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POROSITY ESTIMATIONS OF MIXED ASSEMBLAGES OF SOLID PARTICLES WITH DIFFERENT PACKING CHARACTERISTICS

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The authors' theory of porosity estimation from particle size distribution was modified to make allowances for the size and species dependences of packing characteristics of particles. The new version is examined together with other existing theories by using well known relations and past experimental data. It is shown that the estimates by the modified theory are almost consistently in good agreement with the known data whereas the other theories examined show theoretical deficiencies that sometimes give rise to large deviations for mixtures of particles of remarkably different sizes. Some possible problems are also discussed in association with the new assumption introduced.

Introduction

Packing porosity is one of the most basic geometrical properties associated with an assemblage of solid particles. It is well known experimentally that the fractional void volume of a packing depends on the size distribution of the particles involved. A method of predicting porosity from the particle size distribution was proposed by the present authors.⁵⁾ It has

since been developed for and examined with mixtures of solid particles which have a similar packing characteristic.⁶⁻⁸⁾

It is often observed, however, that the packing porosities of sized particles vary with the sizes and/or species of the particles. Suzuki *et al.*¹¹⁾ proposed a theory of porosity estimation which allowed for the size dependence of the void fraction of uniform spheres. Although not explicitly mentioned, their theory was based on the premise that the porosity of a packing is equal to an average, defined by them, of the

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partial void fraction around a particle. They observed good agreement between computed estimates and experiments. From a theoretical point of view, however, it seems essential to examine if the premise is generally correct. Cross *et al.*²⁾ modified our original theory to allow for the size dependence of the packing characteristics. But the authors' original version⁵⁾ cited by them had not generally described the possible effect of macropores. The effect was analysed in our recent paper.⁷⁾

In this paper we extend our theory to mixtures of solid particles which have different packing characteristics. The new version developed in the paper will be examined, together with the other existing theories, by using well-known relations and past experimental data.

1. Extension of the Authors' Theory

Our considerations start with a simplified model for the coordination number (see Fig. 1). The central sphere is in direct contact with neighboring spheres having the average diameter, $\bar{D}$, of particles. Denoting the surface porosity by ε_A , which is the fractional void area on a spherical surface of diameter $(D + \bar{D})$, gives a simple expression for the coordination number, $C(D)$, around a specified sphere of diameter D . That is,

$$C(D) = 16(1 - \varepsilon_A) \{ (D + \bar{D}) / (2\bar{D}) \}^2 \quad (1)$$

where

$$\bar{D} = \int_0^\infty D f(D) dD \quad (2)$$

Here $f(D)$ is the number frequency size distribution of particles.

When the solid particles of a mixture have different packing characteristics, it is generally supposed that the surface porosity of the model becomes a function of sizes and species of both the specified particle and the neighboring spheres. Here, a species means a particle characteristic other than size, the variations of which cause differences between packing porosities of spheres of identical size. Little is known, however, as to how the coordination number around a sphere varies with the size and species of neighboring spheres. For simplicity, therefore, it is assumed here that the surface porosity of the model depends only on the size D and the species j of the specified particle and does not depend on the properties of the surrounding particles. This assumption can be written as

$$\varepsilon_A = \varepsilon_{Aj}(D) \quad (3)$$

This is the only assumption that is added to our previous theory. It means that the coordination number of the theory should be dealt with as a function of both the size and species of a specified particle. That

Fig. 1. A simplified model for the coordination number. Hatching indicates the spherical shell considered in the theory

is,

$$C_j(D) = 16 \{ 1 - \varepsilon_{Aj}(D) \} \{ (D + \bar{D}) / (2\bar{D}) \}^2 \quad (4)$$

The logical framework of our theory will be mentioned briefly. According to the simplified model of Fig. 1, every hypothetical sphere of diameter $(D + \bar{D})$ has to share the space partly in common with the other hypothetical spheres. The space commonly occupied, $V_m(D)$, is restricted within the spherical shell hatched in Fig. 1. That is,

$$V_m(D) = \pi(D + \bar{D})^3/6 - \pi(D \sim \bar{D})^3/6 \quad (5)$$

where

$$(D \sim \bar{D}) = \begin{cases} 0 & \text{for } D \leq \bar{D} \\ D - \bar{D} & \text{for } D > \bar{D} \end{cases} \quad (6)$$

