HEAT INPUT-RESPONSE IN COOLING TOWER—ZEROTH MOMENTS OF TEMPERATURE VARIATIONS

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For a countercurrent cooling tower subjected to a thermal disturbance by inlet air or water, theoretical equations were solved in terms of zeroth moments of air and water temperatures in the tower. Experiments were carried out by changing the temperature of inlet water at the tower top. The zeroth moments of water and air temperatures at either end of the tower predicted the relationships between the air-film and water-film heat transfer coefficients, which were then compared with the heat transfer coefficients measured from steady-state operation.

Introduction

Much attention has been focused, as reviewed by Wakao and Kaguei,⁸⁾ on the application of dynamic heat/mass input-response techniques to determine rate parameters of transfer processes in packed beds. For example, Smith *et al.*^{1,3,7)} determined rate parameters in a chromatographic column from moments of a response signal of a tracer species imposed at the column inlet.

The heat transfer coefficients of cooling towers have so far been measured only from steady-state operations by McAdams *et al.*,⁵⁾ Yoshida and Tanaka,⁹⁾ Hensel and Treybal,⁴⁾ and Nori and Ishii.⁶⁾ The objective of this work is to apply dynamic techniques to a cooling tower and to examine information on the rate parameters obtainable from the zeroth moments of air and water temperatures.

1. Theoretical

1.1 Assumptions and basic equations

The assumptions used in the theoretical model are listed below:

- (i) Air and water flow countercurrently, each in plug flow mode, in an adiabatic cooling tower.
- (ii) Packings are completely wetted by water so that heat and mass transfer takes place in the same area of the air–water interface.
- (iii) Specific heat of humid air, latent heat of vaporization of water, and flow rates of air and water remain unchanged in the tower.
- (iv) The Lewis equation relates the heat and mass transfer coefficients for air at the air–water interface:

$$\frac{h_G}{k_y} = C_G \quad (1)$$

(v) No temperature gradient exists across the water film covering solid packing.

The basic equations are then derived from unsteady state heat and mass balance.

for air temperature:

$$\varepsilon_G \rho_G C_G \frac{\partial T_G}{\partial t} = -G C_G \frac{\partial T_G}{\partial x} - h_G a (T_G - T_i) \quad (2)$$

for air humidity:

$$\varepsilon_G \rho_G \frac{\partial Y}{\partial t} = -G \frac{\partial Y}{\partial x} + k_y a (Y_i - Y) \quad (3)$$

for water temperature:

$$\begin{aligned} \varepsilon_L \rho_L C_L \frac{\partial T_L}{\partial t} = & L C_L \frac{\partial T_L}{\partial x} - h_L a (T_L - T_i) \\ & - h_p a_p (T_L - (T_p)_{\text{Surface}}) \end{aligned} \quad (4)$$

at the air–water interface:

$$h_G a (T_G - T_i) + h_L a (T_L - T_i) = k_y a \lambda (Y_i - Y) \quad (5)$$

where h_L and h_p are, respectively, the heat transfer coefficients of water at the air–water interface and on the solid packing surface. Also, note that a is the surface area of air–water contact and a_p is the surface area of solid packing, both in a unit volume of packed tower.

In the system under consideration, a one-shot thermal disturbance is given, in terms of a change in inlet water or air temperature, to a cooling tower operated at steady state. At $t = \infty$ or in sufficiently long time, however, the temperatures of air and water and the air humidity in the tower will become,

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respectively, the same as those at $t=0$, i.e., before the disturbance was imposed. Let us express the air and water temperatures and the air humidity as

$$T_G = T_G^\infty + T'_G \quad (6)$$

$$T_L = T_L^\infty + T'_L \quad (7)$$

$$Y = Y^\infty + Y' \quad (8)$$

where superscript ∞ stands for values at steady state, and T'_G , T'_L and Y' are the variations due to the disturbance.

