SIMULATION OF SPATIOTEMPORAL BUBBLE BEHAVIOR IN A FREELY BUBBLING THREE-DIMENSIONAL FLUIDIZED BED

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Key Words: Reactor Design, Fluidized Bed, Bubble Behavior, Simulation Modelling, Coalescence

Based on experimental observations, some of which were carried in our laboratory, a simple computer simulation model or a simulator to predict the spatiotemporal behavior of bubbles in freely bubbling three-dimensional gas-fluidized beds of Group B particles is developed. In this simulator a few criteria have superseded the more sophisticated models for the treatment of interaction and coalescence of bubbles and hence it requires less computational effort. For both laboratory- and large-scale fluidized beds the spatial distribution of bubble diameter as well as that of bubble frequency at any level above the distributor and at various superficial gas velocities were satisfactorily estimated from qualitative and quantitative standpoints. The operation of the simulator was extended to predict the distribution of the fluidizing gas between the continuous and discontinuous phases and the change of coalescence frequency with the height above the distributor. Good agreement with related data reported in the literature was obtained as well.

Introduction

Simulation is the art by which a computer-aided examination can be performed for countless applications. For the simulation of a system its features are represented by a model. In the model there are always some parameters. Their values can be determined either by corresponding experimental studies or by theoretical analyses but they must ultimately be evaluated by experimental observations. The simulation can be extended to verify the effect of some parameters on the behavior of the system and/or to predict the behavior of some properties that are difficult to measure experimentally.

It is already accepted that the global behavior of fluidized beds is mainly influenced by the spatiotemporal behavior of the bubble phase. For simulating the behavior of a freely bubbling fluidized bed, computer simulation models or simulators had been developed.\(^4,12,16,17\) Based on the assumptions given by Clift and Grace,\(^2\) Nguyen et al.\(^17\) introduced a simulation model in which the effect of each bubble on all other bubbles was considered. Farrokhahalee \textit{et al.}\(^4\) simplified the approach used by Nguyen \textit{et al.}\(^17\) and the bubbles were assumed to grow as they rose at a constant rate and also by coalescence. Horio \textit{et al.}\(^12\) based on the treatment given by Clift and Grace,\(^3\) which is more general than their previous one,\(^2\) developed a simulation model in which the mutual effect between the lower and the upper bubbles was also considered. All these models are based on the effect of bubble interaction on the movement of bubbles. Recently, Mori \textit{et al.}\(^16\) introduced a steady-state model to simulate the time-averaged spatial distribution of bubbles. However, these reports did not give the actual spatial distribution of bubble size or that of bubble frequency, although their actual distribution gives insightful understanding of the behavior of the bed.

The prediction of bubble behavior is the first stage
of the more general case for the prediction of the performance of fluidized-bed reactors. All the previously mentioned models are difficult to extend to reactor modelling, as it includes a set of equations, simple or not, that have to be solved. Also, the numerical manipulation is a time consumer. From this viewpoint, a simpler model will be appreciated, especially when it can approximate the observed spatiotemporal behavior of bubbles. Then it can easily be extended to the reaction case. In this paper a simple computer simulation model that can predict the spatiotemporal bubble behavior in laboratory- and large-scale freely bubbling fluidized beds is introduced.

1. Simulator Basis and Structure

Figure 1 shows a flow chart of the simulator. The main subroutines are outlined in the following subsections. Meanwhile, the simulator was programmed by TIT (The Intelligen T)13) and executed by the microcomputer PC 9801-F2 (NEC, 128 K bytes RAM).

1.1 Initialization

In this subroutine the operating conditions are given to the computer. Then the jet height \( h_j \), equivalent bubble diameter \( D_{Bi} \), and frequency \( f_{Bi} \) for successive initial bubble generation in the whole bed can be calculated.

