The reported lateral dispersion of fluidized solids is accounted for by the bubbling bed model and an additional postulate which views solids being pushed aside by the rising bubbles, then drawn into their wakes and mixed there.

Brötz studied the lateral movement of fluidized solids by measuring the rate of approach to uniformity of two types of solids originally separated by a partition plate in a shallow rectangular fluidized bed. The mixing was described by a diffusion-type model with lateral dispersion coefficient $D_{sr}$.

Gabor and Mori and Nakamura used essentially the same approach and Table 1 shows the range of reported $D_{sr}$ values from these experiments.

In a bubbling fluidized bed the many rising bubbles are viewed to be the main cause for solids movement, pushing solids aside as they rise (lateral dispersion) and dragging a wake of solids up the bed (axial dispersion). For a given bed and operating conditions the recently proposed bubbling bed model describes all aspects of bed behaviour (gas movement, bubble frequency and velocity, etc.) in terms of one parameter, the effective bubble size $d_{b}$. Thus it should be possible to account for the reported axial and radial dispersion of solids in terms of the bubbling bed model. The present paper extends the relationship to the radial movement of solids and shows that the reported radial dispersion coefficients can be explained satisfactorily by the bubbling bed model.

**Relationship of the Bubbling Bed Model to the Radial Dispersion of Solids**

Consider a vigorously bubbling fluidized bed where the gas velocity entering the bed is $u_{0}$ and where the minimum fluidizing velocity is $u_{mf}$. Then the bubbling bed model relates the bubble velocity $u_{b}$, bubble fraction in the bed $\varepsilon$ and mean voidage in the bed $\varepsilon_{f}$ to the effective bubble size $d_{b}$ by

$$u_{b} = \frac{u_{0} - u_{mf}}{\varepsilon_{f}} = u_{0} - u_{mf} + \frac{1}{2} (u_{b} - u_{mf})^{2/3}$$

and

$$1 - \varepsilon_{f} = \left(1 - \varepsilon_{mf}\right) (1 - \delta)$$

The rising bubbles are surrounded by a cloud of circulating gas. The thickness of this cloud ring is approximated by

$$\delta = \frac{u_{b} (1 - \delta)}{2 u_{b} (1 - \delta)_{mf}}$$

Referring to Fig. 1, postulate a mechanism for the lateral mixing of solids as follows:

As a bubble rises it pushes aside the emulsion ahead of it. The solids close enough to the bubble enter its cloud and are drawn into the wake following the bubble. Complete mixing of solids occurs in the wake giving rise to a lateral movement. Solids further from the rising bubble are displaced somewhat as the bubble passes by and then return close to their original position. Lateral movement of these solids is negligible. This mechanism is closely related to the mechanism chosen to explain the axial dispersion of solids.

The Einstein random walk expression conveniently gives the dispersion coefficient for the lateral dis-

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Bed</th>
<th>Gas</th>
<th>Solids</th>
<th>$u_{0}$ [cm/sec]</th>
<th>Range of $D_{sr}$ [cm$^2$/sec] observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brötz (1956)</td>
<td>rectangular section</td>
<td>air</td>
<td>sand 0.2~0.5mm</td>
<td>1~2.2u_{mf}</td>
<td>0.1~3</td>
</tr>
<tr>
<td>Gabor (1964)</td>
<td>rectangular 17.8cm×4.62cm picked with steel spheres or brass cylinders</td>
<td>N$_2$</td>
<td>copper and nickel shot 0.088~0.33mm</td>
<td>u_{mf}/s=15.3~107</td>
<td>0.6~2</td>
</tr>
<tr>
<td>Mori and Nakamura (1965)</td>
<td>rectangular 90cm×30cm</td>
<td>air</td>
<td>polyvinyl chloride 0.555mm</td>
<td>u_{mf}=29.5</td>
<td>5~140</td>
</tr>
</tbody>
</table>

* Received on May 18, 1968
** Oregon State University, Oregon

JOURNAL OF CHEMICAL ENGINEERING OF JAPAN
placement of particles in terms of the mean square distance moved \((\Delta r)^2\). Thus for this two dimensional movement,

\[
D_{tr} = \frac{\text{fraction of solids which mix}}{\text{mean square distance moved}} \times \frac{\text{distance moved}}{\text{time interval considered}}
\]

\[
= \frac{1}{4} \left( \frac{\text{fraction of solids which mix per unit time}}{\text{time interval considered}} \right) \text{(4)}
\]

Let us evaluate the terms in this expression. First, the volume of emulsion entering the wake of a single bubble per unit time is that passing through the cloud at the major cross section of the bubble, or

\[
\pi d_b^2 \cdot \text{volume of emulsion entering per unit time (5)}
\]

Hence the fraction of bed solids entering bubble wakes and intermixing per unit time is

\[
\frac{\text{number of bubbles}}{\text{volume of emulsion entering in the bed}} \times \text{a wake per unit time}
\]

\[
= \frac{\delta V_{bed}}{V_b} \left( \frac{\pi d_b^2 \cdot \text{volume of emulsion}}{(1 - \delta) V_{bed}} \right)
\]

where

\[V_b = \frac{\pi}{6} d_b^3\]

