GAS HOLDUP AND VOLUMETRIC LIQUID-PHASE MASS TRANSFER COEFFICIENT IN SOLID-SUSPENDED BUBBLE COLUMNS

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The effects of column dimensions, gas velocity and the properties of liquid and solid particles on the gas holdup \( \varepsilon_g \) and the volumetric liquid-phase mass transfer coefficient \( k_{La} \) in the solid-suspended bubble column of liquid-solid batch operation were studied experimentally. The presence of suspended solid particles in the bubble column reduces values of \( \varepsilon_g \) and \( k_{La} \), and their reduction by an addition of solid particles to the column is high in the transition regime and low in the heterogeneous flow regime. Based on these observations, empirical equations for \( \varepsilon_g \) in transition flow and in heterogeneous flow, and an empirical equation for \( k_{La} \) applicable to the above two flow regimes are proposed.

Introduction

The solid-suspended bubble column is widely used as a three-phase slurry reactor in industrial chemical processes. In the column of liquid-solid batch operation the liquid is a fluidizing medium, and the solid particles are suspended by bubble agitation. To design a column of this type as a slurry reactor, the gas holdup \( \varepsilon_g \) and the volumetric liquid-phase mass transfer coefficient \( k_{La} \) should be known.

A number of research works have been done on \( \varepsilon_g \) and \( k_{La} \). These works show that both \( \varepsilon_g \) and \( k_{La} \) increase with increasing gas velocity and decreasing solid concentration, but the effect of solid concentration on \( \varepsilon_g \) and \( k_{La} \) becomes small at high gas velocity or in a column of large diameter. However, the effects of properties of liquid and solid on \( \varepsilon_g \) and \( k_{La} \) are not clear in these works.

The purpose of this study is to clarify experimentally the effects of column dimensions, gas velocity and the properties of liquid and solid particles on \( \varepsilon_g \) and \( k_{La} \) in the solid-suspended bubble column of liquid-solid batch operation.

1. Experimental

The experimental apparatus used in this work is shown in Fig. 1. The dimensions of plexiglass columns and the perforated plates used as gas distributors are shown in Table 1. Figure 2 shows the details of gas distributors. A perforated plate is covered with stainless steel wire gauze of 300 mesh to prevent the solid particles from settling down through the holes of the perforated plate, and the holes are oriented in triangular pitches.

The liquids used in this work were demineralized water and aqueous solutions of glycero, glycol, barium chloride and sodium sulfate. The operating temperature was kept at 298.2 ± 0.5 K. Table 2 shows the physical properties of liquids, and Table 3 shows those of glass and bronze spheres used in this work.
The average gas holdup $\varepsilon_G$ was calculated with Eq. (1), using the data of the static slurry height $H_L$ and the height $H_F$ of the aerated slurry which were determined by visual observation.

$$\varepsilon_G = \frac{(H_F - H_L)}{H_F}$$ (1)

The physical absorption of oxygen in the air by the liquid was employed to determine $k_{L\alpha}$. The details of this method are similar to those discussed in the previous paper. The operations of desorption and absorption of oxygen were carried out four times, changing the period $t$ of supplying the air to the column, and then a relation of the concentration $c$ of dissolved oxygen vs. $t$ was obtained. By assuming complete mixing of liquid and constant oxygen concentration in the air in the column, a material balance of oxygen in the liquid gives Eq. (2):

$$k_{L\alpha} = \frac{2.303(1 - \varepsilon_G - \varepsilon_S)}{t} \log \frac{c_t - c}{c_i - c_o}$$ (2)

where $c_i$ and $c_o$ are the saturated and initial concentrations, respectively, of dissolved oxygen. A linear relationship of $\log \frac{(c_t - c)}{(c_i - c_o)}$ vs. $t$ was obtained, and $k_{L\alpha}$ was evaluated from the slope of this line and Eq. (2).

The above experiments of $\varepsilon_G$ and $k_{L\alpha}$ were carried out in the range of gas velocity where all the solid particles were in suspension. Similar experiments of $\varepsilon_G$ and $k_{L\alpha}$ were done in the column without solid particles.

