DETERMINATION OF SPECIFIC RESISTANCE OF HIGHLY COMPRESSED CAKE BY USE OF EXPRESSION DATA

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Specific resistance α , which is one of the fundamental properties of particulate beds, is usually determined by the compression permeability test.^{1,4)} However, this method is rather tedious, especially in determining an α -value of highly compressed cake, where a liquid should be permeated into the cake for a long time. A previous paper²⁾ presented a new technique for obtaining permeability data in a moderate pressure range based on a filtration experiment. In this paper, we shall present a convenient method for obtaining a local α -value of highly compressed cake, using both numerical solutions of the consolidation equation and empirical expression data of homogeneous semisolid materials.

1. Theory

If the creep effect of the material is negligible, the consolidation equation can be represented by the form³⁾

$$\frac{\partial p_s}{\partial \theta_c} = C_e \left\{ \frac{\partial^2 p_s}{\partial \omega^2} - \frac{1}{\alpha} \left(\frac{d\alpha}{dp_s} \right) \left(\frac{\partial p_s}{\partial \omega} \right)^2 \right\} \quad (1)$$

$$C_e = \frac{1}{\mu \rho_s \alpha (-de/dp_s)} \quad (2)$$

where p_s is the local solid compressive pressure; θ_c , the consolidation time; C_e , the local modified consolidation coefficient and a function of p_s ; ω , the volume of solid per unit sectional area between an arbitrary position in cake and the filter medium; α , the local specific resistance of cake; μ , the viscosity of liquid; ρ_s , the true density of solid, and e , the local void ratio of cake. Eq. (1) can be rewritten by using the following variables:

$$\phi = p_s/p \quad (3)$$

$$z = i\omega/\omega_0 \quad (4)$$

$$T_c^* = i^2 C_e^* \theta_c / \omega_0^2 \quad (5)$$

$$C = C_e \frac{p}{\alpha} \frac{d\alpha}{dp_s} \quad (6)$$

in a dimensionless form as

$$\frac{\partial \phi}{\partial T_c^*} = \frac{C_e}{C_e^*} \frac{\partial^2 \phi}{\partial z^2} - \frac{C}{C_e^*} \left(\frac{\partial \phi}{\partial z} \right)^2 \quad (7)$$

Here p is the expression pressure; i , the number of drainage surfaces; ω_0 , the total solid volume in cake per unit sectional area; T_c^* , the dimensionless consolidation time; and C_e^* , the local C_e -value at the drainage surface ($p_s = p$).

If compression permeability data of α and e can be represented in the following form:

$$\alpha = \alpha_0 p_s^n \quad (8)$$

$$e = E_0 - C_c \ln p_s \quad (9)$$

then Eq. (7) can be rewritten in the form

$$\frac{\partial \phi}{\partial T_c^*} = \phi^{1-n} \frac{\partial^2 \phi}{\partial z^2} - n \phi^{-n} \left(\frac{\partial \phi}{\partial z} \right)^2 \quad (10)$$

Numerical solutions of Eq. (10) are summarized in terms of the average consolidation ratio, U_c , which is a measure of the extent of consolidation.

$$U_c = \frac{\int_0^1 (e_1 - e) dz}{\int_0^1 (e_1 - e_\infty) dz} = \frac{\int_0^1 \ln(p_s/p_{s1}) dz}{\int_0^1 \ln(p/p_{s1}) dz} \quad (11)$$

Here e_1 denotes the initial void ratio of homogeneous semisolid material; e_∞ , the equilibrium void ratio of compressed cake under the expression pressure p ; and p_{s1} , the pre-consolidation pressure for obtaining a homogeneous semisolid material of void ratio e_1 . Since the solution ϕ of Eq. (10) is a function of T_c^* , z , n and p/p_{s1} , the $U_c - T_c^*$ relation of semisolid material depends only on the values of n and p/p_{s1} .

Based on Eqs. (10) and (11), the dimensionless consolidation time T_{c50}^* required for attaining 50% of U_c at each n and p/p_{s1} is shown in Fig. 1. In the

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Fig. 1. Relationship between T_{c50}^* , p/p_{s1} and n

numerical computations of Eq. (10), the interval of dimensionless distance Δz was set at 0.1 and that of dimensionless time ΔT_c^* was set at 0.0001. Since the relation between $\log T_{c50}^*$ and $\log(p/p_{s1})$ at each n is almost linear, as shown in the figure, they can be represented by the following formula.