Denoting the average number of hypothetical spheres concerned with the common occupation of space by $\bar{n}$, the space which should be allocated to a specified sphere of diameter D , $V_c(D)$, can be described as

$$V_c(D) = \pi(D \sim \bar{D})^3/6 + V_m(D)/\bar{n} \quad (7)$$

Where no macropores exist in a packing, the total bulk volume V_{BT} , of the packing should be equal to the summation of $V_c(D)$ to all the particles of the packing. That is,

$$V_{BT} = \int_0^\infty V_c(D) N f(D) dD \quad (8)$$

where N is the total number of solid particles of the packing. The overall average porosity of a packing, $\bar{\varepsilon}$, is related to both the total bulk volume above and the total solid volume of particles, V_{ST} , as

$$\bar{\varepsilon} = 1 - V_{ST}/V_{BT} \quad (9)$$

where

$$V_{ST} = \int_0^\infty \pi(D^3/6) N f(D) dD \quad (10)$$

The average number $\bar{n}$ of Eq. (7) can approximately be evaluated when its dependences on the sizes and species of particles are discarded. Defining the aver-

age porosity, $\varepsilon_{mj}(D)$, of a spherical shell by Eq. (12) leads to the following volume balance equation regarding the total solid space of the packing.

$$\begin{aligned} & \int_0^\infty \frac{\pi}{6} (D \sim \bar{D})^3 N f(D) dD \\ & + \frac{1}{\bar{n}} \int_0^\infty \sum_{j=1}^s (1 - \varepsilon_{mj}(D)) V_m(D) N f_j(D) dD \\ & = \int_0^\infty \frac{\pi}{6} D^3 N f(D) dD \end{aligned} \quad (11)$$

where

$$\varepsilon_{mj}(D) = 1 - \left[\frac{\pi}{6} \{D^3 - (D \sim \bar{D})^3\} + C_f(D) \frac{\pi}{12} \bar{D}^3 \left(1 - \frac{3}{8} \frac{\bar{D}}{D + \bar{D}} \right) \right] / V_m(D) \quad (12)$$

$$f(D) = \sum_{j=1}^s f_j(D) \quad (13)$$

Here, s is the total number of different species of the mixture and $f_j(D)$ is the sub-fraction of $f(D)$ which belongs to the species j .

The above is a brief of the theoretical framework of our method. A macropore in the theory means the void space which could exist apart from solid surfaces at a distance greater than half an average diameter of particles. The effect of macropores was taken up in our recent paper.⁷⁾ A set of working equations for porosity estimation is summarized for a discrete size distribution as follows. By numbering the size fractions from the largest to the smallest in a mixture,

$$\bar{\varepsilon} = 1$$

$$\frac{\sum_{i=1}^m D_i^3 f_i}{\max \text{ of } \sum_{i=1}^p \left[(D_i \sim \bar{D})^3 + \frac{1}{\bar{n}} \{ (D_i + \bar{D})^3 + (D_i \sim \bar{D})^3 \} \right] f_i} \quad (14)$$

where $1 \leq p \leq m$, and

$$\bar{n} = 1$$

$$\begin{aligned} & + 2\bar{D} \frac{\sum_{i=1}^p \sum_{j=1}^s (1 - \varepsilon_{Aj}(D_i)) (D_i + \bar{D})^2 \left(1 - \frac{3}{8} \frac{\bar{D}}{D_i + \bar{D}} \right) f_{ji}}{\sum_{i=1}^p \{ D_i^3 - (D_i \sim \bar{D})^3 \} f_i} \end{aligned} \quad (15)$$

$$\bar{D} = \frac{\sum_{i=1}^p (D_i f_i)}{\sum_{i=1}^p f_i} \quad (16)$$

$$f_i = \sum_{j=1}^s f_{ji} \quad (17)$$

Here m is the total number of size components of a mixture, f_{ji} is the fractional number of particles belonging to the i -th size fraction and species j , and f_i is the number fraction of the i -th component.

