DISCRIMINATION OF SHAPE OF PARTICLES SEPARATED BY INCLINED ROTARY DISK AND ITS EVALUATION BY LINEAR DISCRIMINANT FUNCTION

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Mullite beads (2.38–2.83 mm in diameter) whose shapes are irregular but close to spherical were repeatedly separated with an inclined rotary disk. The disk was tilted at 4.5° to the horizontal and rotated at 6.5 rpm. After 10 separations, the beads were finally classified into two groups: highly spherical particles and others. The shapes of these particle projections were analyzed and several linear discriminant functions (LDF) for particle shape were determined. The degree of sphericity and separation efficiency were evaluated and the effects of shape indices on the separation and discrimination were studied. Variances of sphericity between the classified groups of particles are larger than those of surface roughness and elongation. The shapes of mullite beads are satisfactorily discriminated with an LDF composed of two indices on global sphericity. The separation efficiency can also be evaluated by using the same LDF.

Introduction

An inclined rotary disk has been used to obtain highly spherical particles from a mixture of various kinds of particles of different shapes.^{1,7,8)} However, the spherical particles separated are not always highly spherical, but show distributions of shape.⁴⁾ Therefore, to evaluate the efficiency of shape separation, it is important to describe particle shape more accurately.

Particle shape has been described by various parameters, such as sphericity, circularity, elongation, bulkiness factor and Fourier descriptors.³⁾ Among these parameters, some correlations have been found by using principal component analysis, and the particles can be classified with two principal components for the shape.²⁾ Therefore, the particle shape should be treated in multivariate space of the parameters.

In this study, statistical evaluation of particle shapes which are approximately spherical is carried out through repeated separations with an inclined rotary disk. Discriminant analysis⁶⁾ is applied to the shape indices⁹⁾ of the classified groups of particles, and several linear discriminant functions are determined. The linear discriminant function is regarded as a measure of the shape. The influence of effective shape indices on the separation and discrimination of the shape are discussed.

1. Repeated Separation with Inclined Rotary Disk

1.1 Experimental

The inclined rotary disk used in this experiment was similar to that of Sugimoto *et al.*,⁸⁾ except that its surface was plated with chromium.⁴⁾ The schematic diagram of the rotary disk is shown in Fig. 1. Particles were fed onto the disk at a feed rate of 25 g/min. The disk was tilted at 4.5° to the horizontal and rotated at 6.5 rpm. Spherical particles can roll down nearly in a straight line on the disk. On the other hand, non-spherical particles cannot roll and are carried along the spiral scraper which is fixed and does not rotate with the disk.^{4,8)}

The procedure of separation is illustrated in Fig. 2. Particles rolling down at n -th separation are denoted by P_n , and the others are denoted by R_n . P_n particles were then sorted again. This procedure was repeated. After the separation was carried out 10 times, P_{10} particles obtained were regarded as "highly spherical" and are denoted by "S." The sum of all R_n particles were regarded as "non-spherical" and are denoted by "NS." The feed particles (P_0) were finally classified into two groups, S and NS, to which discriminant analysis was applied.

Particles used were mullite beads sieved in 2.38 mm–2.83 mm size range. Figure 3 shows photographs of particles which are not sorted (P_0 particles). A few elongated particles are observed, but the shapes of the mullite beads are approximately spherical.

Particle shape was measured in the same way as in previous works.^{2,4,5)} Profiles of particle projections

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Fig. 1. Experimental apparatus (inclined rotary disk).

Fig. 2. Procedure of repeated separation.

were read by a digitizer and processed.⁵⁾ The shape indices calculated for the analysis were the ratios of representative diameters (circularity, etc.), coefficients of variation of statistical diameters and elongation.⁹⁾ These definitions are listed in Table 1.