2. Results and Discussion

2.1 Flow regimes in bubble flow

Under the usual operating conditions for the bubble column, bubble flow and slug flow are observed. Furthermore, three flow regimes are recognized in bubble flow: the homogeneous flow, transition and heterogeneous flow regimes. Attempts have been made to determine the flow regime in bubble flow by using the $U_G$ vs. $\varepsilon_G$ curve. Marrucci's equation and the Koide et al. equation for $\varepsilon_G$ in homogeneous flow and the Akita et al. equation for $\varepsilon_G$ in the heterogeneous flow of the air-water two-phase system. The region between the curves predicted by Marrucci's equation and the Akita et al. equation corresponds approximately to the region where the $U_G$ vs. $\varepsilon_G$ curve for the transition flow lies. The observed values for $\varepsilon_G$ shown in Fig. 3 indicate that most experiments were carried out in the transition regime and the heterogeneous flow regime.

2.2 Experimental results of $\varepsilon_G$ and $k_{L\alpha}$

As shown in Fig. 4, both $\varepsilon_G$ and $k_{L\alpha}$ increase with increasing $U_G$ and decrease with increasing solid concentration $c_S$ and solid density. However, the effect of $c_S$ on $\varepsilon_G$ and $k_{L\alpha}$ becomes less pronounced as
the hole diameter $\delta$ of the perforated plate is increased.

Figure 5 shows the interpolated values $e_g$ and $k_{La}$ at $U_g = 0.03-0.15\, \text{m/s}$ from experimental data. This figure indicates that the terminal velocity $V_t$ of a single particle has no effect on $e_g$, but $k_{La}$ decreases with increasing $V_t$ at low gas velocity.

Figure 6 shows that values for $e_g$ and $k_{La}$ in the column of $\delta = 1\, \text{mm}$ and without solid particles decrease when $D_T$ is larger than 0.2 m. This might be due to the change of the flow regime from transition to heterogeneous flow. In the heterogeneous flow regime, where most of the data except those for $D_T = 0.10-0.14\, \text{mm}$ and $\delta = 1\, \text{mm}$ in Fig. 6 were observed, neither $e_g$ nor $k_{La}$ is affected by $D_T$, and the effect of $c_s$ on $e_g$ and $k_{La}$ becomes less pronounced.

Figure 7 shows the interpolated values of $e_g$ and $k_{La}$ at $U_g = 0.03-0.15\, \text{m/s}$ from experimental data. Both $e_g$ and $k_{La}$ decrease with increasing static slurry height $H_s$ in the range of $H_s < 1\, \text{m}$. However, the effect of $H_s$ on $e_g$ and $k_{La}$ becomes small in the range
Fig. 7. Effect of static slurry height on \( \varepsilon_g \) and \( k_{La} \).

Figures 8 and 9 show that \( \varepsilon_g \) decreases with increasing liquid viscosity and surface tension. However, the effects of these liquid properties on \( \varepsilon_g \) is reduced as \( \delta \) increases. Figures 8 and 9 show also that \( k_{La} \) decreases with increasing liquid viscosity and surface tension. The reduction of \( k_{La} \) value by an addition of solid particles is higher in the more viscous liquid, as shown in Fig. 9.

Figure 10 shows that both \( \varepsilon_g \) and \( k_{La} \) in the aqueous solutions of inorganic electrolytes are larger than those in water.

Summing up the above observations on the effect of solid particles on \( \varepsilon_g \) and \( k_{La} \), the presence of solid particles in liquid might enhance bubble coalescence into larger bubbles\(^1\) and therefore reduce values of \( \varepsilon_g \) and \( k_{La} \). The reduction of \( \varepsilon_g \) and \( k_{La} \) values by an addition of solid particles to liquid is high in the transition regime and in highly viscous liquid, and low in the heterogeneous flow regime.