1.2 Steady state

Equations (2)–(5) are then split into two sets of equations, one for steady state and the other for the variations. The former is given by

$$GC_G \frac{dT_G^\infty}{dx} + h_G a (T_G^\infty - T_i^\infty) = 0 \quad (9)$$

$$G \frac{dY^\infty}{dx} - k_y a (Y_i^\infty - Y^\infty) = 0 \quad (10)$$

$$LC_L \frac{dT_L^\infty}{dx} - h_L a (T_L^\infty - T_i^\infty) = 0 \quad (11)$$

$$h_G a (T_G^\infty - T_i^\infty) + h_L a (T_L^\infty - T_i^\infty) = k_y a \lambda (Y_i^\infty - Y^\infty) \quad (12)$$

Note that T_L is equal to T_p at steady state, i.e., $T_L^\infty = T_p^\infty$. Equation (4) is then converted into Eq. (11). Equations (9)–(12) are usually solved by the conventional graphical method, or analytically²⁾ provided that the saturation humidity Y_i may be expressed as a quadratic equation of temperature.

1.3 Response to a thermal disturbance

The variations are expressed as

$$\varepsilon_G \rho_G C_G \frac{\partial T'_G}{\partial t} = -GC_G \frac{\partial T'_G}{\partial x} - h_G a (T'_G - T'_i) \quad (13)$$

$$\varepsilon_G \rho_G \frac{\partial Y'}{\partial t} = -G \frac{\partial Y'}{\partial x} + k_y a (\dot{Y}'_i - Y') \quad (14)$$

$$\varepsilon_L \rho_L C_L \frac{\partial T'_L}{\partial t} = LC_L \frac{\partial T'_L}{\partial x} - h_L a (T'_L - T'_i) - h_p a_p (T'_L - (T'_p)_{\text{Surface}}) \quad (15)$$

$$h_G a (T'_G - T'_i) + h_L a (T'_L - T'_i) = k_y a \lambda (Y'_i - Y') \quad (16)$$

Let us integrate Eqs. (13)–(16) with respect to time. Because $T'_G = T'_L = Y' = 0$ at $t=0$ and at $t=\infty$, we have

$$GC_G \frac{dT_G^*}{dx} + h_G a (T_G^* - T_i^*) = 0 \quad (17)$$

$$G \frac{dY^*}{dx} - k_y a (Y_i^* - Y^*) = 0 \quad (18)$$

$$LC_L \frac{dT_L^*}{dx} - h_L a (T_L^* - T_i^*) = 0 \quad (19)$$

$$h_G a (T_G^* - T_i^*) + h_L a (T_L^* - T_i^*) = k_y a \lambda (Y_i^* - Y^*) \quad (20)$$

in which T_G^* , T_L^* , T_i^* , Y^* and Y_i^* are, respectively, the zeroth moments of T'_G , T'_L , T'_i , Y' and Y'_i , i.e., the integrations with respect to time from $t=0$ to ∞ : T_G^* and L^* , for example, are

$$T_G^* = \int_0^\infty T'_G dt \quad (20a)$$

$$T_L^* = \int_0^\infty T'_L dt \quad (20b)$$

Note that the third term on the right-hand side of Eq. (15) vanishes when it is integrated with time. This comes from the fact that heat transfers from the water film into the solid packing, but later transfers from the packing into the water film again:

$$\int_0^\infty (T'_L - (T'_p)_{\text{Surface}}) dt = 0 \quad (21)$$

Equations (17)–(20) are then Laplace transformed with respect to x :

$$GC_G (s\bar{T}_G^* - (T_G^*)^I) + h_G a (\bar{T}_G^* - \bar{T}_i^*) = 0 \quad (22)$$

$$G (s\bar{Y}^* - (Y^*)^I) - k_y a (\bar{Y}_i^* - \bar{Y}^*) = 0 \quad (23)$$

$$LC_L (s\bar{T}_L^* - (T_L^*)^I) - h_L a (\bar{T}_L^* - \bar{T}_i^*) = 0 \quad (24)$$

