ENTRAINMENT AND ELUTRIATION FROM
FLUIDIZED BEDS

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A simple model is proposed to account for the entrainment of solids and elutriation (selective removal) of fines from fluidized beds. This model fits the reported findings, provides physical meaning to the parameters of the reported experimental correlations, and indicates the pertinent factors controlling the removal of solids from vigorously bubbling beds.

Introduction

For design of fluidized beds we need to know the rate of entrainment of solids from a bed, and its variation with location of the exiting gas stream. This paper deals with these phenomena. Let us first define a number of terms used in connection with this aspect of fluidization.

A fluidization vessel usually consists of two zones, a dense bubbling phase having a more or less distinct upper surface separating it from a lean or dispersed phase. The section of the vessel between the surface of the dense phase and the exiting gas stream from the vessel is called the freeboard and its height is called the freeboard height, H. The purpose of the freeboard is to allow the solids to separate from the gas stream, and as its height is increased entrainment lessens. Eventually a freeboard height is reached above which entrainment becomes nearly constant. This is called the transport disengaging height, TDH.

For a freeboard height less than the TDH, the size distribution of solids in the freeboard changes with position. When the gas stream exits above the TDH then both the size distribution and entrainment rate become nearly constant and are given by the saturation capacity of the gas stream under pneumatic transport conditions. Elutriation refers to the selective removal of the fines from a mixture, and this may occur either below or above the TDH.

Previous Investigations

Representative investigations into the various aspects of entrainment and elutriation can be classified as follows:

a. Entrainment at or above the TDH based on the saturation carrying capacity of the gas stream; by Zenz and Weil16, Lewis et al.4, Blyakher and Pavlov23, Andrews19.

b. Entrainment below the TDH and the influence of the properties of the dense phase on this entrainment; by Zenz and Weil16, Lewis et al.4, Blyakher and Pavlov23, Andrews19.

c. Elutriation above and below the TDH; by Leva4, Osberg and Charlesworth7, Yagi and Aochi13, Thomas et al.10, Wen and Hashinger12, Sanari and Kunii15.

Let us discuss briefly the reported findings of these phenomena. Later these various aspects will be tied together with a simple model.

Entrainment at or above the TDH

Under entrainment conditions Zenz and Weil16 envisioned the bed as a saturation feed device such that the freeboard above the TDH is a pneumatic conveying tube for the transportation of solids. For a single size of solids experiments in pneumatic conveying show that there exists a maximum particle concentration which can be held in suspension by a flowing gas without collapse of the solids into a dense slugging mass. According to Zenz and Weil16 this limiting condition is given by a correlation which can then be used to estimate the entrainment rate $F_s$ [gms/sec] above the TDH. For a bed of fine solids of wide size distribution under flow conditions where $u_s > u_c$ for almost all sizes of solids, Zenz and Weil16 propose an approximate procedure to estimate entrainment.

Entrainment below the TDH

Bubbles of gas rise through the dense phase, erupt at the surface and project solids into the freeboard above. Assuming that the energy of particles at the surface of the dense phase follows the Maxwell-Boltzmann distribution, Andrews19 calculated that the entrainment rate should decrease exponentially with freeboard height.

In their study Zenz and Weil16 noted that the intermittent bursting action of bubbles causes sharp
velocity fluctuations just above the surface of the dense bed. These fluctuations dissipate with height and the gas velocity smoothes to the average velocity at the TDH.

Blyakher and Pavlov\(^2\) studied the effects of grids positioned in the freeboard of conical vessels. Their observations show that such grids effectively smooth out the velocity fluctuations above the dense bed, reduce entrainment at given freeboard height and sharply lower the TDH.

Lewis et al.\(^5\) made a thorough study of the complex interaction of the many variables on entrainment. Their experiments were limited to beds of narrow cuts of fine particles (11 sizes, 4 different materials) and they examined in particular the role of the dense phase on entrainment. Their findings are briefly summarized below and in Fig. 1.

For a given freeboard height, \(H\), the bulk density of dispersed phase decreases with height, and raising the freeboard increases the bulk density at any level. When the freeboard height is greater than the TDH or high enough so that entrainment is negligible then the bulk density at any level becomes maximum \(\rho_R\) and these conditions are referred to as complete reflux.