Next by statistical considerations the mean square distance between any two points chosen at random within a circle of diameter \(d_s\) can be shown to be\(^{(13)}\)

\[(\Delta r)^2 = \frac{1}{4} d_s^2\]

Replacing Eqs.(1), (3), (6) and (7) in Eq.(4) gives for the lateral dispersion coefficient

\[
D_{tr} = \frac{3}{16} \left( \frac{\delta}{1 - \delta} \right) \frac{u_{mf} \cdot d_b}{\text{sec}}
\]

(8)

Interpretation of the Reported Experimental Data

From experiments in a shallow bed Gabor\(^{(2)}\) calculated \(D_{tr}\) values as reported in Table 2. To compare these values with predictions of the bubbling bed model a bubble size in the bed must be known. In the absence of such information let us choose a reasonable bubble size and see if the trends in calculated \(D_{tr}\) with a change in gas velocity follow the reported values. For Gabor's shallow bed choose \(d_b = 1.5\) cm. Then Eqs.(1), (2) and (8) give \(D_{tr}\) as a function of \(u_o\) as shown in Table 2.

In a similar manner Table 3 compares theory with experiments of Mori and Nakamura\(^{(6)}\) in larger beds. The reported bubble size falls in the range of values extracted from the model.

The change in calculated \(D_{tr}\) with \(u_o\) follows the observed trends.

Conclusion

When simultaneous measurements of \(D_{tr}\) and \(d_s\) become available then the postulate of this paper can be tested and modified accordingly. Until then the equation derived should not be used for predictive purposes but should be taken as suggestive of trends and illustrative of the type of analysis which is possible with the bubbling bed model.

Nomenclature

\[D_{tr} = \text{lateral dispersion coefficient of solids} \quad [\text{cm}^2/\text{sec}]\]

\[d_b = \text{effective bubble size} \quad [\text{cm}]\]

\[d_p = \text{particle size} \quad [\text{cm}]\]

\[g = \text{acceleration of gravity} \quad [\text{cm}/\text{sec}^2]\]

\[(\Delta r)^2 = \text{mean square lateral displacement of particles} \quad [\text{cm}]\]

\[\Delta t = \text{time increment} \quad [\text{sec}]\]

\[u_o = \text{velocity of rising bubbles} \quad [\text{cm}/\text{sec}]\]

\[u_{mf} = \text{velocity of rise of a bubble with respect to the bed solids ahead of it} \quad [\text{cm}/\text{sec}]\]

\[u_{mf} = \text{minimum fluidizing velocity} \quad [\text{cm}/\text{sec}]\]
DEAD ZONE PILED ON THE PLATE IMMERSED IN A FLUIDISED BED

SHIN-ICHI MAKISHIMA AND TAKASHI SHIRAI
Research Laboratory of Resources Utilization, Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo

Movements of solid particles were observed by photography around a fixed plate immersed in a two dimensional fluidised bed, and were found to be very complicated, making the boundary zone of solids on the plate different from the freely fluidised zone.

The height of the fixed dead zone in the boundary zone was studied experimentally with different conditions of gas velocity and gas distributor. Judging from those observed results, the fluidised bed should be treated as a fluid also having the property of solid beds.

I. Introduction

Reuter\(^1\) (1966) presumed that there might be a "boundary layer" around a fixed body immersed in a fluidised bed, judging from the existence of drag force upon it. Rietema\(^2\) (1967) has recently proposed the "mechanical stress theory" of fluidised beds and applied it to the extra pressure drop and the critical diameter of bubbles.

The movements of solid particles and gas bubbles around a body immersed in fluidised beds are often important for the analysis of the practical characteristics of fluidised beds, such as heat or mass transfer from the immersed surfaces and the distribution of stresses in fluidised beds.

The purposes of this paper are to investigate the inhomogeneity of the movements of solid particles and gas bubbles around an immersed body and further the distribution of stresses, whose results will give a different view of fluidised beds from the assumption previously prevailing that the fluidised bed was inviscid.

2. Experimental Apparatus and Procedures

The experimental apparatus was a two dimensional fluidised bed (9 mm width x 1000 mm height x 380 mm length) with two planes of 380 mm length (3 mm thick glass plates) and the planes (9 mm width walls of wood). The particles used were FCC catalyst, whose diameters ranged from 100 to 200 mesh, angle of repose was 29.4 degrees, minimum fluidisation velocity was 0.62 NTP cm/sec and the bulk density was 0.90 gr/cc. Air was supplied from underneath through a multiorifice plate distributor with 19 holes of 1 mm\(^2\) or filter-paper distributor.

The plate was fixed horizontally at the center of the solids bed and at the level of 0~200 mm height from the bottom. Then, white and black solid particles were charged in the apparatus layer by layer. After the solids bed was fluidised at a certain velocity, photographs of movements of solid particles were taken by a 6\(\times\)6 cm\(^2\) camera through the transparent glass wall, enlarged and analysed.

---

Received on June 7, 1968

2. Rietema, J. (1967)