It is already known that the initial bubbles are generated at a distance above the distributor plate. For a perforated plate this distance is the jet height, which depends on the manner in which gas is distributed. The equivalent bubble diameter and the jet height are calculated by the equations proposed by Khattab and Ishida.15) The equivalent bubble diameter is then converted to the diameter of a realistic cap-shaped bubble with \( D_B = 1.5D_{Bi} \), on volume basis. The frequency of the initial bubble is calculated from the equation

\[
f_{Bi} = A(U_0 - K_iU_{mf})/V_{Bi}
\]

where \( K_i \) is the leakage factor at the jet region, whose value is specified by the user. Grace and Clift7) summarized most of the studies in which the visible bubble flow rate was directly measured or estimated. The value of \( K_i \) was found to be larger than unity, especially in the jet region.15,18,21) This was also confirmed by direct measurement of the interstitial gas velocity.14) However, the value of the leakage factor differs from system to system and within the system itself.7) To ascertain a value of \( K_i \) is difficult as it depends on many factors. But the results of Pyle et al.18) for Ballotini 15 \( U_{mf} = 1.2 \text{ cm/s} \) and that of Werther21) for sand particles \( U_{mf} = 1.8 \text{ cm/s} \) showed a leakage of about 40\% of the superficial gas flow in the jet region. Also, Grace in a recent review9) of simulation methods and reactor models stated that more accurate predictions are obtained if one chooses leakage equal to 70\% of \( (U_0 - U_{mf})A \) for the Group B particles. Then 40\% leakage, i.e. \( K_iU_{mf} = 0.4U_0 \), is applied to the beds of sand and glass beads in the jet region based on the experimental study presented in a previous paper.15) The results of that study indicated that the leakage in a bed of alumina particles is less than that for sand or glass beads even when their minimum fluidization velocities are in close range and bed size is the same. Hence 30\% leakage is assumed for alumina particles. At present, the effect of particle properties on the value of \( K_i \) is not clear. Nguyen et al.17) and Farrokhlaee et al.4) assumed about 60\% leakage, while Horio et al.12) and Mori et al.16) neglected the leakage and assumed the application of the simple two-phase theory. The consideration of leakage is quite important. Otherwise we cannot simulate the bubble frequency nor its change with height above the distributor. The leakage may also have an important role in reactor simulation, because quite a lot of gas goes to the emulsion phase at the bottom of the bed, where good contact between gas and solid is achieved.

The time interval \( \Delta t_i \) between the formation of two successive initial bubbles is calculated as \( 1/f_{Bi} \). In the operation of the simulation a time increment \( \Delta t \) was chosen as \( z\Delta t_0 \), where \( z \) is an integer, so that the initial bubble will rise about one-fifth of its equivalent diameter in this time increment. Multiplication by \( z \) is just to decrease the execution time, and it was confirmed that this has no effect on the data to be predicted. Therefore, a number of bubbles equal to \( z \) will be generated simultaneously at the beginning of each time increment.

1.2 Bubble generation

The bubbles are generated from the holes of the distributor randomly at a certain height specified by the jet height. The location of a bubble \( (x_i, y_i, z_i) \) is
that of the center of its equivalent sphere. Location, diameter and volume of each bubble existing in the bed can be stored in a data buffer in the computer memory. This data buffer allows easy access, exchange or erasure of the data.

1.3 Bubble rising

By this subroutine all the bubbles are allowed to rise with a velocity corresponding to their size and shape. Hatano et al.\textsuperscript{10} distinguished the three kinds of bubbles that exist in a freely bubbling bed, i.e., normal bubble, accelerated bubble and slug. The normal bubble is a bubble-cap shaped one, while the accelerated bubble is an elongated one with aspect ratio larger than unity and the slug is a bubble which covers the bed diameter. The average of the bubble rising velocity coefficients \( k_B \) for these kinds were found to be 0.71, 2 and 0.35 respectively. Consequently a flag is introduced to identify the shape of the bubbles in the data buffer. Here, a coefficient of 0.75 is taken for normal bubbles, half that value for slugs and double it for accelerated bubbles. The new height of a bubble is given by the equation

\[
z + k_B g D_B^0.5 \Delta t
\]

The intersection of the bubbles with the wall of the bed is checked by a sub-subroutine. Bubbles are shifted toward the axis of the bed until their surfaces touch the inner surface of the wall. Although this assumption seems to affect the bubble behavior, our examination showed that the bimodal behavior of bubble diameter and frequency discussed in section 2 can be obtained without this assumption. However, based on the experimental observation presented in a previous paper\textsuperscript{10} we set this assumption.

1.4 Bubble order and bubble eruption

Since the coalescence between bubbles takes place and rising velocities of bubbles change with the increase in their diameters, the data of bubbles in the data buffer must be renewed and rearranged according to their heights at the end of each time increment. Meanwhile, bubble data are printed out as the bubble’s center passes a level at which the behavior of the bubbles has to be determined. The data of a bubble are erased from the data buffer as its center passes the bed level.

