A NOTE FOR SOLVING THE EXPRESSION EQUATION BY THE METHOD OF MOMENT*

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In the previous report³⁾ the method of moment was employed to solve the basic differential equation of forced expression. No explanation, however, was given as to the validity of using the larger root β for the non-dimensional critical time θ_{cri} .

It is shown in this complementary report that theoretically the value of β is restricted to the larger root of Eq. (17)³⁾ with the range of $0 < K < \sqrt{3}/2$.

Some comments on the diffusion process with different boundary conditions are also given.

In the previous report³⁾ the value of β , which is the non-dimensional critical time θ_{cri} , was determined as the larger positive root of the quadratic equation:

$$24F(0)\alpha\beta^2 + (\alpha - 108F(0))\beta + 3/2 = 0 \quad (17)^{3)}$$

without any discussion about the choice.

Some discussion about the choice of β by the original investigator H. Fujita was reproduced in the text by Crank^{2a)}, and recently Ames¹⁾ stated that it was not so rigorous.

It is the purpose of this report to discuss the problem because the Ames description is not clear.

From Eq. (17')³⁾, one obtains

$$\sqrt{\beta} = (1 \pm \sqrt{1 - 2K^2/3})/4K \quad (A)$$

where K is defined by Eq. (26'')³⁾ and is the non-dimensional form of K_e .

In Fig. 1 the square of the larger $\sqrt{\beta}$ of Eq. (A) is shown by the line $(A)_L$ and the square of the smaller by the line $(A)_S$. These lines coincide at the point P.

The physical meaning of A_2 is visualized from Eq. (18)³⁾ as

$$\varepsilon(1, \theta) = 1 - A_2(\theta) \quad (18')$$

That is, A_2 relates to ε at $v=1$ in the later stage and it increases as time elapses. Consequently ϕ becomes positive and ϕ is clearly positive by definition (Eq. (19')³⁾).

On the other hand from Eqs. (16)³⁾ and (19')

$$q(1) = \alpha/30 \quad (19'')$$

Inserting Eq. (19'') into Eq. (20)³⁾ yields

$$\phi(1) = 14(\alpha\beta + \alpha/12 - 9/2)/15 \quad (20')$$

As $\phi(1)$ is positive, one obtains

$$\alpha\beta + \alpha/12 - 9/2 > 0 \quad (20'')$$

Using Eq. (26')³⁾ and noting that K and β must be positive from their physical meaning, the above relation becomes

$$\beta^{3/2} - \frac{\beta}{2K} + \frac{\sqrt{\beta}}{12} - \frac{1}{96K} > 0 \quad (20''')$$

Inserting the larger $\sqrt{\beta}$ of Eq. (A) into Eq. (20'''), one obtains

$$\sqrt{1 - 2K^2/3} > 0 \quad (B)$$

and therefore

$$K < \sqrt{3}/2 \quad (B')$$

If the smaller $\sqrt{\beta}$ of Eq. (A) were inserted, the relation becomes $\sqrt{1 - 2K^2/3} < 0$, and this is not allowed.

Thus it is clear that the value of β must be the larger positive root of Eq. (17) and the range of K is $0 < K < \sqrt{3}/2$.

The above restriction on K comes solely from the boundary conditions (9)³⁾ and (10)³⁾ and does not depend on the method of moment.

The range of β restricted by the relation (20''') is shown by the shaded area in Fig. 1 in which the broken line itself is to be excluded.

When the expression coefficient is constant, α is 30 from Eq. (16) and the roots of Eq. (17) are 1/12 and 1/40, then the values of K become 1.155 and 1.186 respectively from Eq. (17'). The set ($\beta=1/12$, $K=1.155$) satisfies the relation (20''') as shown by the point Q in Fig. 1, but the set ($\beta=1/40$, $K=1.186$) does not (cf. the point R). Therefore, the value of β must be the larger positive root of Eq. (17).

The same procedure is applied for the diffusion process

Fig. 1 The range of β with varying K

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with the following boundary conditions^{2b)}:

$$\begin{aligned} c &= 0, & t &= 0, & 0 < x < 1 \\ c &= 1, & x &= 1, & t > 0 \\ \partial c / \partial x &= 0, & x &= 0, & t > 0 \end{aligned}$$

In this case $\sqrt{\beta}$ is found to be the larger root of Eq. (9.228)^{2a)}, given as

$$\sqrt{\beta} = \frac{1 + \sqrt{1 - 2K^2/3F(1)}}{4K} \quad (C)$$

and the required range of K is $0 < K < \sqrt{3F(1)/2}$.

Notation

c = non-dimensional concentration of diffusing substance [—]
 $F(1)$ = non-dimensional coefficient of diffusion, $F(c)$, at $c=1$

t = non-dimensional time of diffusion process [—]
 x = non-dimensional distance in opposite direction of diffusion from phase boundary [—]

Note: The other symbols are the same as those in the previous report³⁾

Literature cited

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- 2b) Crank, J.: *ibid.*, 178
- 3) Satoh, T. and K. Atsumi: *J. Chem. Eng. Japan*, 3, 92 (1970)