At complete reflux the bulk density at any level \(l\) above the dense bed is given by

\[
\rho_l = \rho_{R0} e^{-al}
\]  

(1)

where \(a\) is a constant and \(\rho_{R0}\) is the bulk density of lean phase just above the surface of the dense bed.

At conditions other than complete reflux Lewis et al.\(^\text{5,}\) found that the bulk density is some fixed value less than at complete reflux and that this is independent of level in the bed, thus

\[
\rho_l - \bar{\rho} = \text{constant throughout the freeboard}
\]

(2)

Qualitatively the rate of entrainment varies the same way as does the bulk density of the lean phase. Thus at given \(u_0\) the entrainment varies with freeboard height by

\[
F = \mathcal{F} e^{-aH}
\]

(3)

where \(\mathcal{F}\) is a constant whose significance is treated later and \(a\) is the same constant as in Eq.(1).

Examples of numerical values of \(a\) and \(\mathcal{F}\) are given in Table 1.

The overall effect of gas velocity and freeboard height on entrainment was correlated by Lewis et al. by

\[
\frac{F}{A_d u_0} = Be^{-(\log u_0 + a + H)} \quad [\text{gm/cm}^2]\]

(4)*

where \(B\) is given in Table 2 and

\[
b = 8.86 \times 10^3 \rho_{d5}^{1/2} \quad [\text{cm/sec}]
\]

Elutriation

Elutriation refers to the selective removal of fines by entrainment from a bed consisting of a mixture of particle sizes. Now, the rate of elutriation of solids of size \(d_p\) from a mixture is characterized by the net upward flux of this size of solid as follows,

\[
\text{Rate of removal of solids of size } \frac{dW(d_p)}{dt} = \kappa \frac{W(d_p)}{W} \quad [\text{gms/cm}^2\text{sec}]
\]

(5)

If \(W(d_p)\) is the weight of solids of size \(d_p\) and \(W\) is the weight of all solids in the bed, then Eq.(5) becomes, in symbols,

\[
\frac{dW(d_p)}{dt} = \kappa \frac{W(d_p)}{W}
\]

(6)

where \(\kappa\) (gms/cm\(^2\)sec) is an elutriation (rate) constant. In a mixture \(\kappa\) varies with size of solids; a large value of \(\kappa\) corresponds to a rapid removal rate for fine solids, and \(\kappa = 0\) means that the particular size of solid is not removed at all by entrainment.

The elutriation rate is also defined by

\[
\text{Rate of removal of solids of size } \frac{dW(d_p)}{dt} = \kappa \frac{W(d_p)}{W}
\]

(7)

* The original equation for Eq.(4), namely Eq.(11) in ref. 5, gives \(F = Be^{-(\log u_0 + a + H)}\). Since this contradicts Eq. (1) in ref. 5 which has \(Fe^{-aH}\), the original equation seems to have left out the square brackets. The corrected equation is shown here.

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Table 1  Numerical values of \(a\) and \(\mathcal{F}\) taken from Lewis et al.\(^5\)

<table>
<thead>
<tr>
<th>Solids</th>
<th>(a_{0.45} [\text{sec}^{-1}])</th>
<th>(\mathcal{F}/A_d [\text{gms/cm}^2\text{sec}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075mm glass</td>
<td>0.6-0.8</td>
<td>0.07-0.11 ((u_0=52) cm/sec)</td>
</tr>
<tr>
<td>0.070mm cracking</td>
<td>0.6-0.4</td>
<td>0.15 ((u_0=40) cm/sec)</td>
</tr>
</tbody>
</table>

* \(L_n\) is the height of static bed.

Table 2  Values of \(\mathcal{F}[\text{gms/cm}^2]\) in Eq.(4) from Lewis et al.\(^5\)

<table>
<thead>
<tr>
<th>Column diam.</th>
<th>cracking catalyst</th>
<th>glass spheres</th>
</tr>
</thead>
<tbody>
<tr>
<td>[cm]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>0.070mm</td>
<td>0.075mm</td>
</tr>
<tr>
<td>7.6</td>
<td>0.081</td>
<td>0.053</td>
</tr>
<tr>
<td>14.6</td>
<td>0.020</td>
<td>0.083</td>
</tr>
</tbody>
</table>

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Fig. 1 Density of lean phase as a function of level in the bed and the freeboard height
Comparing definitions we see that

\[ \kappa = \kappa^* \frac{A_1}{W} \]  

\( \kappa(\text{sec}^{-1}) \) is also called an elutriation constant. It varies proportionately with the sectional area of the bed and inversely with bed height, while \( \kappa^* \) is unaffected by these changes as long as the quality of fluidization remains the same. In most applications either constant can be used, however in certain steady operations dealing with particle growth or shrinkage \( \kappa \) is preferred since it can be used directly in the governing population balance equations.