Number of particles sampled for the analysis was about 100 for S and NS particles respectively and was 50–100 for P_n particles. Number of points on the profile of a particle was about 200 and that used for calculation was 128.⁵⁾

1.2 Results of separation

The weight ratios of P_n particles to P_0 and P_{n-1} particles give yields of spherical particles after n separations and at n -th separation, respectively. They are denoted by E_n and ΔE_n and are defined by the following expression:

$$\Delta E_n = W_{P_n} / W_{P_{n-1}} \quad (1)$$

$$E_n = W_{P_n} / W_0 = \prod_{n'=1}^n \Delta E_{n'} \quad (2)$$

where W_{P_n} , $W_{P_{n-1}}$, and W_0 are the weights of P_n , P_{n-1} and P_0 particles respectively. Figure 4 shows the changes of E_n and ΔE_n with n . For $n \geq 5$, ΔE_n does not change.

Fig. 3. Photographs of mullite beads which are not sorted (P_0 particles).

Table 1. Definition of shape indices

1)	$\psi_{HL} = D_H / D_L$, $\Psi_{HL} = \log(1 - \psi_{HL})$
2)	$\psi_{FL} = \bar{D}_F / D_L$, $\Psi_{FL} = \log(1 - \psi_{FL})$
3)	$\psi_{HF} = D_H / \bar{D}_F$, $\Psi_{HF} = \log(1 - \psi_{HF})$
4)	$\psi_{ML} = \bar{D}_M / D_L$, $\Psi_{ML} = \log(1 - \psi_{ML})$
5)	$\sigma_F^2 = \frac{1}{\pi} \int_0^\pi (D_F(\theta) - \bar{D}_F)^2 d\theta / \bar{D}_F^2$, $\Sigma_F = \log \sigma_F$
6)	$\sigma_M^2 = \frac{1}{\pi} \int_0^\pi (D_M(\theta) - \bar{D}_M)^2 d\theta / \bar{D}_M^2$, $\Sigma_M = \log \sigma_M$
7)	$Z = D_{F\pi/2} / D_{F\min}$, $Z = \log(Z - 1)$

where

$$D_H = \sqrt{4A/\pi}, \quad D_L = L/\pi$$

$$\bar{D}_M = \frac{1}{\pi} \int_0^\pi D_M(\theta) d\theta, \quad \bar{D}_F = \frac{1}{\pi} \int_0^\pi D_F(\theta) d\theta$$

A : projected area

L : perimeter

The cumulative distribution of shape indices Ψ_j , transformed by the following Equations:

$$\Psi_j = \log(1 - \psi_j) \quad (\text{for } j=1-4 \text{ in Table 1}) \quad (3)$$

$$\Psi_j = \log \psi_j \quad (\text{for } j=5 \text{ and } 6 \text{ in Table 1}) \quad (4)$$

$$\Psi_j = \log(\psi_j - 1) \quad (\text{for } j=7 \text{ in Table 1}) \quad (5)$$

were plotted on a normal probability chart. Typical distributions are shown in Figs. 5, 6 and 7. In Eqs. (3)–(5), ψ_j 's are defined by the equations in Table 1 and subscript j denotes the shape index number in the table. Tsubaki *et al.*¹⁰⁾ suggested that Ψ_j 's obey the normal distribution. The experimental results in these figures show that there are nearly linear relations and

Fig. 4. Change of yields of spherical particles with number of separations.

Fig. 5. Change of distributions of shape index ψ_{HF} ($=\log(1-\psi_{HF})$) with separation.

that the distributions of Ψ_j of P_n particles are approximately expressed by the normal distribution. As the separation proceeded, the distributions of the shape indices became narrower. The medians of indices of S particles were smaller than for any other P_n 's. By comparing Ψ_j 's of P_n particles with those of a circle measured by the digitizer, the fraction of highly spherical particles among P_n particles increases as n increases, and S particles become closer to spheres.

A large amount of non-spherical particles were sorted in the first or the second separation, and no noticeable changes in shape indices are found for $n \geq 5$.

2. Determination of Linear Discriminant Function

By using shape index Ψ_{ij} ($j=1, N$), a linear discriminant function (LDF), f_i for a mullite particle i , is given by⁶⁾

$$f_i = \sum_{j=1}^N w_j (\Psi_{ij} - \bar{\Psi}_j) \quad (6)$$

Fig. 6. Change of distributions of shape index Σ_F ($=\log \sigma_F$) with separation.