$$h_G a (\bar{T}_G^* - \bar{T}_i^*) + h_L a (\bar{T}_L^* - \bar{T}_i^*) = k_y a \lambda (\bar{Y}_i^* - \bar{Y}^*) \quad (25)$$

where $\bar{T}_G^*$, $\bar{T}_L^*$, $\bar{T}_i^*$, $\bar{Y}^*$ and $\bar{Y}_i^*$ are, respectively, the Laplace transforms of T_G^* , T_L^* , T_i^* , Y^* and Y_i^* with respect to x . $\bar{T}_G^*$, for example, is

$$\bar{T}_G^* = \int_0^\infty T_G^* \exp(-sx) dx \quad (25a)$$

Here, let us simply assume that Y_i is a linear function of T_i :

$$Y_i = A + BT_i \quad (26)$$

Consequently we obtain

$$Y'_i = BT'_i \quad (27)$$

and

$$\bar{Y}_i^* = B\bar{T}_i^* \quad (28)$$

With the aid of Eqs. (1) and (28), $\bar{T}_G^*$ and $\bar{T}_L^*$ are obtained from Eqs. (22) to (25). The inverse transformations then give

$$(T_G^*)^I = (T_G^*)^I \exp\left(-\frac{h_G a X}{GC_G}\right) + \frac{U}{V} \left(1 - \exp\left(-\frac{h_G a X}{GC_G}\right)\right) + \frac{Q(P-H)}{H\left(1 - H \frac{GC_G}{h_G a}\right)} (1 - \exp(-HX)) \quad (29)$$

$$(T_L^*)^{\text{II}} = (T_L^*)^{\text{I}} + \frac{Q(P-H)}{H\left(1 + H \frac{LC_L}{h_L a}\right)} (1 - \exp(-HX)) \quad (30)$$

$$H = \frac{h_G a}{GC_G} \left(1 - \frac{GC_G}{LC_L} V\right) \left/ \left(1 + \frac{h_G a}{h_L a} V\right) \right. \quad (30a)$$

$$P = \left(\frac{(T_L^*)^{\text{I}}}{GC_G} - \frac{U}{LC_L} \right) \left/ \left(\frac{(T_L^*)^{\text{I}}}{h_G a} + \frac{U}{h_L a} \right) \right. \quad (30b)$$

$$Q = \left((T_L^*)^{\text{I}} + \frac{h_G a}{h_L a} U \right) \left/ \left(1 + \frac{h_G a}{h_L a} V \right) \right. \quad (30c)$$

$$U = \frac{\lambda}{C_G} (Y^*)^{\text{I}} + (T_G^*)^{\text{I}} \quad (30d)$$

$$V = 1 + \frac{\lambda B}{C_G} \quad (30e)$$

2. Experimental

2.1 Apparatus and procedures

Experiments were carried out with a forced-draft cooling tower of the countercurrent type. The tower, made of polystyrene foam, was packed with corrugated stainless steel wire (0.22 mm)-netting. The height of the packed tower was 50 mm and the inside diameter 75 mm. 20°C water was distributed by gravity on the top of the packed tower. Air was pumped by a centrifugal fan into dehumidifying columns (a molecular sieve column and a silica gel column) and, after flowing in a water-jacketed pipe (to keep the air temperature constant), introduced to the bottom of the tower. The air flowing out of the tower was then introduced to a gas meter, while the water flow rate was measured at the tower bottom. At both ends of the tower, thermistors were used to measure the air and water temperatures and a humidity-measuring element (Shibaura Electronic Co., Tokyo, Model CHS-1-H1) to measure the air humidities. The experimental conditions and the steady-state data are listed in Table 1.

When the cooling tower had been operated under steady-state condition, the inlet 20°C water was shifted to a stream of 12°C water for about 950 s and again back to the 20°C water (water flow rate unchanged). During the run the temperature and the humidity of the inlet air remained unchanged so that $(T_G^*)^{\text{I}} = 0$ and $(Y^*)^{\text{I}} = 0$. Figure 1 shows the temperatures of inlet water $(T_L^{\text{II}})^{\text{exptl}}$, outlet water $(T_L^{\text{I}})^{\text{exptl}}$ and outlet air $(T_G^{\text{II}})^{\text{exptl}}$ recorded with time. Note that superscript I stands for the tower bottom and II the tower top. Note that the thermistors' time-constant is so small (about 0.1 s) that no substantial time delay is assumed to be involved in the temperature-time curves shown in Fig. 1.

