'$ = rising flow rate in stripping section
- $\varepsilon$ = exergy, $\Delta e$ = exergy change
- $\lambda$ = latent heat

### Subscripts

- $b$ = bottom stage
- $c$ = condenser or side-cooler
- $cj$ = $j$-th stage with cooler
- $f$ = feed stage
- $ha$ = energy accepting
- $r$ = reboiler or side-boiler
- $rfj$ = $j$-th stage with boiler
- $sep$ = separation
- $t$ = top stage
- $v$ = vapor

### Literature Cited