$$T_{c50}^* = 0.196(p/p_{s1})^{f(n)} \quad (12)$$

$$f(n) = -0.0379n^2 - 0.288n + 0.324 \quad (13)$$

$$(0.1 \leq n \leq 2.0, \quad 2 \leq p/p_{s1} \leq 100)$$

On the other hand, combining Eqs. (2), (5), (8) and (9), T_{c50}^* is expressed by

$$T_{c50}^* = \frac{i^2 \theta_{c50} p_{s1}^{1-n}}{\mu \omega_0^2 \rho_s \alpha_0 C_c} (p/p_{s1})^{1-n} \quad (14)$$

where θ_{c50} is the real consolidation time for attaining 50% of U_c . Therefore, combining Eqs. (12) and (14) yields

$$\frac{i^2 \theta_{c50} p_{s1}^{1-n}}{\mu \omega_0^2 \rho_s \alpha_0 C_c} = 0.196(p/p_{s1})^{f(n)+n-1} \quad (15)$$

That is, for semisolid materials at the same pre-consolidation pressure p_{s1} ,

$$\frac{i^2 \theta_{c50}}{\mu \omega_0^2} \propto (p/p_{s1})^{f(n)+n-1} \quad (16)$$

because ρ_s , C_c , α_0 , n and p_{s1} are constant for these materials. Accordingly, obtaining θ_{c50} from constant-pressure expression data, and plotting $(i^2 \theta_{c50}/\mu \omega_0^2)$ and (p/p_{s1}) on logarithmic graph paper, we can find the value of n from the slope of the plots.

$$\text{slope} = f(n) + n - 1$$

$$= -0.0379n^2 + 0.712n - 0.676 \quad (17)$$

The local specific resistance, α , corresponding to the

Fig. 2. Empirical determination of n

expression pressure p , is obtained from Eqs. (2), (5), (9) and (12), thus:

$$\alpha = \frac{i^2 \theta_{c50} p}{\mu \omega_0^2 \rho_s C_c \{0.196(p/p_{s1})^{f(n)}\}} \quad (18)$$

2. Discussion

The experimental materials used in this study were Korean kaolin ($\rho_s = 2600 \text{ kg/m}^3$) and Victorian brown coal ($\rho_s = 1480 \text{ kg/m}^3$). These materials were slurried and placed in a compression permeability cell.^{1,4)} Compression permeability data α and e were measured^{1,4)} under solid compressive pressure p_s of 981 kPa–6.86 MPa. ($C_c = 0.161$ for Korean kaolin and 0.458 for brown coal.) These slurried materials were also pre-consolidated under pressure p_{s1} of 98.1–196 kPa, resulting in homogeneous semisolid materials. Then these semisolid materials were expressed under constant-pressure p of 981 kPa–6.86 MPa, thereby obtaining θ_{c50} .

Figure 2 shows the empirical relation between $(i^2 \theta_{c50}/\mu \omega_0^2)$ and (p/p_{s1}) . It can be seen from the figure that the slopes of plots of Victorian brown coal are independent of the pre-consolidation pressure p_{s1} . From the slopes of plots in the figure, we can find that $n = 0.88$ for Korean kaolin and 1.30 for Victorian brown coal. **Figure 3** shows the $\alpha - p_s$ relation calculated from Eq. (18). The agreement between calculated and experimental α -values (obtained by the permeability test) is satisfactory. (The maximum error of the calculated value compared to the experimental one is 18% for Korean kaolin and 35% for Victorian brown coal.)

A future research topic would be to obtain the α -value of material which fails to obey Eqs. (8) and (9).

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Fig. 3. Comparison of theoretical and experimental permeability data

Nomenclature

C_c	= empirical constant for compression data	[—]
C_e	= local modified consolidation coefficient	[m ² /s]
C_e^*	= C_e -value at the drainage surface ($p_s = p$)	[m ² /s]
E_0	= empirical constant for compression data	[—]
e	= local void ratio	[—]
e_1	= initial void ratio	[—]

e_∞	= equilibrium void ratio under expression pressure p	[—]
i	= number of drainage surfaces	[—]
n	= compressibility coefficient	[—]
p	= expression pressure	[Pa]
p_s	= local solid compressive pressure	[Pa]
p_{s1}	= pre-consolidation pressure where equilibrium void ratio e_1 is attained	[Pa]
T_c^*	= dimensionless consolidation time	[—]
T_{c50}^*	= dimensionless consolidation time for attaining 50% of U_c	[—]
U_c	= average consolidation ratio	[—]
z	= dimensionless distance	[—]
α	= local specific resistance	[m/kg]
α_0	= empirical constant for permeability data	[m/(kg·Pa ⁿ)]
θ_c	= consolidation time	[s]
θ_{c50}	= consolidation time for attaining 50% of U_c	[s]
μ	= viscosity of liquid	[Pa·s]
ρ_s	= true density of solid	[kg/m ³]
ω	= volume of solid per unit sectional area between an arbitrary position in cake and a filter medium	[m]
ω_0	= total solid volume in cake per unit sectional area	[m]

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