WALL EFFECT ON THE BEHAVIOUR OF SINGLE BUBBLES RISING IN HIGHLY VISCOUS LIQUIDS

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The wall effect on the terminal velocity and the shape of single bubbles rising in highly viscous liquids has been investigated experimentally. The terminal velocities of single bubbles affected by the column wall are correlated with two dimensionless equations between \( ReM0.48(1 - \lambda^{1.5}) \) and Eötvös number \( Eo \), where \( \lambda \) is the bubble and column diameter ratio and \( 1 - \lambda^{1.5} \) is the wall effect function, which was also used for relatively low-viscosity liquids.

The aspect ratio \( h/w \) defining the bubble deformation is found to be a function of \( Re \) and \( \lambda \) for \( Re \geq 0.1 \), while it is a function of \( \lambda \) only for \( Re < 0.1 \). The bubble deforms from the sphere in the vicinity of \( Eo = 6 \), which corresponds to the border of dimensionless equations of bubble rising velocity.

Introduction

The behaviour of bubbles rising in highly viscous liquids is very important in designing gas-liquid contact equipment such as fermenters. Theoretical investigations in this field have been made on small spherical bubbles for \( Re < 1 \) \(^{9,16} \) and spherical bubbles at intermediate Reynolds number \(^{3,4,10} \) and bubbles deformed from spherical shape. \(^{19} \) Some experimental works have been done, \(^{1,2,15} \) and recently a generalized correlation chart of Reynolds number versus Eötvös number for bubbles rising in Newtonian liquids of large extent was proposed. \(^{2,7} \) These studies have been carried out under conditions where the size of the experimental apparatus has negligible effect or none at all on bubble motion. However, the rising velocities of bubbles are affected to a certain extent in practice by column size.

Studies of the wall effect on the rising velocity of single bubbles in highly viscous liquids seem to be few \(^{6,17} \) compared with those for relative low-viscosity liquids. \(^{11,15,20-22} \) For the latter, Tsuge et al. \(^{20,21} \) showed that by using the wall correction factor for a solid sphere falling in liquids, \(^{14} \) the rising velocities of single bubbles can be correlated by dimensionless equations.

The aims of this paper are to study experimentally the wall effect on the rising velocity and the shape of single bubbles rising freely in highly viscous liquids. The rising velocities obtained and the existing data are correlated with dimensionless groups by considering the wall effect function. The wall effect on the shape of single bubbles is also correlated with dimensionless groups.

1. Experimental Apparatus and Procedure

The experimental apparatus consists of an acrylic test tank of cross section \( 0.15 \times 0.15 \text{ m}^2 \) and \( 0.4 \text{ m} \) in height. To circulate constant-temperature water, a square box made of acrylic resin was placed around the tank.

The gas used was dry air in order to prevent mass transfer between gas and liquid in the tank. Single bubbles were introduced by hypodermic syringes or enema syringes and bubble volumes were varied from \( 1 \times 10^{-9} \) to \( 8 \times 10^{-5} \text{ m}^3 \).

A 16 mm high-speed cine-camera (Locam) was used to film single bubbles rising between levels of 0.05 and 0.15 m beneath the liquid surface, where it was confirmed that the rising bubbles reached terminal velocity. Measurements of terminal velocity, volume and shape of bubbles were made by frame-to-frame analysis with a film analyser.

To study the wall effect on the behaviour of single bubbles, six Pyrex glass tubes of inside diameters 0.099, 0.080, 0.0506, 0.0405 and 0.0305 m respectively, were inserted in the tank. The tube ends were polished flatly to prevent the exchange of liquid between inside and outside of tubes when they were put vertically on the plate in the tank.