2.3 Correlation of \( \varepsilon_g \)

In the following correlation of \( \varepsilon_g \) and \( k_{La} \), data observed in the columns with \( H_L \geq 1.0 \text{ m} \) were used. Firstly, \( \varepsilon_g \) in bubble columns without solid particles were correlated with experimental conditions, respectively, for heterogeneous flow and the transition regimes. Secondly, the correlation of \( \varepsilon_g \) in each flow regime was modified to include the effect of solid particles on \( \varepsilon_g \).\(^*\)

1) \( \varepsilon_g \) in the heterogeneous flow regime  In the heterogeneous flow regime \( U_L \) and \( \delta \) have no effect on

\* \( \varepsilon_g \) data for \( c_s > 0 \) obtained in a column where the transition regime was observed for \( c_s = 0 \) were classified as those in the transition regime.
the least square method for water and aqueous solutions of glycerol and glycol:

\[
\frac{e_{GH}}{(1 - e_{GH})^2} = 0.277 \left( \frac{U_G \mu_L}{\sigma_L} \right)^{0.918} \left( \frac{g \mu_L^4}{\rho_L \sigma_L^2} \right)^{-0.252}
\]  

(3)

For the aqueous solutions of inorganic electrolytes the coefficient of the right-hand side in Eq. (3) was 0.364 instead of 0.277.

The effect of solid particles on reducing \( e_G \) value increases with increasing \( c_s \), \( \rho_s \) and \( \mu_L \) and with decreasing \( D_T \) and \( U_G \). Therefore, an equation in the form of Eq. (4) was assumed and the numerical constants of the denominator in Eq. (4) were decided by the direct search method using data observed in the heterogeneous flow regime.

\[
\frac{e_{GH}}{(1 - e_{GH})^2} = \frac{A \left( \frac{U_G \mu_L}{\sigma_L} \right)^{0.918} \left( \frac{g \mu_L^4}{\rho_L \sigma_L^2} \right)^{-0.252}}{1 + 4.35 \left( \frac{c_s}{\rho_s} \right)^{0.748} \left( \frac{\rho_s - \rho_L}{\rho_L} \right)^{0.887} \left( \frac{D_T U_G \rho_L}{\mu_L} \right)^{-0.168}}
\]  

(4)

\( A = 0.277 \) for water and aqueous solutions of glycerol and glycol, and \( A = 0.364 \) for aqueous solutions of inorganic electrolytes. The average error in estimating \( e_G \) by Eq. (4) was 5.7% for 474 data in the experimental ranges of \( 0.14 \text{ m} \leq D_T \leq 0.30 \text{ m}, \ 1.64 \times 10^{-4} \leq (U_G \mu_L/\sigma_L) \leq 2.92 \times 10^{-2}, \ 1.69 \times 10^{-11} \leq (g \mu_L^4/\rho_L \sigma_L^2) \leq 2.84 \times 10^{-6}, \ 0 \leq (c_s/\rho_s) \leq 0.08, \ 1.12 \leq (\rho_s - \rho_L)/\rho_L \leq 7.80 \) and \( 3.15 \times 10^2 \leq (D_T U_G \rho_L/\mu_L) \leq 4.82 \times 10^4 \). Figure 11 shows that \( e_G \) values estimated by Eq. (4) agree well with those observed experimentally.

2) Correlation of \( e_G \) in the transition regime

Sakata et al. have shown that the \( U_G \) vs. \( (e_G T)/(e_G H) \) curve for the bubble column without solid particles has a maximum value of \( (e_G T)/(e_G H) - 1 \) at a certain gas velocity \( U_G \). Figure 12 shows that \( (e_G T)/(e_G H) \) has a maximum value and the \( U_G \) vs. \( (e_G T)/(e_G H) - 1 \) curve is similar to a log normal distribution curve, where \( (e_G H) \) is calculated by Eq. (3) at the same gas velocity as \( (e_G T) \) is observed. Therefore, a simulation of the \( U_G \) vs. \( (e_G T)/(e_G H) - 1 \) curve by Eq. (5) was attempted.

\[
(\frac{(e_G T)}{(e_G H)} - 1) = M \exp \left[ -\ln \left( \frac{U_G}{U_G C} \right)^2 \right]
\]  

(5)

where \( M \) is the maximum value of \( (e_G T)/(e_G H) - 1 \) at \( U_G C \) shown in Fig. 12. As it has been shown that \( (M+1) \) and \( U_G C \) increase with increasing number \( N \) and pitch \( P \) of holes in the gas distributor and with decreasing \( \delta \) and \( D_T \), \( M \) and \( U_G C \) were correlated with \( N, P, \delta, D_T \) and the physical properties of liquids by Eqs. (6) and (7), respectively.