For a particular case of the packing of single-size spheres of the size and species, D_i and j , respectively, the packing porosity, $\bar{\varepsilon}_{oj}(D_i)$ ($= \bar{\varepsilon}$), becomes

$$\bar{\varepsilon}_{oj}(D_i) = 1 - \{ 1 + 13(1 - \varepsilon_{Aj}(D_i))/2 \} / 8 \quad (18)$$

This equation means that the surface porosities in the theory can be known from experimental data on the packing porosities of identical particles.

Another expression for the average porosity will be derived for later discussions. According to Eqs. (7), (8), (10) and (11), the total bulk volume of the packing without macropores can be described as

$$V_{BT} = V_{ST} + \frac{1}{\bar{n}} \int_0^\infty \sum_{j=1}^s \varepsilon_{mj}(D) V_m(D) N f_j(D) dD \quad (19)$$

Let us define the average porosity of a hypothetical sphere, $\varepsilon_j(D)$, by the following equation.

$$\varepsilon_j(D) = 1 - \frac{(\pi D^3/6) + C_f(D)(\pi \bar{D}^3/12) \left(1 - \frac{3}{8} \frac{\bar{D}}{D + \bar{D}} \right)}{\pi (D + \bar{D})^3/6} \quad (20)$$

Then

$$\varepsilon_{mj} V_m(D) = \varepsilon_j(D) \cdot \pi (D + \bar{D})^3/6 \quad (21)$$

Combining Eqs. (19) and (21) gives

$$V_{BT} - V_{ST} = \frac{1}{\bar{n}} \int_0^\infty \sum_{j=1}^s \varepsilon_j(D) \cdot \frac{\pi}{6} (D + \bar{D})^3 N f_j(D) dD \quad (22)$$

Dividing both sides by V_{BT} ($= V_{ST}/(1 - \bar{\varepsilon})$) results in

$$\bar{\varepsilon} = \frac{1 - \bar{\varepsilon}}{\bar{n}} \int_0^\infty \sum_{j=1}^s \varepsilon_j(D) (D + \bar{D})^3 f_j(D) dD \bigg/ \int_0^\infty D^3 f(D) dD \quad (23)$$

This is another expression for the average porosity of a packing without macropores. For a discrete size distribution of particles of a same species it is re-written as

$$\bar{\varepsilon} = \frac{1 - \bar{\varepsilon}}{\bar{n}} \sum_{i=1}^m [\{ \varepsilon(D_i) v_i \} \{ (D_i + \bar{D})^3 / D_i^3 \}] \quad (24)$$

The number fractions in the theory can be calculated from the volume fractions of constituent particles by assuming that the particles are of spherical shape.

2. Discussion

In the preceding section we extended our original theory to a more general version of porosity estimation. The modified version may be applied to mixtures of solid particles having different packing characteristics. When all the particles have the same

packing characteristic, all the equations derived in the paper reduce exactly to the corresponding equations in our previous papers.⁵⁻⁸⁾ Except for the possible effects of species and of macropores, the working equations in this paper are identical with those of Cross *et al.*²⁾ It may be that species dependences are intimately associated with the surface properties of particles such as the microscopic geometries, the adhesional and/or the frictional properties. When original powders are of identical size but have different packing characteristics, we would have to consider the species dependences for estimation of the mixture porosities. In the following discussion sameness of species will be assumed, where the suffix *j* of the paper should be cancelled out by use of Eq. (17).

As was mentioned already, on the other hand, the theory of Suzuki *et al.*¹¹⁻¹³⁾ was based on the premise that the packing porosity is equal to an average of the partial void fraction, ε_i , around a particle. That is,

$$\bar{\varepsilon} = \sum_{i=1}^m \varepsilon_i v_i \quad (25)$$

where ε_i is related to their model for coordination number. The physical meaning of ε_i of Eq. (25) is similar to that of $\varepsilon(D_i)$ of Eq. (24), which can be used when the existence of macropores is discarded. It does not seem likely, however, that Eq. (24) generally reduces to an equation similar to Eq. (25).