Table 1. Experimental conditions and steady-state data

Height of tower	$X = 0.05$ m
Flow rate of air	$G = 0.239$ kg/m ² s
Flow rate of water	$L = 0.518$ kg/m ² s
At the tower top	
Water temperature	$(T_L^{\text{II}})^{\text{II}} = 20^\circ\text{C}$
Air temperature	$(T_G^{\text{II}})^{\text{II}} = 15.3^\circ\text{C}$
Air humidity	$(Y^{\text{II}})^{\text{II}} = 0.01005$
At the tower bottom	
Water temperature	$(T_L^{\text{I}})^{\text{I}} = 17^\circ\text{C}$
Air temperature	$(T_G^{\text{I}})^{\text{I}} = 14^\circ\text{C}$
Air humidity	$(Y^{\text{I}})^{\text{I}} = 0.00029$

Fig. 1. Input signal $(T_L^{\text{II}})^{\text{exptl}}$ and response signals $(T_L^{\text{I}})^{\text{exptl}}$ and $(T_G^{\text{II}})^{\text{exptl}}$.

2.2 Results and discussion

1) Steady state With the steady-state data listed in Table 1 the heat transfer coefficients are found by conventional graphical stepwise calculation to be $h_L a = 30,500$ W/m³K and $h_G a = 10,700$ W/m³K. Also, the calculation shows that T_i in the tower under this steady-state condition ranges from 11°C (at the tower bottom) to 17°C (tower top). Figure 1 indicates that during the period of about $800 < t < 1000$ s the temperatures change only slightly. If we assume this is also steady state the heat transfer coefficients in this period ($Y^{\text{II}} = 0.0075$) are found to be $h_L a = 44,000$ W/m³K and $h_G a = 13,000$ W/m³K, and T_i is found to vary from 8°C (tower bottom) to 11°C (tower top).

2) Dynamic Therefore, T_i in the dynamic period is estimated to be in the range between about 8 and 17°C. The coefficient B of Eq. (26) is then considered to be 0.00078 K⁻¹. Also, from the areas under the curves shown in Fig. 1 the zeroth moments are found to be $(T_L^{\text{II}})^{\text{exptl}} = -7692$ Ks, $(T_L^{\text{I}})^{\text{exptl}} = -6292$ Ks and $(T_G^{\text{II}})^{\text{exptl}} = -4761$ Ks. Note that the zeroth moments are all negative because of the cold water pulse being imposed on the tower.

With the following data: $B = 0.00078$ K⁻¹, $C_G = 1.02$ kJ/kg K, $C_L = 4.18$ kJ/kg K, $G = 0.239$ kg/m²s, $L = 0.518$ kg/m²s, $(T_G^{\text{I}})^{\text{I}} = 0$, $(T_L^{\text{I}})^{\text{I}} = -6292$ Ks, $(Y^{\text{I}})^{\text{I}} = 0$, $X = 0.05$ m and $\lambda = 2490$ kJ/kg Eqs. (29) and

Fig. 2. $h_L a$ – $h_G a$ relationships predicted from dynamic operation; steady-state $h_L a$ and $h_G a$ data calculated with $(T_G^\infty)^{\text{II}} = (T_G^\infty)^{\text{I}}_{\text{exptl}} + \Delta_G$ and $(T_L^\infty)^{\text{I}} = (T_L^\infty)^{\text{I}}_{\text{exptl}} + \Delta_L$:

Key	Δ_L [°C]	Δ_G [°C]
a	0	0
b	0.2	0
c	–0.2	0
d	0	0.2
e	0	–0.2

(30) are calculated to find what $h_L a$ and $h_G a$ values give $(T_G^*)^{\text{II}}$ of Eq. (29) and $(T_L^*)^{\text{II}}$ of Eq. (30), respectively, equal to $(T_G^*)^{\text{II}}_{\text{exptl}} \times (1 + \delta_G)$ and $(T_L^*)^{\text{II}}_{\text{exptl}} \times (1 + \delta_L)$. Hence, δ_G and δ_L are possible experimental errors and they are temporarily assumed to be ± 0.02 , i.e., $\pm 2\%$ errors. In Fig. 2 the $h_L a$ – $h_G a$ relationship thus obtained from Eq. (30) is indicated in a hyperbolic-type contour: the upper line of the contour corresponds to δ_L of -0.02 and the lower line to $\delta_L = 0.02$. The narrow contour predicted from Eq. (29) lies along the upper line of the contour obtained from Eq. (30). In any case, the two contours do not cross each other so that $h_L a$ and $h_G a$ values are not determined.

The steady-state data are also considered to contain some experimental errors. Four cases are then assumed: either $(T_L^\infty)^{\text{I}}$ or $(T_G^\infty)^{\text{II}}$ has the errors of $\pm 0.2^\circ\text{C}$. It is found, as shown in Fig. 2, that the errors of $\pm 0.2^\circ\text{C}$ in $(T_L^\infty)^{\text{I}}$ do not cause an appreciable difference in the calculation of $h_L a$ and $h_G a$, while those in $(T_G^\infty)^{\text{II}}$ are magnified in the calculation. However, the $h_L a$ and $h_G a$ data all fall right in the contour predicted from Eq. (30). Considering some experimental errors such as $\pm 0.2^\circ\text{C}$, the $h_L a$ and $h_G a$ values may be determined from the steady-state measurement to be (refer to Fig. 2) $h_L a = 28,000 - 33,000 \text{ W/m}^3\text{K}$ and $h_G a = 10,000 - 11,000 \text{ W/m}^3\text{K}$.

It is found that the individual heat transfer coefficient values are not determined from a single heat input-response measurement by changing inlet water temperature at the column top. Equations (29) and (30) may be used, however, to predict the dynamic

behavior of a cooling tower responding to a thermal disturbance. In the tower, the theory says that the thermal disturbance attenuates as an exponential function of axial distance.

Conclusions

For a cooling tower subjected to a thermal disturbance by inlet air or water, theoretical equations were solved in terms of zeroth moments of variations of the air and water temperatures. A heat input-response measurement predicted a relationship between the air-film and water-film heat transfer coefficients, but their values were determined from the data of steady-state operation only.

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Nomenclature

a	= air-water contact area per unit volume of tower	[m ⁻¹]
a_p	= solid packing surface area per unit volume of tower	[m ⁻¹]
B	= given in Eq. (26)	[K ⁻¹]
C_G	= specific heat of humid air	[J/(kg K)]
C_L	= specific heat of water	[J/(kg K)]
G	= mass velocity of air	[kg/(m ² s)]
H	= defined by Eq. (30a)	[m ⁻¹]
h_G	= heat transfer coefficient of air at air-water interface	[W/(m ² K)]
h_L	= heat transfer coefficient of water at air-water interface	[W/(m ² K)]
h_p	= heat transfer coefficient of water at solid packing surface	[W/(m ² K)]
k_y	= mass transfer coefficient of water vapor at air-water interface	[kg/(m ² s)]
L	= mass velocity of water	[kg/(m ² s)]
P	= defined by Eq. (30b)	[m ⁻¹]
Q	= defined by Eq. (30c)	[K s]
T_G	= air temperature	[K]
T_i	= temperature at air-water interface	[K]
T_L	= water temperature	[K]
T_p	= solid temperature	[K]
T^*	= integration of T' with respect of t	[K s]
$\bar{T}^*$	= Laplace transform of T^* with respect to x	[K s m]
t	= time	[s]
U	= defined by Eq. (30d)	[K s]
V	= defined by Eq. (30e)	[—]
X	= height of packed tower	[m]
x	= axial distance variable	[m]
Y	= air humidity	[kg-water/kg-air]
Y_i	= saturation humidity	[kg-water/kg-air]
Y^*, Y_i^*	= integrations, respectively, of Y' and Y_i' with respect to t	[s]
$\bar{Y}^*, \bar{Y}_i^*$	= Laplace transforms, respectively, of Y^* and Y_i^* with respect to x	[s m]
α_p	= thermal diffusivity of solid packing	[m ² /s]
δ_G, δ_L	= errors	[—]
ε_G	= air volume fraction	[—]
ε_L	= water volume fraction	[—]