Fig. 7. Change of distributions of shape index Z ($=\log(Z-1)$) with separation.

f_i is selected as a function to give a new measure of shape. In Eq. (6), N is the number of indices from 1 to 7 and $\bar{\Psi}_j$ is the mean value of $\bar{\Psi}_j$, given by:

$$\bar{\Psi}_j = (\bar{\Psi}_j^{(S)} + \bar{\Psi}_j^{(NS)})/2 \quad (7)$$

where

$$\bar{\Psi}_j^{(\alpha)} = \left(\sum_{i \in \alpha} \Psi_{ij} \right) / m^{(\alpha)} \quad (\alpha = S \text{ or } NS) \quad (8)$$

$\bar{\Psi}_j^{(\alpha)}$ is the mean value of Ψ_j in group α , and α stands for either S or NS. $m^{(S)}$ and $m^{(NS)}$, which are the numbers of samples, are 98 and 96, respectively. Weighting coefficient w_j in Eq. (6) was determined by Fisher's method⁶⁾ in the following way.

Table 2. Results of discrimination by one shape index

	Ψ_{HL}	Ψ_{FL}	Ψ_{HF}	Ψ_{ML}	Σ_F	Σ_M	Z
$\bar{\Psi}_j^{(S)}$	-1.876	-2.318	-2.079	-1.750	-1.748	-1.398	-1.318
$\bar{\Psi}_j^{(NS)}$	-1.550	-2.103	-1.714	-1.420	-1.400	-1.028	-0.940
d_j	-0.326	-0.215	-0.365	-0.329	-0.348	-0.369	-0.378
v_{jj}	0.099	0.118	0.105	0.107	0.116	0.205	0.157
D_j^2	1.074	0.393	1.217	1.008	1.043	0.666	0.828
$P_r(S \rightarrow NS)$	0.229	0.302	0.135	0.25	0.198	0.281	0.239
$P_r(NS \rightarrow S)$	0.357	0.429	0.367	0.408	0.388	0.378	0.388
Ψ_j of circles	-2.197	-2.678	-2.366	-2.207	-2.063	-2.088	-1.691

To make the difference between LDF of S and that of NS large, the ratio of variance of LDF between groups to that within a group, Δ^2 , given by

$$\Delta^2 = \left(\sum_{j=1}^N w_j d_j^2 \right) / \left(\sum_{j=1}^N \sum_{k=1}^N w_j w_k S_{jk} \right) \quad (9)$$

must be maximized. In Eq. (9), d_j is the difference of mean $\bar{\Psi}_j$ between S and NS, i.e.

$$d_j = \bar{\Psi}_j^{(S)} - \bar{\Psi}_j^{(NS)} \quad (10)$$

and S_{jk} is the total sum of products of deviations from the respective means:

$$S_{jk} = \sum_{\alpha=S}^{NS} \sum_{i \in \alpha} m^{(\alpha)} (\Psi_{ij} - \bar{\Psi}_j^{(\alpha)}) (\Psi_{ik} - \bar{\Psi}_k^{(\alpha)}) \quad (j, k=1, N) \quad (11)$$

Differentiation of Eq. (9) with respect to w_j gives

$$\sum_{j=1}^N w_j v_{jk} = C d_k \quad C: \text{constant} \quad (k=1, N) \quad (12)$$

from which we obtain

$$w_j = \sum_{k=1}^N v_{jk}^{-1} d_k \quad (j=1, N) \quad (13)$$

v_{jk}^{-1} is an element in the inverse matrix of the common covariance matrix $[v_{jk}]$ of shape index:

$$v_{jk} = S_{jk} / M \quad (14)$$

where

$$M = m^{(S)} + m^{(NS)} - 2 = 194 \quad (15)$$

Thus, at the boundary between two groups in N-dimensional population,

$$f_i = 0 \quad (16)$$

and for a particle i which belongs to either S or NS

$$\begin{aligned} f_i &> 0 \quad i \in S \\ f_i &< 0 \quad i \in NS \end{aligned} \quad (17)$$