First of all, the important points above will be examined by using the well-known relations for binary mixtures, of which some mention was made in our recent paper.⁸⁾ For binary mixtures it has been well established that there is a lower limit of packing porosity corresponding to the composition of the mixture.³⁾ See Fig. 2, where identical porosity values ($\bar{\varepsilon}_0$) equal to 0.4 and 0.6, respectively, were assumed for individual packings (abscissas of 0 and 1) of single-size particles. The limiting curves in Fig. 2 are given as follows:

$$\text{For curve (1)} \quad \bar{\varepsilon} = 1 - (1 - \bar{\varepsilon}_0)/(1 - \bar{\varepsilon}_0 v_1) \quad (26)$$

$$\text{For curve (2)} \quad \bar{\varepsilon} = 1 - (1 - \bar{\varepsilon}_0)/v_1 \quad (27)$$

Equation (26) comes into play when larger particles are dispersed in a bed of large quantities of sufficiently smaller particles. Eq. (27) is applicable when smaller particles are completely embedded in the interstices of the packing of larger particles. Both these limiting equations should be exactly correct as long as each structural component keeps its packing characteristic, and therefore will serve as the criteria for checking of theories.

Returning to the authors' theory, we showed in a recent paper⁸⁾ that, for binary mixtures of particles with an identical packing characteristic, Eq. (14) together with Eqs. (15) to (18) reduces exactly to the

Fig. 2. Examination of theories by use of well-known relations for limiting porosities of binary mixture. Broken lines: by theory of Suzuki *et al.*¹¹⁾

Fig. 3. Densest packing porosity vs. size ratio of binary mixture

above two limiting equations as the size ratio D_2/D_1 approaches null. On the other hand, the theory of Suzuki *et al.*¹¹⁾ leads to a limiting curve drawn for each value of $\bar{\varepsilon}_0$ in Fig. 2 as the size ratio D_2/D_1 becomes negligible. In the calculations, the following relation by Ridgway and Tarbuck⁹⁾ was assumed between $\bar{\varepsilon}_j$ and $\tilde{N}_{cj}$ in their theory.

$$\bar{\varepsilon}_j = 1.072 - 0.1193 \tilde{N}_{cj} + 0.00431 \tilde{N}_{cj}^2 \quad (28)$$

where $\bar{\varepsilon}_j$ and $\tilde{N}_{cj}$ are, respectively, the packing porosity and the coordination number around a particle in a bed of single-size spheres.

The theories can also be checked by examining the densest packing porosities of binary mixtures of various size ratio. Figure 3 shows the predicted porosities for the densest packings for three kinds of packing characteristics, i.e., $\bar{\varepsilon}_0$ equal to 0.4, 0.5 and 0.6, respectively. Figure 3 also plots the following empirical relations, found by Furnas⁴⁾ and Ridgway and Tarbuck¹⁰⁾ respectively.

$$\bar{\varepsilon}_{\min} = \{(1 + \bar{\varepsilon}_0) - y\} / \{(1 + \varepsilon_0)/\varepsilon_0 - y\}$$

Table 1. Differences between estimates ($\bar{\epsilon}_H$) of porosity by the theory of Suzuki *et al.* and those ($\bar{\epsilon}$) by ours