An underlying assumption of Eq.(5) requires independent behavior of the different particle sizes of a mixture. That this is reasonable is suggested by the experiments of Sanari and Kunii\(^\text{9)}\), who found that changing the coarse had no appreciable effect on the entrainment of the fines. Fig. 2 illustrates these findings.

Yagi and Aochi\(^\text{13)}\) and Wen and Hashinger\(^\text{11)}\) both suggested that the velocity factor should be the difference between the superficial gas velocity and the terminal velocity of fine particle \( u_t \). Note both correlations do not include explicitly any term relating to the coarse size. Elutriation has been studied in batch systems\(^\text{4,7,10,11,13)}\) and in steady flow systems\(^\text{8,10)}\).

A Model for the Entrainment of Solids from Dense Fluidized Beds

Lewis et al.\(^\text{5)}\) describe the entrainment process in the following terms. Bursting bubbles of gas project agglomerates of particles into the space above the bed, and as the air flow rate is increased this action becomes more violent with agglomerates projected successively higher into the freeboard. These agglomerates are frequently broken up to form a dispersed phase as well as streams of particles in seeming random motion. Some particles are seen to move upward, others downward.

This behavior is understandable if we note that measurable entrainment occurs only when \( u_t > u_{mf} \). These are conditions where practically all the gas passes through the bed in large bubbles whose velocities are considerably in excess of \( u_t \). These energetic intermittent bursts of gas cause the initial dislodgement of solids and the subsequent breakup of agglomerates.

With this picture as a basis let us develop a rather simple model to account for the various aspects of entrainment from a dense bubbling bed. This is illustrated in Fig. 3. First consider one size of solid being fluidized.

Postulate 1. Three distinct phases are present above the dense phase (in the freeboard)

phase 1: gas stream with completely dispersed solid. These solids are transported pneumatically with velocity \( u_t \).

phase 2: projected agglomerates moving upward with velocity \( u_t \).

phase 3: descending agglomerates and parcels of thick dispersion moving downward with velocity \( u_t \).

Postulate 2. At any level in the bed the rate of dissipation of agglomerates to form dispersed solid of phase 1 is proportional to the concentration of agglomerates of solids at that level.

Postulate 3. Upward moving agglomerates occasionally reverse direction and move downward, and the frequency of this change from phase 2 to phase 3 at any level is proportional to the solids concentration in phase 2 at that level.

If we let \( \phi_1, \phi_2, \phi_3 \) (gms/sec) be the mass flow rate of each phase and \( C_1, C_2, C_3 \) (gm/cm\(^3\)) be the weight of each phase in unit volume of freeboard, then the net upward flow of solids \( F \) is

\[ F = \phi_1 + \phi_2 - \phi_3 \]

where

\[ \phi_1 = A_1C_1u_t, \quad \phi_2 = A_2C_2u_t, \quad \phi_3 = A_3C_3u_t \]

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Also the average concentration of solids or bulk density \( \bar{\rho} \) at any level is

\[
\bar{\rho} = C_i + C_j + C_k
\]

(12)

Now, a mass balance for phase 1 gives

\[
\begin{align*}
\text{(increase of solids)} &= \text{(transfer of solids from phases 2 and 3 into 1)} \\
\text{(decrease of solids)} &= \text{(transfer of solids from phase 2)} \\
\end{align*}
\]

for phase 2

\[
\begin{align*}
\text{(increase of solids in phase 3)} &= \text{(transfer of solids from phase 2 into 3)} \\
\text{(decrease of solids)} &= \text{(transfer of solids from phase 3 to 1)} \\
\end{align*}
\]

To write these mass balances in symbols introduce rate coefficients to apply to postulates 2 and 3, namely \( K \) (sec\(^{-1}\)) for the transfer from phases 2 and 3 to 1, and \( K^* \) (sec\(^{-1}\)) for the transfer from phase 2 to 3. Then the above three mass balances become

\[
\begin{align*}
\frac{dC_1}{dt} &= K(C_i + C_j) \\
- \frac{dC_2}{dt} &= (K + K^*)C_i \\
\frac{dC_3}{dt} &= K(C_j - K^*)C_i
\end{align*}
\]