3. Result of Discrimination and Discussion

3.1 Selection of shape index for LDF

The results of discriminant analysis by each shape index ($j=1, 7$) are shown in Table 2. In this table, D_j^2

is the square of Mahalanobis' distance between S and NS particles, given by

$$D_j^2 = d_j^2 / v_{jj} \quad (18)$$

where v_{jj} is equivalent to a variance of Ψ_j . D_j^2 gives a discrimination efficiency and is a measure of Δ^2 .⁶⁾ D_j^2 of Ψ_{FL} is the smallest of all. A large amount of mullite beads are generally both spherical and convex in shape (Fig. 3) and for these shapes, $\bar{D}_F \approx D_L$, i.e. $\psi_{FL} \approx 1$ and values of Ψ_{FL} , which represents surface roughness, do not differ appreciably between S and NS particles. Therefore, this index is not effective for the discrimination of mullite beads. D_j^2 of elongation is small, since elongated mullite beads rolled down into spherical shape if they rotated around the longer axis of the particle. On the other hand, D_j^2 's of Ψ_{HF} and Ψ_{HL} , which represent global sphericity, are large. These results imply that discrimination of mullite beads with the inclined rotary disk is influenced not by roughness and elongation but by sphericity.

In Table 2, $Pr(S \rightarrow NS)$ and $Pr(NS \rightarrow S)$ represent the probability of misclassification:

$$\begin{aligned} Pr(S \rightarrow NS) &= m(i | f_i < 0, i \in S) / m^{(S)} \\ Pr(NS \rightarrow S) &= m(i | f_i > 0, i \in NS) / m^{(NS)} \end{aligned} \quad (19)$$

where $m(i | f_i < 0, i \in S)$ is the number of S particles assigned to NS particles by the LDF. $Pr(S \rightarrow NS)$ is about 20% and $Pr(NS \rightarrow S)$ is about 40%. As mentioned above, a large amount of non-spherical particles were separated from P_n particles for $n \leq 2$, and the fraction of highly spherical particles among R_n particles rather increased for $n \geq 5$. As a result, the NS particles contained more S particles.

3.2 Discrimination by using LDF

Mahalanobis' distance D^2 between two groups in multivariable space, given by

$$D^2 = \sum_{j=1}^N \sum_{k=1}^N v_{jk}^{-1} d_j d_k \quad (20)$$

generally increases as the number of shape index increases.⁶⁾ This is schematically illustrated in Fig. 8, where $N=2$ and each ellipse represents the dispersion

Fig. 10. Changes of fraction of S particles with separation.

Table 4. Comparison of yields estimated by LDF's with experimental one

N	E_1	x_{P0}	x_{P1}	x_{R1}
2	0.713	0.606	0.697	0.380
5	0.743	0.616	0.702	0.367
Exp.	0.710	—	—	—

(26) with experimental ones are shown in Table 4. There are agreements between them. The LDF is therefore effective in estimating the shape separation efficiency.

Newton's separation efficiency η_N can be calculated by

$$\eta_N = (x_{P1} - x_{P0})(x_{P0} - x_{R1}) / \{x_{P0}(1 - x_{P0})(x_{P1} - x_{R1})\} \quad (29)$$

The calculated value of η_N is about 0.2, which is very small. This is because P_{10} particles were highly spherical, more spherical particles were mingled in R_1 particles under this experimental condition, and the basis of discrimination is quite strict.

Conclusion

Shape analysis was applied to mullite beads classified with the inclined rotary disk. The linear discriminant function (LDF) was determined by the analysis of shape indices. The degree of sphericity and the yield of separation were evaluated by LDF. The following results were obtained:

(1) The degree of sphericity of mullite beads is evaluated by LDF with two shape indices, Ψ_{HL} and Ψ_{HF} , and it is found that the LDF is effective in estimating the shape separation efficiency.

(2) The variance of Ψ_{FL} in separated mullite particles is small, so that Ψ_{FL} is less effective than sphericity for the discrimination of shapes close to spherical.