\[
U_G C H L = 9.59 \left[ \frac{(\delta)}{P} \left( \frac{D_T g \rho_p}{N \sigma_L} \right) \right]^{-0.384} \left( \frac{g \mu_L^4}{\rho_L \sigma_L^2} \right)^{0.384}
\]  

(6)

\[
U_G C H L = 9.59 \left[ \frac{(\delta)}{P} \left( \frac{D_T g \rho_p}{N \sigma_L} \right) \right]^{-0.384} \left( \frac{g \mu_L^4}{\rho_L \sigma_L^2} \right)^{0.384}
\]  

(7)

The effect of solid particles on reducing \( e_G \) value in the transition regime is larger than that in the heterogeneous flow regime, as shown in Fig. 4. This effect increases with increasing \( c_s, \rho_s \) and \( \mu_L \) and with decreasing \( D_T \) and \( U_G \). Therefore, an equation in the form of Eq. (8) was assumed to express the effect of solid particles on \( e_G \). The numerical constants of the denominator in Eq. (8) were decided by the direct search method using data observed in the transition regime.
Fig. 12. Effects of gas velocity, column diameter and gas distributor on \( (\varepsilon_{GT}/\varepsilon_{GH})_{0} - 1 \).

![Fig. 12](image_url)

\[
\varepsilon_{GT} = \frac{(\varepsilon_{GH})_{0}}{1 + 49.1 (\frac{cs}{\rho_s})^{0.519} (\frac{\rho_s - \rho_L}{\rho_L})^{1.14} \left( \frac{D_T U_G \rho_L}{\mu_L} \right)^{-0.496}}
\]

in the experimental ranges of \( 0.10 \leq D_T \leq 0.14 \), \( 1.69 \times 10^{-11} \leq (gU_G^2/\rho_L \sigma_L) \leq 2.84 \times 10^{-6} \), \( 0 \leq (cs/\rho_s) \leq 0.08 \), \( 1.12 \leq (\rho_s - \rho_L)/\rho_L \leq 7.8 \), \( 3.25 \times 10^2 \leq (D_T U_G \rho_L)/\mu_L \leq 2.67 \times 10^4 \), and \( 0.88 \leq (\delta/P)(D_T^2 \rho_P/Ne_L) \leq 5.32 \). The average error in estimating \( \varepsilon_G \) by Eq. (8) was 10% for 313 data. Figure 13 shows that \( \varepsilon_G \) estimated by Eq. (8) agree well with those observed experimentally.

As a complete prediction of the flow regime boundary between the transition regime and the heterogeneous flow regime cannot yet be made, it is recommended that Eqs. (4) and (8), respectively, be used for columns of \( D_T \geq 2 \times 10^{-4} \) m and \( D_T < 2 \times 10^{-4} \) m. This criterion of using \( D_T \delta \) value for columns of \( D_T \geq 0.1 \) m is based on the observations of \( \varepsilon_G \) in this work and the previous ones.\(^9\)\(^13\)\(^16\)

### 2.4 Correlation of \( k_L a \)

Akita et al.\(^1\) have shown that a better empirical equation of \( k_L a \) is obtained by using \( \varepsilon_G \) instead of \( U_G \) in the bubble column without solid particles. Then, \( \log k_L a \) is plotted against \( \log \varepsilon_G \) in Fig. 14, and this yields straight lines of the slope of 1.18. Figure 14 shows also that \( D_T \) and the difference in the flow regimes have almost no effect on the \( \varepsilon_G \) vs. \( k_L a \) relation.