1.5 Interaction and coalescence of bubbles

Interaction between an isolated pair of bubbles has been studied theoretically and experimentally,\textsuperscript{2,3,20} and was also observed in a freely bubbling bed.\textsuperscript{10} It was found that the upper bubble strongly affected the movement of the lower one, while the upper was insignificantly affected by the lower. Toei et al.\textsuperscript{20} reported vertical critical spacing beyond which interaction and coalescence do not occur. In the analysis of Grace and Clift\textsuperscript{2,3} any pair of bubbles more or less affect the movement of each other and coalescence will occur after a period of time depending on the relative bubble sizes. A criterion analogous to that of Toei et al.\textsuperscript{20} was used in the present simulation model to check interaction and coalescence. In Fig. 2(a) there are two vertically aligned bubbles and two ranges are defined as

\[
\begin{align*}
0.75 D_{Bu} < Z < 1.5 D_{Bu} \\
0.75 D_{Bu} > Z > 0
\end{align*}
\]

where \( Z \) is the vertical distance between the centers of the upper and lower bubbles. For non-vertically aligned bubbles, shown in Fig. 2(b), a critical horizontal distance \( X \) is considered in addition to the previous conditions. \( X \) is the only parameter that has to be determined by comparing the predicted data with the experimental ones.

In the acceleration range the velocity of the lower bubble is taken to be twice its normal velocity. This is in agreement with the data presented by Toei et al.\textsuperscript{20} and is not far from the analysis given by Clift and Grace,\textsuperscript{2,3} especially when the elongation of the lower bubble is considered. In addition, the condition for coalescence, \( 0.75 D_{Bu} \), means that the nose of the lower bubble moves a reasonable distance in the wake of the leading one before coalescence. On the other hand, when the upper bubble is fairly smaller than the lower one and the horizontal distance \( X \) is greater than its critical distance \( X_c \), interaction and coalescence do not occur. Instead, the lower bubble rises faster and the relative movement of the upper will be downward, shown by the curved path in Fig. 2(c) as explained by Rowe.\textsuperscript{19} In the present simulation such relative movement is simplified by the path shown as the dashed line.

In the coalescence range, the lower bubble is assumed to be suddenly absorbed by the upper one. Unlike Horio et al.,\textsuperscript{12} the position of the composite bubble is that of the upper one just for simplicity. Botterill et al.\textsuperscript{1} observed that on coalescence the volume of the composite bubble was not the simple sum of the original volumes. Grace and Venta\textsuperscript{8} investigated this phenomenon and reported a representative figure of their data, Fig. 1 in their paper,\textsuperscript{8} for the change in volumes of bubbles during coalescence. In their report, the maximum volume achieved is 10 to 20\% for non-vertically aligned bubbles and 2 to 14\% for vertically aligned bubbles in excess of the final volume of the composite bubble. Assuming that point A in their figure,\textsuperscript{8} in which this maximum is 20\%, is the initial sum of the two bubble volumes, this maximum is 31\% in excess of the initial sum and then the final composite volume becomes around 9\% in excess of the initial sum. Hence, by considering the ranges they reported and to use one value for all
cases, 6% volume increase is assumed in the jet region in the present work. Werther\(^{21}\) deduced that rising bubbles effectively degas the dense phase and at higher bed levels, depending on the bed diameter, the gas content of the dense phase finally corresponds to that at \(U_{mf}\). Hence the percentage of volume increase \(P\) in Eq. (3) may be represented by Eq. (4):

\[
V_{\text{comp}} = (V_1 - h V_2)(1 - hP) \quad (3)
\]

\[
\beta = 0.06(K - 1)/(K_1 - 1) \quad (4)
\]

where \(V_{\text{comp}}\) is the volume of the composite bubble generated by the coalescence of the two bubbles 1 and 2. \(K_1\) is the leakage factor in the jet region and \(K\) is that at any level. The above equations indicate that the bubble flow rate is not conserved during the rise of bubbles. Since this rate is represented by \(A(U_o - KU_{mf})\), the value of \(K\) can be calculated from the bubble data in this model. This value of \(K\) will change the value of \(\beta\) by Eq. (4). For simplicity, calculation of \(K\) is performed at 0.1 m height intervals above the distributor, because calculation of the bubble flow rate is time-consuming. Between two levels \(K\) is assumed to be constant and has the value at the lower level. Nguyen \textit{et al.}\(^{17}\) assumed a volume increase of 15% without checking whether the emulsion phase is defluidized or not. Horio \textit{et al.}\(^{12}\) and Mori \textit{et al.}\(^{16}\) assumed the application of the simple two-phase theory and hence there was no bubble volume increase, while, Farrokhalaee \textit{et al.}\(^{4}\) assumed that the bubble volume increases at a rate consistent with their correlation.