λ	= latent heat of vaporization of water	[J/kg]
ρ_G	= density of humid air	[kg/m ³]
ρ_L	= density of water	[kg/m ³]

<Superscripts>

I	= tower bottom
II	= tower top
∞	= steady state
	= variation

<Subscripts>

G	= air
L	= water

Literature Cited

- 1) Andrieu, J. and J. M. Smith: *Chem. Eng. J.*, **20**, 211 (1980).
- 2) Fahim, M. A., R. S. Al-Ameeri and N. Wakao: *Chem. Eng. Comm.*, **36**, 1 (1985).
- 3) Hashimoto, N. and J. M. Smith: *Ind. Eng. Chem. Fundam.*, **12**, 351 (1973).
- 4) Hensel, S. L., Jr. and R. E. Treybal: *Chem. Eng. Prog.*, **48**, 362 (1952).
- 5) McAdams, W. H., J. B. Pohlentz and R. C. St. John: *Chem. Eng. Prog.*, **45**, 241 (1949).
- 6) Nori, S. and T. Ishii: *Chem. Eng. Sci.*, **37**, 487 (1982).
- 7) Schneider, P. and J. M. Smith: *AIChE J.*, **14**, 762 (1968).
- 8) Wakao, N. and S. Kagui: "Heat and Mass Transfer in Packed Beds," Gordon and Breach, N.Y., pp. 264-295 (1982).
- 9) Yoshida, F. and T. Tanaka: *Ind. Eng. Chem.*, **43**, 1467 (1951).

OIL-PHASE PERMEATION BEHAVIOR OF O/W EMULSION THROUGH A POROUS POLYTETRAFLUOROETHYLENE MEMBRANE

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Oil-in-water emulsions consisting of drops smaller than 10 μm were permeated through porous polytetrafluoroethylene membranes under conditions where only the oil phase permeated. Factors influencing the oil permeation flux were experimentally investigated. They are drop size, stirring velocity during the permeation experiment, volume fraction of the oil phase, and surfactant concentration, among others. When the maximum drop size was doubled, the oil flux became almost ten times larger. Effects of stirring velocity during the permeating experiment and of volume fraction of the oil phase were very large. For the case where the surfactant (SDS) concentration was 2.6 mol/m³, the oil flux was proportional to the 4th power of the stirring velocity and the 3.4th power of the volume fraction of the oil phase. As the surfactant concentration was increased up to the critical micelle concentration, the oil flux decreased to a very small value.

Introduction

The demulsifying process of emulsions composed of very small drops, smaller than 10 μm , has recently received much attention from the viewpoint of tertiary oil recovery, emulsion-type liquid membrane treatment, waste water treatment for ships and so on. Conventional separators utilizing the density difference between the oil and drop phases are not effective for emulsions of such small drops. Electrical demulsifying methods are now being actively investigated as a demulsifying technique for emulsions of

very small drops, using an electric pulse of thousands of volts.^{1,2,3)} Another type of demulsification technique was also proposed which uses porous membranes and which can be operated under rather mild conditions.^{4,5)}

The purpose of this work is to investigate the mechanisms of demulsification using porous membranes.

1. Experimental Apparatus and Method

A schematic diagram of the experimental apparatus is shown in Fig. 1. The main part is a stirred tank 6 cm in diameter and 8 cm high. It is fitted with a turbine stirrer with six blades to mix the contents. Details of

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