Therefore, the following empirical equation of \( k_L a \) for the bubble columns without solid particles was obtained by dimensional analysis and the least square method, using \( \varepsilon_G \) instead of \( U_G \), where the exponent of the Schmidt number \( (\mu_L/\rho_L D_T) \) was assumed to be 0.5 as was done by Akita et al.\(^1\)

\[
k_L a = \frac{2.11 \left( \frac{\mu_L}{\rho_L D_T} \right)^{0.500} \left( \frac{g \mu_L^2}{\rho_t \sigma_L^3} \right)^{-0.159} \varepsilon_G^{1.18}}{1 + 1.47 \times 10^4 \left( \frac{cs}{\rho_s} \right)^{0.612} \left( \frac{V_i}{D_T \sigma} \right)^{0.486} \left( \frac{D_T \sigma P}{\mu_L} \right)^{-0.477} \left( \frac{D_T U_G \rho_L}{\mu_L} \right)^{-0.343}}
\]

The average error of estimating \( k_L a \) by Eq. (9) was 15% for 106 data.

The effect of solid particles on the reduction of \( k_L a \) value decreases with increasing \( U_G \), \( D_T \) and \( \sigma_L \) and with decreasing \( V_i \), \( cs \), \( \rho_s \) and \( \mu_L \). The dependence of \( k_L a \) on \( \varepsilon_G \) in the column with solid particles is similar to that in the column without solid particles, as shown in Fig. 14. Therefore, an equation in the form of Eq. (10) was assumed to modify Eq. (9) by taking the effect of solid particles on \( k_L a \) into consideration. The numerical constants of the denominator in Eq. (10) were decided by the direct search method.\(^3\)

![Fig. 13](image_url)

![Fig. 14](image_url)
in the experimental ranges of $3.71 \times 10^2 \leq (\mu_L/\rho_L D_T) \leq 1.06 \times 10^6$, $1.69 \times 10^{-11} \leq (\rho_L/\rho_L \sigma_L)^2 \leq 2.84 \times 10^{-6}$, $0.0291 \leq \varepsilon_L \leq 0.253$, $0 \leq (\varepsilon_S/\rho_S) \leq 0.08$, $2.18 \times 10^{-4} \leq (V_4/\rho_D \theta) \leq 2.84 \times 10^{-2}$, $1.36 \times 10^3 \leq (D_T^3 \theta_D/\sigma_L) \leq 1.22 \times 10^4$ and $3.16 \times 10^2 \leq (D_T U_D/\mu_L) \leq 4.18 \times 10^4$. Figure 15 shows that $k_L a$ values estimated by Eq. (10) agree relatively well with those observed experimentally. The average error in estimation was 20% for 475 data.

### 2.5 Comparison of this work with the previous ones

1) Bubble column without solid particles

Figure 9 shows that $e_G$ values in the heterogeneous flow regimes agree well with the Akita et al. equation, but $k_L a$ values are much larger than those predicted by the Akita et al. equation for columns with a single orifice plate as a gas distributor.

The equation of $U_{GC}$ derived from Eq. (7) for water,

$$U_{GC} = 1.45(PN/3D_T^{0.52})$$

agrees well with the Sakata et al. result,

$$U_{GC} = 1.55(PN/3D_T^{0.5})$$

which is obtained from their graphical representation of $U_{GC}$. Sakata et al. also give a boundary in a graph where the flow regime changes from the transition to the heterogeneous. If $[(e_G^t)/(e_G^h) - 1] = 0.1M$ is assumed to give the boundary of flow regime transition from transition flow to heterogeneous flow, Eq. (5) gives the equation for the boundary, $U_G/U_{GC} = 4.56$. The calculated values of $U_G$ by this equation for water, above which the flow is heterogeneous, agree roughly with the Sakata et al. result.

2) Bubble column with solid particles

Kara et al. have given empirical equations for $e_G$ in the transition flow of air-water-coal and dried mineral ash, and different values are given to the constants in the equations for slurries of different particle size. However, $e_G$ values estimated from their empirical equation based on 70um particles agree well with $e_G$ observed in the column of $D_T = 0.14$ m and $\delta = 2.5$ mm, but are smaller than those observed in the column of $D_T = 0.14$ m and $\delta = 1$ mm.

Kojima et al. have proposed an empirical equation for $e_G$ in columns with a porous plate as a gas distributor, and the equation might be applicable to $e_G$ in the homogeneous flow regime but not to $e_G$ observed in this work. The effects of $U_G$, $\delta$, $D_T$ and $c_S$ on $e_G$ and $k_L a$ observed by Kato et al. are similar to those observed in this work. However, the values for $e_G$ and $k_L a$ are much larger than those observed in this work, since sodium sulfite aq. solution used in their work has a high frothing ability.