As in other studies,\(^{4,12,16,17}\) splitting was not treated here. Splitting is common in particles of group A as classified by Geldart.\(^5\) This model for aggregative fluidized beds cannot be applied to extremely high-pressure beds, where the interface between bubbles and emulsion phases becomes unclear.

2. Application of the Simulator

2.1 Determination of horizontal critical distance \(X_c\)

The present computer simulation model was applied to predict the spatiotemporal behavior of bubbles for the following two cases. The first is a previous study in our laboratory\(^{10}\) and the second is that of GOLFERs.\(^6\) To define \(X_c\), comparison with the data of the first case has been made since it provides the spatial distribution of actual bubble sizes at different bed levels for various superficial gas velocities \(U_o\).

In Fig. 3 the predicted bubble-diameter distribution densities for different values of \(X_c\) are compared to those observed experimentally at \(U_o = 8 U_{mf}\) and at \(L_d = 0.5\) m. When \(X_c\) is less than 0.75 \(D_{bu}\), the distribution gives smaller average bubble diameter than that of the experimental data. For \(X_c = 0.8 D_{bu}\), most of the bubbles are large, giving a larger average bubble diameter. Meanwhile, the distribution for \(X_c = 0.75 D_{bu}\) is close to the experimental one. The same conclusion was also obtained for other cases. The spatial distribution of two kinds of frequency, \(f_B\) and \(f_{Bc}\), for the cases given in Fig. 3 are shown in Fig. 4. As defined in a pervious paper,\(^{10}\) \(f_B\) is the frequency detected by each probe while \(f_{Bc}\) is the frequency detected by each probe for the bubbles whose centers passed that probe. When \(X_c = 0.75 D_{bu}\) the distribution is in good agreement with that experimentally observed, which is also shown in Fig. 4. However, the higher values predicted for \(f_B\) and \(f_{Bc}\) at \(X_c\) smaller than 0.75 \(D_{bu}\) are due to the presence of many small or medium-sized bubbles, while, when \(X_c\) becomes 0.8 \(D_{bu}\) larger values of \(f_B\) were obtained since most of the bubbles are large and will be counted by many probes. Based on such figures, the criterion \(X_c = 0.75 D_{bu}\) is a reasonable assumption, especially when it is compared with the overlap region which results from the analysis of Clift and Grace.\(^{3}\) This criterion is used in all cases of the following applications.

2.2 Simulation of a laboratory-scale fluidized bed

Bed materials and fluidization conditions of the system are given in a previous paper.\(^{10}\) In all cases the simulation was allowed to proceed for a 12-s fluidization period. The data in the first 2 s were discarded to avoid the effect of startup. The rest was found to be a satisfactory period for obtaining reproducible behavior. Meanwhile, the sampling period of the observed data was 4.96 s.

1) Sand particles In Fig. 5(a) and (b) comparison between the predicted and the observed bubble diameter distribution at various gas velocities and heights is shown. Hirama's equation\(^{11}\) for the mean bubble diameter is represented by broken lines to facilitate the comparison. For \(U_o = 3 U_{mf}\) there are many bubbles whose diameters are from 2 to 4 cm and relatively larger bubbles rise in the region between the wall and the bed axis, especially at low values of \(L_d\), giving rise to bimodal distribution. This tendency is simulated quite well. For larger superficial gas velocities \(U_o\) coalescence takes place more frequently,
giving rise to bubbles of larger diameters, which at higher levels rise around the bed axis. Also at higher gas velocities the predicted bubble diameters are in consistent agreement with the observed data.

The flow of bubbles at lower bed levels generally assumes bimodal behavior and the flow at higher levels shifts to monomodal behavior. In the literature, the distributions of bubble frequency in the previous simulators were not discussed or were not simulated satisfactorily. Meanwhile, Figs. 6(a) and (b) indicate that such transition can be simulated successfully. This is achieved by taking into account the initial leakage factor $K_t$ and the changes in the leakage factor $K$ with height above the distributor.