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Nomenclature

D	= diameter	[m]
D^2	= square of Mahalanobis' distance	[—]
d_j	= difference of mean value of shape index j between groups	[—]
$E_n, \Delta E_n$	= overall yield and yield of spherical particles at n -th separation (see Eqs. (1) and (2))	[—]
F_1	= test statistic	[—]
f_i	= value of linear discriminant function for particle i	[—]
M	= defined by Eq. (15)	[—]
m	= number of particles	[—]
N	= number of shape indices	[—]
n	= number of separations	[—]
P_n, R_n	= particles rolled into spherical side and transported to non-spherical side at n -th separation	[—]
Pr	= misclassification probability (see Eqs. (19) and (20))	[—]
S, NS	= P_n particles and particles obtained by accumulation of R_n particles at $n=10$	[—]
S_{jk}	= total sum of squares and products of deviation from the respective means	[—]
v_{jk}	= common covariance of shape index (see Eq. (14))	[—]
W_0	= weight of P_0 particles	[kg]
W_{P_n}	= weight of P_n particles	[kg]
w_j	= weighting coefficient of linear discriminant function for shape index j	[—]
x	= fraction of S particles	[—]
Z	= elongation	[—]
Δ^2	= measure of discrimination	[—]
Z	= $\log(1 - Z)$	[—]
η_N	= Newton's separation efficiency	[—]
Σ_F	= $\log \sigma_F$	[—]
Σ_M	= $\log \sigma_M$	[—]
σ	= coefficient of variation of statistical diameter	[—]
Ψ	= shape index transformed by Eqs. (3), (4) and (5) (see Table 1)	[—]
ψ	= shape index (see Table 1)	[—]
<Subscripts>		
F, H, L, M	= Feret's, projected area, perimeter and Martine's diameter	
i	= particle i	
j, k	= shape index j, k	
n, n'	= number of separations	
P, R	= spherical side and non-spherical side	
S, NS	= P_{10} particles and other particles	
<Superscripts>		
(α)	= group α	
$(P_n), (S), (NS)$	= P_n particles, P_{10} particles and other particles	
$-, =$	= mean and mean of means	

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MODIFIED DERIVATIVE DECOUPLING CONTROL OF NONLINEAR MULTIVARIABLE PROCESS

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Key Words: Process Control, Noninteracting Control, Nonlinear Multivariable Process, Derivative Decoupling Control, Mixing Tank, Liquid Level, Temperature, Optimal Control, Inverse Nyquist Array

A method is developed for on-line noninteracting control of a nonlinear multivariable process to handle constraints on the control variables based on the derivative decoupling control approach. An extension of the proposed modified derivative decoupling control method to load changes (unknown disturbance) is also treated. This modified method is investigated for noninteracting control of laboratory-scale mixing tanks in series both by digital simulation and by experiment using an on-line microcomputer. Liquid levels and temperature in tanks mixing hot and cold water inflow streams are controlled. The results are compared with those obtained by instantaneously optimal control and by the inverse Nyquist array technique. Both simulated and experimental performances of the modified derivative decoupling controller are found to be better in comparison with those obtained by the controller based on the inverse Nyquist array. Further, experiments showed that the modified derivative decoupling controller can reduce interaction to a negligible level.

Introduction

Complex industrial processes are of an essentially multivariable nature and would demand treatment as such if a simpler approach were not adequate in most cases. However, due to the presence of interactions in the multivariable system, controllers tuned for a single loop often must be retuned by trial and error to avoid destabilizing the closed-loop responses. It has been noted in the literature that multivariable control design techniques⁴⁾ do provide controller synthesis strategies which directly treat the interactions among multiple inputs and outputs, while the concept is complex in structure and application. Besides the complexity, almost no technique except optimal con-

trol theory can effectively handle constraint conditions on the control variables from the theoretical point of view.

Liu³⁾ presented an approach for noninteracting control based on the decoupling of the state derivatives satisfying the given constraints on control variables. Hutchinson and McAvoy²⁾ applied this derivative decoupling method to the servo control of an experimental heat exchanger system in which the objective was to control the two state variables by manipulation of two inputs. However, a detailed examination of this control method has revealed the general difficulties encountered in obtaining the controller equations.^{1,2,5)} In addition, this derivative decoupling control method is limited to a setpoint change.

This paper is concerned with a method to overcome

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