The liquids used were glycerol and corn syrup, whose average physical properties and Morton number \( M \) are shown in Table 1. Physical properties of liquids were measured, while the surface tension of...
Table 1. Physical properties of liquids

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Temp. [K]</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$\mu$ [Pa·s]</th>
<th>$\sigma$ [N/m]</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycerol</td>
<td>288.0</td>
<td>$1.265 \times 10^3$</td>
<td>2.08</td>
<td>$6.24 \times 10^{-6}$</td>
<td>5.97 \times 10^2</td>
</tr>
<tr>
<td></td>
<td>293.0</td>
<td>$1.262 \times 10^3$</td>
<td>1.37</td>
<td>$6.34 \times 10^{-6}$</td>
<td>1.07 \times 10^2</td>
</tr>
<tr>
<td></td>
<td>298.0</td>
<td>$1.258 \times 10^3$</td>
<td>0.865</td>
<td>$6.26 \times 10^{-6}$</td>
<td>1.78 \times 10^2</td>
</tr>
<tr>
<td></td>
<td>303.0</td>
<td>$1.256 \times 10^3$</td>
<td>0.565</td>
<td>$6.21 \times 10^{-6}$</td>
<td>3.31</td>
</tr>
<tr>
<td>Corn Syrup</td>
<td>288.0</td>
<td>$1.393 \times 10^3$</td>
<td>10.7</td>
<td>$7.54 \times 10^{-6}$</td>
<td>2.13 \times 10^5</td>
</tr>
<tr>
<td></td>
<td>293.0</td>
<td>$1.392 \times 10^3$</td>
<td>6.89</td>
<td>$6.90 \times 10^{-6}$</td>
<td>4.83 \times 10^4</td>
</tr>
<tr>
<td></td>
<td>296.5</td>
<td>$1.399 \times 10^3$</td>
<td>6.52</td>
<td>$6.14 \times 10^{-6}$</td>
<td>5.47 \times 10^4</td>
</tr>
</tbody>
</table>

glycerol was taken from a handbook.12)

2. Results and Discussion

2.1 Rising velocity of single air bubbles

Figures 1(a), (c) and (d) show the effect of column inside diameter $D$ on the rising velocities in glycerol and corn syrup, and Fig. 1(b) shows the effect of $M$ on the rising velocity in glycerol in the case of $D = 0.0592$ m. The rising velocity of a bubble of given diameter decreases with decreasing $D$ and increasing $M$. During the present experiments the bubble trajectories were always rectilinear, as has been observed previously.6,13)

To correlate the rising velocities of single bubbles, the following equation is used:

$$u = \phi(d, \rho, \mu, \sigma, g, D)$$

(1)

Dimensional analysis yields

$$\phi(Re, Eo, M, \lambda) = 0$$

(2)

where $Re = du/p/\mu$, $Eo = gpd^2/\sigma$, $M = g/\mu/\rho d^3$ and $\lambda = d/D$. By assuming that the wall effect can be separated from the effect of the other variables, especially $M$, Eq. (2) is rewritten as follows:

$$\phi_0(Re, Eo, M) \cdot \phi_1(\lambda) = 1$$

(3)

where $\phi_1(\lambda)$ is a wall effect function and becomes 1 when $\lambda$ approaches 0. Therefore, $\phi_0(Re, Eo, M) = 1$ describes the bubble behaviour when free from the wall effect.

2.1.1 Correlations with negligible wall effect

Based on the result of dimensional analysis, the experimental data for glycerol and corn syrup in the region of $\lambda < 0.1$, which can be considered to be nearly free from the wall effect, are correlated by use of the dimensionless groups $ReM^{0.48}$ and $Eo$ as shown in Fig. 2. The experimental results by Kojima et al.13) in the region of $\lambda < 0.1$ are in good agreement with the present data. Both sets of data can be correlated by the following dimensionless equations for $\lambda < 0.1$ and $3.31 \leq M \leq 2.13 \times 10^5$, and the mean deviation of the data from these equations is $\pm 25\%$.

$$0.5 < Eo < 6 : ReM^{0.48} = 0.069Eo^{1.55}$$

(4)
6 \leq Eo \leq 200: \quad ReM^{0.48} = 0.106Eo^{1.31} \quad (5)

To compare with previous works, Hadamard-Rybczynski's Eq. (6) for spherical bubbles and the correlation of Tadaki et al., Eq. (7), which is applicable for Re < 200 and for purified liquids,\(^{18}\) are transformed respectively to Eqs. (8) and (9).

\[
CD = 16/Re \quad (6)
\]
\[
CD = 18.5/Re^{0.82} \quad (7)
\]
\[
ReM^{0.48} = 0.0833M^{-0.02}Eo^{1.5} \quad (8)
\]
\[
ReM^{0.48} = 0.108M^{0.056}Eo^{1.27} \quad (9)
\]

As is clear from Fig. 2, Eqs. (8) and (9) show good agreement with the present correlations for the range 1 < M < 10^6 and bubble shapes begin to deviate from spherical shape in the vicinity of Eo = 6, which will be discussed later.