(1) Input data

No.	Origin	D_1 ($\bar{\epsilon}_o(D_1)$)	D_2 ($\bar{\epsilon}_o(D_2)$)	D_3 ($\bar{\epsilon}_o(D_3)$)
1	Fig. 5 ⁽¹¹⁾	0.230 (0.387)	0.115 (0.386)	
2		0.460 (0.385)	0.163 (0.381)	
3		0.650 (0.370)	0.163 (0.381)	
4		0.650 (0.378)	0.081 (0.388)	
5	Fig. 6 ⁽¹¹⁾	0.230 (0.382)	0.115 (0.375)	
6		0.460 (0.392)	0.163 (0.379)	
7		0.650 (0.378)	0.163 (0.379)	
8		0.650 (0.378)	0.081 (0.384)	
9	Fig. 7 ⁽¹¹⁾	0.920 (0.382)	0.324 (0.383)	0.194 (0.385)
10	Fig. 8 ⁽¹¹⁾	0.920 (0.382)	0.650 (0.370)	0.460 (0.385)
11	Fig. 9 ⁽¹¹⁾	0.650 (0.370)	0.460 (0.385)	0.324 (0.383)
12	Fig. 10 ⁽¹¹⁾	0.650 (0.378)	0.460 (0.392)	0.324 (0.382)
13	Fig. 11 ⁽¹¹⁾	0.920 (0.452)	0.650 (0.473)	0.324 (0.490)
14	Fig. 12 ⁽¹¹⁾	0.920 (0.452)	0.650 (0.473)	0.460 (0.485)
15	Fig. 13 ⁽¹¹⁾	0.920 (0.447)	0.650 (0.457)	0.460 (0.462)
16	Fig. 5 ⁽¹²⁾	12.7 (0.420)	9.6 (0.440)	6 (0.420)
17	Fig. 6 ⁽¹²⁾	28 (0.360)	14 (0.365)	7 (0.365)
18	Fig. 7 ⁽¹²⁾	20 (0.409)	12 (0.387)	9 (0.388)
19	Fig. 8 ⁽¹²⁾	5.2 (0.550)	3.7 (0.540)	2.7 (0.525)
20	Fig. 9 ⁽¹²⁾	1.55 (0.432)	0.55 (0.432)	0.194 (0.432)
	Fig. 3 ⁽¹³⁾	Log-Normal ($\ln \sigma_g = 0.05, 0.10, \dots, 0.70$)		
21		Glass beads (Tapping)		
22		(Rodding)		
23		Crushed glass (Tapping)		
24		(Rodding)		
	Fig. 5 ⁽¹³⁾	Log-Uniform ($a = 1.0, 1.5, \dots, 7.0$)		
25		Glass beads (Tapping)		
26		(Rodding)		
27		Crushed glass (Tapping)		
28		(Rodding)		
	Fig. 7 ⁽¹³⁾	Rosin-Rammler ($n = 2, 3, \dots, 15$)		
29		Glass beads (Tapping)		
30		(Rodding)		
31		Crushed glass (Tapping)		
32		(Rodding)		
	Fig. 9 ⁽¹³⁾	Andreasen ($Q = 0.2, 0.3, \dots, 1, 2, \dots, 5$)		
33		Glass beads (Tapping)		
34		(Rodding)		
35		Crushed glass (Tapping)		
36		(Rodding)		

(2) Porosity difference = $|\bar{\epsilon}_H - \bar{\epsilon}|$ and number of the affairs

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	9	6	6	2	9	6	6	3	52	63	63	63	57	63	63	63
0.01	0	3	1	5	0	3	1	4	7	0	0	0	6	0	0	0
0.02	0	0	1	1	0	0	0	1	2	0	0	0	0	0	0	0
0.03	0	0	1	1	0	0	2	1	1	0	0	0	0	0	0	0
0.04	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.06																
No.	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
0	52	63	63	33	9	10	14	14	12	13	13	13	12	12	13	13
0.01	6	0	0	17	4	4	0	0	1	0	0	0	1	1	1	1
0.02	2	0	0	10	1	0	0	0	0	0	0	0	1	1	0	0
0.03	3	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0.06																
No.	33	34	35	36												
0	10	9	9	9												
0.01	2	3	2	2												
0.02	2	2	4	3												
0.03	3	3	2	3												
0.04	1	1	0	0												
0.05	1	1	0	0												
0.06																

all of them we carried out computations by both theories. For mixtures of the two- and three-size components, the individual compositions of constituent particles were varied by ten percent on volume basis and several values were assumed for the distribution constants of the continuous size distributions. Special attention was paid to the differences between the estimates by the two theories. **Table 1** summarizes the input data used for calculations and the number of affairs which resulted in the porosity differences designated. It is found in Table 1 that the differences between the two estimates are not so great, all of them being less than about five percent. The rather small differences may be ascribed to the relatively small ratios, D_m/D_1 , of the extreme sizes of the particles involved.