(13)

At the freeboard height, \( H \), the gas stream leaves the vessel, so there should be no downward flow, \( \mathcal{Z}_3 = 0 \) hence

\( C_3 = 0 \) at \( l = H \)

(14)

Letting the flow rate of solids projected from the bed surface be \( \mathcal{Z}_3 \) we then have at this level

\( C_i = 0 \) and \( C_j = \mathcal{Z}_3/A_i \) at \( l = 0 \)

(15)

Solving Eq.(13) with the boundary conditions of Eqs.(14) and (15) gives

\[
1 - F_i/\mathcal{Z}_3 = 1 - e^{-aH}
\]

(16)

where

\[
F_i = \frac{\left(1 + \frac{u_1}{u_2}\right)K}{1 + \left(1 + \frac{u_1}{u_2}\right)K^*}
\]

(17)

\[
a = \frac{K^*}{u_1} \left[1 + \left(1 + \frac{u_1}{u_2}\right)K^*/K^*\right]
\]

(18)

and \( F_i \) is the mass flow rate corresponding to the saturation capacity of the flowing gas stream.

For conditions of normal entrainment much solid is projected from and returns to the bed, thus

\[
F_i/\mathcal{Z}_3 \ll 1
\]

(19)

hence from Eq.(17)

\[
F_i/\mathcal{Z}_3 \approx \left(1 + \frac{u_1}{u_2}\right)K^*/K^*
\]

(20)

For the freeboard height \( H \) appreciably less than the TDH Eq.(16) reduces to

\[
F = \mathcal{Z}_3 e^{-aH}
\]

(21)

\[
a = \frac{K^*}{u_2} \left(1 + \frac{F_i}{\mathcal{Z}_3}\right) = \frac{K^*}{u_2}
\]

(22)

This expression provides a physical interpretation of the parameters \( \mathcal{Z}_3 \) and \( a \) reported by Lewis et al.\(^5\) in Eq.(3), and also suggests how they may be affected by the gas flow rate. Thus if it is reasonable to take \( u_2 \) proportional to \( u_2 \) then

\[
a \approx \frac{1}{u_2} \text{ or } au_2 \equiv \text{constant}
\]

(23)

The above relation may justify the experimental findings of constant \( au_2 \) for changing \( u_2 \) reported by Lewis et al.\(^5\).

Also comparing Eq.(4) to Eq.(21) gives

\[
\mathcal{Z}_3 = A_i u_2 B e^{-aH/(u_2)^2}
\]

(24)

This model also gives the bulk density of solids in the freeboard. Thus for fluidizing conditions where \( F_i/\mathcal{Z}_3 \ll 1 \) the concentration at any height for conditions of total reflux can be found to be

\[
\rho_0 = \frac{\mathcal{Z}_3}{A_i} \left(1 + \frac{1}{u_2} e^{-aH}\right)
\]

(25)

For a height of freeboard \( H \) the average concentration or bulk density of solid at all levels \( l \) is related to \( \rho_0 \) at that same level by

\[
\rho_0 = \frac{\mathcal{Z}_3}{A_i} \frac{1}{u_2} e^{-aH}
\]

(26)

These two equations are of the same form as Eqs. (1) and (2) which are given by Lewis et al.\(^5\). In particular Eq.(26) indicates that throughout the freeboard the concentration of solids is a fixed amount less than the corresponding value at infinite reflux.

**Application of the Entrainment Model to Elutriation Phenomena**

Extending this model to a binary system and assuming the weight fraction of fines in the projected agglomerates \( x_i \) is the same as in the bed, Eq.(16) is modified to give

\[
\frac{(F_i)_{\text{elas}}}{F_i} = 1 - \left[1 - \frac{F_i}{\mathcal{Z}_3}ight] (1 - e^{-aH})
\]

(27)

On the other hand Eq.(5), written for the one size being elutriated, becomes

\[
(F_i)_{\text{elas}} = \kappa^* A_i X_i
\]

(28)

Combining Eqs.(27) and (28) gives

\[
\kappa^* = \frac{\mathcal{Z}_3}{A_i} \left[1 - \left(1 - \frac{F_i}{\mathcal{Z}_3}\right)(1 - e^{-aH})\right]
\]