2.1.2 Correlations with wall effect

By referring to Eqs. (4) and (5), the present data for \(\lambda \geq 0.1\) are plotted as relations between \(ReM^{0.48}/Eo^{1.55}\) and \(\lambda\) for Eo < 6 and between \(ReM^{0.48}/Eo^{1.31}\) and \(\lambda\) for 6 \leq Eo as shown in Fig. 3(a) and (b). With increasing \(\lambda\), \(ReM^{0.48}/Eo^{1.55}\) and \(ReM^{0.48}/Eo^{1.31}\) decrease, which means that the rising velocities of single bubbles are retarded by the wall effect.

Two of the authors\(^{20,21}\) showed that the wall effect function \(1 - \lambda^{1.5}\) is suitable for relatively low-viscosity liquids for \(3.7 \times 10^{-14} < M < 1.5 \times 10^{-2}\) and \(\lambda \leq 0.5\). By applying \(1 - \lambda^{1.5}\) to highly viscous liquids, the present experimental data for \(\lambda \geq 0.1\) are well correlated using \(ReM^{0.48}/(1 - \lambda^{1.5})\) and Eo as shown in Fig. 4, and this is expressed by the following equations.

\[1 < Eo < 6: \quad ReM^{0.48}/(1 - \lambda^{1.5}) = 0.069Eo^{1.55} \quad (10)\]
\[6 \leq Eo < 700: \quad ReM^{0.48}/(1 - \lambda^{1.5}) = 0.106Eo^{1.31} \quad (11)\]

The mean deviation of the data from these equations is \(\pm 25\%\). This result gives support to the assumption used to obtain Eq. (3), which is that the wall effect function \(\phi(\lambda)\) is not a function of M over this range. Equations (10) and (11) are shown as a solid line in Fig. 3(a) and (b), respectively. Clift et al.\(^{5}\) proposed a similar correction factor \((1 - \lambda^{2})^{1.5}\) for bubbles and drops for Eo < 40, Re > 200 and \(\lambda < 0.6\), but the range of M was not defined.

The data of Kojima et al.\(^{13}\) and Angelino\(^{13}\) for 0.1 \leq \lambda \leq 0.6 and Eo \leq 200 are adequately correlated by Eq. (11) as shown in Fig. 5. However, Angelino's data for Eo \geq 200 deviate from our correlation, which may be due to the bubble shape.

The drag coefficient, \(C_D\), for spherical cap bubbles was proposed theoretically for Re > 150, Eo \geq 40 and \(\theta = 0.278\) rad\(^{5}\):

\[C_D = 8/3 \quad (12)\]
Equation (12) is reduced to Eq. (13):

$$ReM^{0.48} = M^{0.23}Eo^{0.75}/\sqrt{2}$$

The agreement between Eq. (13) and Angelino’s data for $Eo > 200$ shown in Fig. 5 suggests that their bubbles were of spherical cap shape ($\theta < \pi/2$). The bubbles in the present study are of ellipsoidal cap shape ($\theta = \pi/2$), as shown in Fig. 6(a).

Bubbles in the region of $\lambda > 0.6$ are called slugs or Taylor bubbles. White et al. proposed the following equation for the slug velocity independent of inertia effect and surface tension for $EoD=pgD^2/\sigma > 70$ and $FrD=u/\sqrt{gD}<0.05$:

$$u = 0.0096pgD^2/\mu$$

The present data for slugs in the region of $EoD > 70$ and $FrD < 0.05$ agree closely with Eq. (14) as shown by the solid lines in Fig. 1(c) and (d).

2.2 Bubble shape

2.2.1 Bubble shape with negligible wall effect

The bubble shape in the case of negligibly small wall effect is a function of both $Re$ and $M$ for low-$M$ liquids ($M < 0.1$), while it is a function of $Re$ only for high-$M$ liquids ($M > 3$).