Tables 2 and **3** show individual comparisons of theoretical estimates with the numerical values of porosities measured by Ridgway and Tarbuck⁽¹⁰⁾ and by Westman and Hugill,⁽¹⁴⁾ respectively. These data are particularly favorable to the present discussion because the two mixtures are very different from each other in the gaps in the sizes of particles, i.e., the former is small and the latter is large. The final columns of Tables 2 and 3 show the calculated porosities in which the effect of macropores was discarded in our theory, and therefore those values coincide with the estimates by the theory of Cross *et al.*⁽²⁾ As can be seen in the summarized **Table 4**, the

where (29)

$$y = 1 - 2.62(D_2/D_1) + 1.62(D_2/D_1)^2$$

$$\bar{\epsilon}_{\min} = \bar{\epsilon}_o - \bar{\epsilon}_o(1 - \bar{\epsilon}_o)\{1 - 2.35(D_2/D_1) + 1.35(D_2/D_1)^2\}$$
(30)

It should be noted that binary mixtures of particles having the same packing characteristic cannot take values of porosity smaller than a critical value, ϵ_{cr} , which is determined by equating Eqs. (26) and (27) as

$$\epsilon_{cr} = \bar{\epsilon}_o^2$$
(31)

It can be recognized from Fig. 3 that our estimates are again in consistent agreement with Eqs. (29), (30) and (31).

Suzuki *et al.*⁽¹¹⁻¹³⁾ examined their theory by using their own experimental data and those of others. For

Table 2. Comparison of theoretical estimates with experimental data by Ridgway and Tarbuck¹⁰⁾
 D_1 ($\bar{\epsilon}_o(D_1)$) = 20 mm (0.409), D_2 ($\bar{\epsilon}_o(D_2)$) = 12 mm (0.387), D_3 ($\bar{\epsilon}_o(D_3)$) = 9 mm (0.388)

v_1	v_2	Exp. data	Theoretical estimates by		
			Suzuki <i>et al.</i>	Ours	Cross <i>et al.</i>
0.0101	0.0675	0.385	0.387	0.387	0.387
0.0373	0.1394	0.380	0.385	0.386	0.386
0.1132	0.0477	0.377	0.383	0.383	0.383
0.0803	0.2334	0.378	0.383	0.384	0.384
0.159	0.1558	0.371	0.381	0.381	0.381
0.2401	0.0796	0.368	0.379	0.378	0.378
0.1679	0.2921	0.373	0.380	0.381	0.381
0.3672	0.1025	0.363	0.377	0.375	0.375
0.1133	0.4187	0.375	0.382	0.382	0.382
0.2960	0.2382	0.368	0.378	0.377	0.377
0.1068	0.5659	0.379	0.382	0.383	0.383
0.1978	0.4776	0.374	0.381	0.381	0.381
0.3033	0.3725	0.368	0.380	0.378	0.378
0.4387	0.2413	0.361	0.378	0.375	0.375
0.5392	0.1390	0.359	0.378	0.373	0.373
0.6130	0.1324	0.349	0.380	0.374	0.374
0.1279	0.6906	0.372	0.383	0.384	0.384
0.2630	0.5554	0.364	0.382	0.381	0.381
0.3867	0.4337	0.361	0.381	0.379	0.379
0.5173	0.3041	0.360	0.382	0.378	0.378
0.6622	0.1578	0.355	0.383	0.377	0.377
0.6709	0.1712	0.361	0.384	0.378	0.378
0.0727	0.8136	0.376	0.385	0.385	0.385
0.0475	0.9125	0.381	0.386	0.386	0.386
0.0754	0.8293	0.379	0.385	0.385	0.385
0.2128	0.6922	0.372	0.384	0.383	0.383
0.3521	0.5335	0.370	0.376	0.382	0.382
0.4832	0.4230	0.366	0.384	0.381	0.381
0.6275	0.2770	0.364	0.386	0.381	0.381
0.7894	0.1148	0.373	0.391	0.384	0.384

estimates by the other theories indeed agree well with the data for the mixture of small size difference but give rise to large deviations at several compositions for the mixture with a large gap in sizes. On the other hand, the estimates by the authors' theory are consistently in good agreement with the measured data irrespective of the sizes of the particles involved.