### Conclusions

The presence of suspended solid particles in the bubble column reduces values for the gas holdup $e_G$ and the volumetric mass transfer coefficient $k_L a$. The reduction of $e_G$ and $k_L a$ values by an addition of solid particles is high in the transition regime and low in the heterogeneous flow regime. Based on these observations, empirical equations for $e_G$ in the transition and the heterogeneous flow regimes, and an empirical equation for $k_L a$ applicable to the above two flow regimes are proposed.

### Nomenclature

- $a$ = specific gas-liquid interfacial area based on aerated slurry volume [m$^{-1}$]
- $c$ = concentration of dissolved oxygen [mol m$^{-3}$]
- $c_S$ = $\rho_{SLS}(\varepsilon_G + \varepsilon_S)$, average solid concentration [kg m$^{-3}$]
- $D_L$ = diffusivity of dissolved oxygen [m$^2$ s$^{-1}$]
- $D_T$ = column diameter [m]
- $d_p$ = diameter of solid particle [m]
- $g$ = gravitational acceleration [m s$^{-2}$]
- $H$ = column height [m]
- $H_s$ = level of aerated slurry during operation [m]
- $H_L$ = static slurry height above gas distributor [m]
- $k_L$ = liquid-phase mass transfer coefficient [m s$^{-1}$]
- $k_L a$ = volumetric liquid-phase mass transfer coefficient based on aerated slurry volume [m$^{-1}$]
- $M$ = $((\varepsilon_G^t)/(\varepsilon_G^h) - 1)$ at $U_{GC}$ [s$^{-1}$]
- $N$ = number of holes in gas distributor [—]
- $P$ = pitch of holes in gas distributor [m]
- $t$ = time [s]
- $U_G$ = gas velocity based on cross section of column and average static pressure in column [m s$^{-1}$]
- $U_{GC}$ = gas velocity where maximum value of $((\varepsilon_G^t)/(\varepsilon_G^h) - 1)$ is observed [m s$^{-1}$]
- $V_t$ = terminal velocity of a single particle in stagnant liquid [m s$^{-1}$]
- $\delta$ = hole diameter in gas distributor [m]
- $e_G$ = gas holdup [—]
- $e_G^h$ = gas holdup for heterogeneous flow regime [—]
- $e_G^t$ = gas holdup for transition regime [—]
- $k_L$ = liquid holdup [—]
- $e_S$ = solid holdup [—]
- $\mu_L$ = liquid viscosity [Pa s]
- $\rho_L$ = liquid density [kg m$^{-3}$]
Flow characteristics in a channel with symmetric wavy wall for steady flow

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Key Words: Fluid Mechanics, Friction Factor, Wall Shear Stress, Flow Visualization, Electrochemical Method, Finite Element Method

Flow characteristics in a channel with a symmetric wavy wall were investigated by calculations and experiments. The channel used has a geometry similar to that of the Oxford membrane blood oxygenator. The flow regime covered ranged from laminar to turbulent flow.

The variation of pressure drop and wall shear stress with the Reynolds number was elucidated by the behavior of the circulated vortex formed at the diverging cross section of the channel.

Introduction

The channel or tube with wavy wall is one of several devices employed for enhancing the heat and mass transfer efficiency of processes having high Peclet numbers, such as compact heat exchangers with high heat flux and membrane blood oxygenators in extra-corporeal system, etc.

Chow et al.\(^2\)\(^3\) solved analytically the flow in the channel and tube. Fedkiw et al.\(^5\) obtained the solution for the tube in the creeping flow region by a collocation method. Deibet et al.\(^4\) used a finite difference method for the same geometry beyond the Reynolds number at which inertial effects are important. They also measured the friction factor, and obtained good agreement between experiment and calculation. These results give the stream patterns and friction factors in the channel or tube. There has been, however, little work about the wall shear stress, which is an important factor for the prediction of the heat and mass transfer rates of systems with high Peclet numbers.

In this study we employed a channel with a symmetric sinusoidal wavy wall, which has a geometry...