In the present study, the aspect ratio $h/w$ is used as one of the deformation factors, and the bubble shape for glycerol deforms from sphere to ellipsoidal cap at about $Re = 0.5$ for $\lambda < 0.3$ irrespective of $M$ in the case of $D = 0.169$ m, as shown in Fig. 6(a).

2.2.2 Effect of wall on bubble shape

As shown in Fig. 6(b), bubbles are elongated axially by the wall effect for $\lambda < 0.6$ in the case of $D = 0.0305$ m so that $h/w$ increases with increasing $Re$ and the bubbles become like slugs. Reynolds numbers where bubbles begin to be elongated decrease with increasing $M$. Figure 7 shows the experimental data between $h/w$ and $Re$ for constant $\lambda$ in the range of $3.31 \leq M \leq 2.13 \times 10^3$, which are connected by smooth curves. The values of $h/w$ for a given value of $Re$ are almost equal in the region of $0.1 \leq \lambda \leq 0.3$, while they increase with increasing $\lambda$ if $\lambda$ is larger than 0.4. That is, in this $M$ range the values of $h/w$ are expressed as a function of both $Re$ and $\lambda$ if $Re \geq 0.1$, and of $\lambda$ only if $Re < 0.1$. The results of Kojima et al. and Bhaga et al. under negligible wall effect are also shown in Fig. 7. Their results are a little smaller than ours in the region of $0.1 \leq \lambda \leq 0.3$, which is reasonable considering that their results are free from wall effect while ours are affected to some degree by the wall.

Figures 8(a) and (b) show the relation between $h/w$ and $Eo$ in the cases of $M = 3.31$ and 17.8 for glycerol. The values of $h/w$ begin to decrease in the vicinity of $Eo = 6$, which suggests that the intersection of Eqs. (4) and (5), or Eqs. (10) and (11) for rising velocity of single bubbles, that is, $Eo = 6$, corresponds to the beginning of bubble deformation from spherical shape.

Conclusion

The rising velocity and shape of single air bubbles were investigated experimentally in quiescent highly viscous liquids such as glycerol and corn syrup.

1) The rising velocity of single bubbles for negligible wall effect, $\lambda < 0.1$, is correlated by Eqs. (4) and (5) in the region of $3.31 \leq M \leq 2.13 \times 10^3$.

2) The rising velocity of single bubbles affected by column walls, that is, $0.1 \leq \lambda \leq 0.6$, is correlated in the same $M$ region by Eqs. (10) and (11), where the same wall effect function $1 - \lambda^{1.5}$ as used for relatively low-viscosity liquids is applicable.

3) The bubble deforms from spherical shape in the vicinity of $Eo = 6$, which corresponds to the intersection of dimensionless equations for the bubble rising velocity.
4) The values of $h/w$ are a function of $\lambda$ and $Re$ for $Re > 0.1$ and a function of $\lambda$ only for $Re < 0.1$ in the range of $3.31 \leq M \leq 2.13 \times 10^5$.

**Nomenclature**

- $C_D$ = drag coefficient, $(p g d^4 N d^2)/(6) (\rho u^2/2)(d^2/4) -$ [m$^2$]
- $D$ = column inside diameter [m]
- $d$ = equivalent spherical diameter of a single bubble [m]
- $Eo$ = Eötvös number, $g d^2/\sigma$ [m$^2$]
- $Eo_D$ = Eötvös number, $g d D^2/\sigma$ [m$^2$]
- $Fr_D$ = Froude number, $u/\sqrt{g D}$ [m/s$^2$]
- $g$ = acceleration of gravity [m/s$^2$]
- $h$ = maximum height of bubble [m]
- $M$ = Morton number, $\mu d^2/\rho \sigma^2$ [-]
- $Re$ = Reynolds number, $\rho u d/\mu$ [-]
- $u$ = rising velocity of a single bubble [m/s]
- $u_\infty$ = rising velocity of a single bubble in a liquid of infinite extent [m/s]
- $V_b$ = volume of a single bubble [m$^3$]
- $w$ = maximum width of bubble [m]
- $\theta$ = wake angle [rad]
- $\lambda$ = diameter ratio, $d/D$ [-]
- $\mu$ = liquid viscosity [Pa s]
- $\rho$ = liquid density [kg/m$^3$]
- $\sigma$ = liquid surface tension [N/m]

**Literature Cited**


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