A final examination of the theories will be made with binary mixtures markedly different in packing characteristics. **Figure 4** compares theoretical estimates with the experimental data by Furnas³⁾ and those by Arakawa *et al.*¹⁾ The reported porosities of Furnas can be explained well by the authors' theory and less well by the others examined. However, the data by Arakawa *et al.* cannot well be traced by either of the existing theories compared. In the theoretical development of the new version, on the other hand, one assumption was made regarding the coordination number: that the coordination number around a particle is independent of the packing characteristics of surrounding particles. Although not explicitly stated, the same assumption was also involved in

Table 3. Comparison of theoretical estimates with experimental data by Westman and Huggill¹⁴⁾
 D_1 ($\bar{\epsilon}_o(D_1)$) = 0.176 inch (0.377), D_2 ($\bar{\epsilon}_o(D_2)$) = 0.028 inch (0.382), D_3 ($\bar{\epsilon}_o(D_3)$) = 0.0035 inch (0.425)

v_1	v_2	Exp. data	Theoretical estimates by		
			Suzuki <i>et al.</i>	Ours	Cross <i>et al.</i>
0.000	0.100	0.404	0.398	0.407	0.407
0.000	0.200	0.386	0.371	0.388	0.388
0.000	0.300	0.356	0.346	0.368	0.368
0.000	0.400	0.330	0.321	0.346	0.346
0.000	0.500	0.302	0.300	0.322	0.322
0.000	0.600	0.276	0.281	0.297	0.297
0.000	0.700	0.267	0.268	0.269	0.269
0.000	0.800	0.290	0.266	0.239	0.239
0.000	0.900	0.328	0.287	0.313	0.207
0.100	0.400	0.298	0.285	0.314	0.314
0.100	0.500	0.267	0.265	0.288	0.288
0.100	0.600	0.246	0.251	0.260	0.260
0.100	0.700	0.258	0.247	0.229	0.229
0.100	0.800	0.312	0.266	0.295	0.196
0.200	0.000	0.378	0.345	0.375	0.375
0.200	0.100	0.356	0.319	0.353	0.353
0.200	0.200	0.323	0.293	0.330	0.330
0.200	0.300	0.295	0.270	0.306	0.306
0.200	0.400	0.262	0.249	0.279	0.279
0.200	0.500	0.236	0.234	0.250	0.250
0.200	0.600	0.238	0.228	0.218	0.218
0.200	0.700	0.294	0.245	0.275	0.184
0.200	0.800	0.348	0.343	0.351	0.351
0.300	0.200	0.288	0.256	0.297	0.297
0.300	0.300	0.258	0.234	0.270	0.270
0.300	0.400	0.224	0.217	0.240	0.240
0.300	0.500	0.224	0.209	0.208	0.208
0.300	0.600	0.270	0.224	0.254	0.172
0.400	0.000	0.316	0.267	0.314	0.314
0.400	0.100	0.284	0.242	0.289	0.289
0.400	0.200	0.254	0.219	0.261	0.261
0.400	0.300	0.207	0.201	0.230	0.230
0.400	0.400	0.201	0.192	0.197	0.197
0.400	0.500	0.242	0.204	0.232	0.160
0.400	0.600	0.302	0.308	0.317	0.317
0.500	0.000	0.279	0.228	0.280	0.280
0.500	0.100	0.242	0.205	0.251	0.251
0.500	0.200	0.206	0.185	0.220	0.220
0.500	0.300	0.184	0.174	0.186	0.186
0.500	0.400	0.218	0.184	0.208	0.148
0.500	0.500	0.282	0.294	0.298	0.298
0.600	0.000	0.236	0.191	0.241	0.241
0.600	0.100	0.197	0.170	0.210	0.210
0.600	0.200	0.174	0.157	0.175	0.175
0.600	0.300	0.200	0.165	0.183	0.135
0.600	0.400	0.270	0.283	0.279	0.279
0.700	0.000	0.185	0.156	0.199	0.199
0.700	0.100	0.155	0.141	0.163	0.163
0.700	0.200	0.168	0.147	0.158	0.123
0.700	0.300	0.262	0.277	0.258	0.258
0.800	0.000	0.251	0.126	0.221	0.151
0.800	0.100	0.258	0.129	0.221	0.110
0.800	0.200	0.278	0.281	0.237	0.237
0.900	0.000	0.315	0.113	0.308	0.096
0.900	0.100	0.320	0.304	0.308	0.220

the theories of Suzuki *et al.*¹⁰⁾ and of Cross *et al.*²⁾ In view of the fact that the powders used in the experi-