2) Alumina beads The spatiotemporal behavior of bubbles in a bed of alumina beads ($U_{mf} = 2 \text{ cm/s}$) was predicted at a superficial gas velocity $U_g = 5 U_{mf}$ and compared to the observed behavior in Fig. 7. At the lowest level, $L_d = 0.1 \text{ m}$, the behavior of bubbles is such that there are many small bubbles and the frequency distribution is bimodal. These bubbles, due to coalescence, grow in size and rise around the bed axis at higher bed levels, giving rise to a monomodal distribution of the bubble frequency, $f_B$. However, the frequency $f_{Bc}$ was continually decreasing.

3) Glass beads The simulator was also applied to predict the behavior of bubbles in a bed of glass beads ($U_{mf} = 2.5 \text{ cm/s}$). Consistent agreement between the observed and the predicted data was obtained, as shown in Fig. 8.

2.3 Simulation of a large-scale fluidized bed

GOLFERs had presented the spatiotemporal behavior of bubbles in a $1 \times 1 \text{ m}$ bed of sand particles ($U_{mf} = 3.5 \text{ cm/s}$) at different superficial gas velocities. The simulator was applied to this case as it was exactly applied to the laboratory-scale fluidized bed. The bubble height, $D_B/1.5$, and the bubble frequency, which is analogous to $f_B$, were predicted at the same positions at which the experimental observations were carried out. At each position for certain values of $U_g$ and $L_d$, a range of bubble diameters, then heights, were obtained. Hence, the arithmetic average was taken to represent the bubble height at that position. There was no significant difference between this average and the average obtained on volume basis.

A comparison between the predicted data and the observed data are given in Figs. 9 and 10. Closed keys show the observed data while open keys show the predicted ones. A general review of the figures indicates consistent agreement between them. Careful inspection, of course, shows some deviation which lies within allowable experimental or simulation errors.

2.4 Simulation extension

Since both the bubble diameter and frequency distributions are simulated satisfactorily, this simulator was extended to predict the change of the...
following two properties with height above distributor. The first is the change of coalescence frequency, defined as the number of coalescences per unit time and unit local bed volume. The second is the change in the leakage factor $K$.

The change in coalescence frequency with height above distributor for sand particles in the bed of 0.15 m in diameter is given in Fig. 11. The coalescence frequency has its maximum at a distance 7 to 10 cm above the distributor depending on $U_0$, and this maximum increases with increasing $U_0$. However, at $L_d > 0.2$ m it decreases significantly. This is in agreement with the data reported by Botterill et al.\(^1\) about the change in bubble frequency with height above the distributor. At $L_d > 0.25$ m the coalescence frequency has relatively smaller values for higher $U_0$, this being due to the increased number of coalesced bubbles in

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**Fig. 6.** Bubble frequency distribution for sand particles at $5U_{mf}$.

**Fig. 7.** Spatial distribution of bubble diameter and frequency for alumina particles at $5U_{mf}$.

**Fig. 8.** Spatial distribution of bubble diameter and frequency for glass beads.

**Fig. 9.** Spatial distribution of bubble height for GOLFERs runs.

**Fig. 10.** Spatial distribution of bubble frequency for GOLFERs runs.
the jet region at higher $U_o$ which leads to smaller number of bubbles, for higher $U_o$, at the upper levels.

In Fig. 12 the change in the leakage factor $K$ with height above the distributor is illustrated. The original values are represented by open keys at the corresponding jet heights. The predicted gas distribution between the emulsion and bubble phases changes with height above the distributor. The emulsion phase continues to yield up gas to the bubble phase until finally the gas content in the emulsion phase is that at minimum fluidization. This change proceeds faster at higher superficial gas velocities. This is in accordance with the observation reported for Ballotini 15 by Pyle et al. 18)

In this study, the leakage factor $K$ indicating the ratio of interstitial gas velocity to that at $U_{mf}$ is obtained by the present model and the experimental data for the distributions of bubble diameter and frequency. However, the predicted values of $K$ are so large that the effect of $K$ should be included in the reactor models. So far, there are no reactor models for the fluidized bed in which this effect is taken into consideration. At the same time, to ascertain these values of $K$ by some direct measurement is essential for understanding the behavior of fluidized beds.

**Conclusion**

A fairly simple computer simulation model for predicting spatiotemporal bubble behavior in freely bubbling three-dimensional fluidized beds was developed. Although particle movement was not considered, the simulation satisfactorily predicted the bubble-behavior distribution for both laboratory- and large-scale beds, without need of sophisticated computation means and techniques. The simulation model was also applied to predict the change in the coalescence frequency as well as the distribution of the gas between the bubble and emulsion phases along the height above distributor.