(29)

For very large height of freeboard, Eq.(29) reduces to

\[
\kappa^* = \frac{F_i}{A_i X_i}
\]

(30)

while for conditions of normal entrainment with insufficient height of freeboard, namely for \( \mathcal{Z}_3 \gg F_i \), Eq.(29) becomes

\[
\kappa^* = \frac{\mathcal{Z}_3}{A_i} e^{-aH}
\]

(31)

In all the previously reported elutriation experiments freeboard heights were used which were thought to be sufficiently large for this factor not to influence the elutriation rate. If this assumption were true then according to Eq.(30) the observed mass flux of fines would correspond to the saturation carrying capacity of the gas stream, and \( \kappa^* \) would change inversely proportional to the weight fraction of fines in the bed, or \( X_i \). Fig.4 reported by Wen and Hashinger shows that this assumption is approached in beds consisting primarily of fines, \( X_i > 0.4 \).

On the other hand practically all the findings when
X<0.2 show $\kappa^*$ and $\kappa$ to be independent of $X_s$. This suggests that the freeboard in those experiments was not high enough to eliminate this effect. Thus the reported findings may be expected to be higher than the saturation carrying capacity of the gas stream, or the minimum elutriation expected, and the variation of the elutriation rate with freeboard height may be expected to be approximated by Eq.(31). In this range of conditions any study of elutriation should include the freeboard height as a variable in addition to the other fluidizing conditions normally considered.

Conclusion

A simple model suggested by observation is shown to tie together diverse phenomena related to the removal of solids from fluidized beds such as entrainment and elutriation from fluidized beds, the density of solids in the freeboard, the variation of all these quantities to the freeboard height, the role of the TDH and the saturation carrying capacity of the fluidizing gas.

Eqs.(16)~(18) of this model provide a physical interpretation for the parameters obtained from experimental correlations on entrainment, and also suggest how variations in experimental condition may affect the entrainment.

Finally, Eq.(27) gives physical meanings to the elutriation constant, and shows that it is closely related to the mass flux of total solids projected from the bed as well as the height of freeboard.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>sectional area of bed [cm$^2$]</td>
</tr>
<tr>
<td>$a$</td>
<td>constant defined by Eq.(3) [cm$^{-1}$]</td>
</tr>
<tr>
<td>$C_1, C_2, C_3$</td>
<td>the weight of phase 1, 2 and 3 respectively in unit volume of freeboard [gm/cm$^3$]</td>
</tr>
<tr>
<td>$F$</td>
<td>net upward flow rate of solids [gm/sec]</td>
</tr>
<tr>
<td>$F_s$</td>
<td>saturation carrying flow rate of fine solids [gm/sec]</td>
</tr>
<tr>
<td>$G_0$</td>
<td>flow rate of solids projected from the dense bed surface [gm/sec]</td>
</tr>
<tr>
<td>$G_1, G_2, G_3$</td>
<td>mass flow rate of phase 1, 2 and 3 respectively [gm/sec]</td>
</tr>
<tr>
<td>$H$</td>
<td>height of freeboard [cm]</td>
</tr>
<tr>
<td>$l$</td>
<td>distance from the dense bed surface [cm]</td>
</tr>
<tr>
<td>$u_0, u_{s0}, u_t$</td>
<td>superficial gas velocity based on sectional area of bed, minimum fluidization velocity and terminal velocity respectively [cm/sec]</td>
</tr>
<tr>
<td>$u_1, u_2, u_3$</td>
<td>velocity of solids in phase 1, 2 and 3 [cm/sec]</td>
</tr>
<tr>
<td>$W$</td>
<td>weight of dense bed [gm]</td>
</tr>
<tr>
<td>$X_i$</td>
<td>weight fraction of fines in dense bed</td>
</tr>
<tr>
<td>$\kappa^*$</td>
<td>elutriation constant defined by Eq.(7) [sec$^{-1}$]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>elutriation constant defined by Eq.(5) [gm/cm$^2$-sec]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>bulk density of solids at any level in the freeboard [gm/cm$^3$]</td>
</tr>
<tr>
<td>$\rho^*$</td>
<td>bulk density of solids for conditions of total reflux [gm/cm$^3$]</td>
</tr>
</tbody>
</table>

Literature cited

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