Table 4. Summarized results of Tables 2 and 3Number of the affairs of $\Delta = |\bar{\varepsilon} - \bar{\varepsilon}_{\text{exp}}|$

(1) Data by Ridgway and Tarbuck

	0.00	0.01	0.02	0.03	0.04	0.05	max.	Δ
Suzuki <i>et al.</i>	13	11	5	1	0	0	0.031	
Ours	13	15	2	0	0	0	0.025	
Cross <i>et al.</i>	13	15	2	0	0	0	0.025	

(2) Data by Westman and Hugill

	0.00	0.01	0.02	0.03	0.04	0.05	max.	Δ
Suzuki <i>et al.</i>	15	13	8	8	7	4	0.202	
Ours	22	24	6	1	1	1	0.051	
Cross <i>et al.</i>	20	16	5	0	2	12	0.219	

Fig. 4. Comparison of theoretical estimates with experimental data by Furnas³⁾ and by Arakawa *et al.*¹⁾ Following values were used for calculations. Furnas: $D_1(\bar{\varepsilon}_o(D_1)) = 9.144 \text{ mm}$ (0.415) and $D_2(\bar{\varepsilon}_o(D_2)) = 0.5075 \text{ mm}$ (0.660). Arakawa *et al.*: $D_1(\bar{\varepsilon}_o(D_1)) = 20.1 \mu\text{m}$ (0.617) and $D_2(\bar{\varepsilon}_o(D_2)) = 3.5 \mu\text{m}$ (0.797)

ments of Arakawa *et al.* were very small in size (ca. 3.5 and 20.1 μm) and packed in air environments, the discrepancies observed above seem to be associated with strong interactions between particles and hence with the new assumption introduced in this paper. Further investigations are needed for precise study of that assumption.

Nomenclature

$C_j(D)$	= coordination number as function of diameter D and species j of specified particle	[—]
D	= diameter of particle	[m]
D_i	= diameter of particle of i -th size component	[m]
$\bar{D}$	= average diameter of particles	[m]
$f(D)$	= number frequency size distribution of particles	[1/m]
f_i	= number fraction of particles of i -th component	[—]

$f_j(D)$	= sub-fraction of $f(D)$, belonging to species j	[1/m]
i	= ordinal number regarding component of size of particles $i=1$; largest fraction $i=m$; smallest fraction	[—]
m	= total number of components of sizes of particles	[—]
N	= total number of particles of mixture	[—]
$\bar{n}$	= average number regarding common occupation of space	[—]
p	= dummy variable regarding i	[—]
s	= total number of different species	[—]
V_{BT}	= total bulk volume of packing	[m ³]
$V_c(D)$	= volume of space allocated to specified particle of diameter D	[m ³]
$V_m(D)$	= volume of spherical shell	[m ³]
V_{ST}	= total volume of solid particles	[m ³]
v_i	= volume fraction of particles of i -th component	[—]
v_l	= volume fraction of larger particles of binary mixture	[—]
$\bar{\varepsilon}$	= overall average porosity of packing	[—]
ε_A	= surface porosity	[—]
$\varepsilon_{A,j}(D)$	= surface porosity as function of size D and species j of specified particle	[—]
$\varepsilon_j(D)$	= average porosity of hypothetical sphere of diameter D and species j	[—]
$\varepsilon_{mj}(D)$	= average porosity of spherical shell as function of diameter D and species j of core particle	[—]
$\bar{\varepsilon}_{\text{min}}$	= porosity of densest packing for binary mixture of fixed size ratio	[—]
$\bar{\varepsilon}_o$	= porosity of packing of single-size particles	[—]
$\bar{\varepsilon}_{oj}(D)$	= porosity of packing of identical particles of size D and species j	[—]
ε_{cr}	= critical porosity of binary mixture	[—]

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