**Fig. 12.** Change in leakage factor with height above distributor.

**Nomenclature**

- $A$ = bed cross-sectional area [m$^2$]
- $D_b$ = bubble diameter [m]
- $D_{bi}$ = initial bubble diameter [m]
- $D_h$ = bubble height [m]
- $D_{bl}$ = diameter of lower bubble [m]
- $D_{bu}$ = diameter of upper bubble [m]
- $f_b$ = bubble frequency detected by each probe [s$^{-1}$]
- $f_{bc}$ = bubble frequency detected by each probe for bubbles whose centers passed that probe [s$^{-1}$]
- $f_{bi}$ = initial bubble frequency [s$^{-1}$]
- $g$ = gravitational acceleration [m/s$^2$]
- $h_j$ = jet height [m]
- $K$ = leakage factor
- $k_b$ = bubble rising velocity coefficient
- $L_A$ = height above distributor [m]
- $r/R$ = dimensionless radius
- $\Delta t$ = time increment [s]
- $\Delta t_i$ = $1/f_{bi}$ [s]
- $U_{mf}$ = minimum fluidization velocity [m/s]
- $U_o$ = superficial gas velocity [m/s]
- $V$ = bubble volume [m$^3$]
- $V_{bi}$ = initial bubble volume [m$^3$]
- $V_{comp}$ = composite bubble volume [m$^3$]
- $x$, $y$, $z$ = cartesian coordinate of bubble $i$ [m]
- $X_c$ = horizontal-critical distance [m]
- $Z_c$ = vertical-critical distance [m]
- $x$ = integer number for $\Delta t$
- $\beta$ = percentage volume increase

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MASS TRANSFER IN CREEPING FLOW PAST PERIODIC ARRAYS OF CYLINDERS

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Key Words: Mass Transfer, Fluid Mechanics, Tube Bank, Viscous Flow, Surface Cell Model, Finite Element Method

Creeping flow past a square and an equilateral triangular array of cylinders and mass transfer between the fluid and the surface of the cylinders were studied theoretically. A numerical solution for the drag coefficient of the cylinders was obtained and found to be in good agreement with the asymptotic analytical solutions for very small and very large values of the dimensionless pitch respectively. A numerical solution for the Sherwood number was obtained as a function of the Peclet number, the dimensionless pitch, and the number of rows of cylinders in square and triangular arrays. Theoretical predictions for the drag coefficient of the cylinders and the Sherwood number were compared with approximate analytical solutions based on the free-surface model by Happel.

Introduction

In recent years, mass transfer in a laminar stream flowing past various arrays of thin cylinders at very low velocities has assumed greater importance because of the use of fibers in adsorbers, electrolyzers and hollow-fiber reactors. Relatively many investigations have been made of laminar viscous flow, i.e., the creeping flow past periodic arrays of cylinders. Drummond and Tahia solved the Stokes equation for a viscous fluid in several periodic arrays and obtained asymptotic analytical solutions for the drag on a cylinder in the case of dilute arrays. Sangani and Acirivos solved a similar problem and presented asymptotic analytical solutions for the drag on a cylinder in the case of concentrated square and triangular arrays. However, few studies have been carried out on mass transfer between the surface of cylinders and a laminar stream of viscous fluids flowing past periodic arrays of cylinders at very low velocities, and little information is available.

The present work was undertaken to investigate forced-convection mass transfer in creeping flow past square and triangular arrays of cylinders and to examine the approximate analytical solutions based on the free-surface model, which is convenient for describing mass transfer and for the design of packed beds.

1. Flow Analysis

1.1 Basic equations and boundary conditions

Consider the creeping flow of a viscous fluid past two arrangements of cylinders each of diameter $d$, with pitch $t$ being the distance between the centers of two adjacent cylinders. The first arrangement is the square array whose cross section is a square grid, as shown in Fig. 1. The second is the triangular array whose cross section is an equilateral triangular grid, as shown in Fig. 2. The fluid flows to the right in Figs. 1 and 2 and the superficial velocity of the fluid is $u_m$.

For creeping flow past periodic arrays, the flow pattern around the cylinder is symmetric about the $X$-axis and periodic in the $X$-direction. Therefore the fluid in a typical cell, which is $ABCDEF$ in Figs. 1