MICROMIXING MODELS IN A CONTINUOUS FLOW CHEMICAL REACTOR

TAKEICHIRO TAKAMATSU**, TATSURO SAWADA***
AND NOBORU IZUMOZAKI****

Dept. of Chem. Eng., Kyoto Univ., Kyoto

Dept. of Chem. Eng., Kanazawa Univ., Kanazawa

To display actual values of non-linear reaction in a continuous chemical reactor, micromixing models by Weinstein et al.¹¹⁾ and by the authors⁹⁾ are applied and compared. These models are able to represent the values of micromixing effect by combining the effect of complete segregation and maximum mixedness in proper proportion.

Next, as it might be thought more proper to represent the micromixing effect directly by a mixing model, taking into consideration the flow pattern of the reactor, this is attempted with the mixing model proposed in the previous paper⁷⁾. As a result, it is recognized that the model represents experimental values fairly well, as shown in Fig. 7.

Introduction

Various models have been built to represent quantitatively flow pattern and residence time distribution of fluid elements in a stirred tank. In the case of a 1st order constant temperature reaction, the conversion in a stirred tank reactor can be obtained only from information about the residence time distribution function and reaction kinetics. But as to reaction processes other than the 1st order, determination of conversion is not simple, because of the micromixing effect. For two extreme conditions of micromixing, the concept of complete segregation (C.S.) is considered by Danckwerts¹⁾, and that of maximum mixedness (M.M.) by Zwietering¹³⁾. In the condition of C.S., the fluid elements form discrete lumps and each fluid element is completely mixed, such as in an ideal batch reactor. As a whole the fluid is satisfied with the residence time distribution. In the condition of M.M., the concept of age and life expectation are introduced, and fluid elements having identical life expectation are always mixed uniformly. Actual experimental measurements of a continuous stirred tank reactor indicate, as reported by Worrell et al. and others^{2, 12)}, that the condition is almost at C.S. in the range of low Reynolds number, and that it approaches M.M. as Reynolds number becomes higher. The same result was obtained also by the authors¹⁰⁾. Weinstein and Adler¹¹⁾ have proposed a model combining C.S. and M.M. to show the micromixing effect. Recently, many other similar

models are reported by Mukataka, S. and Kobayashi, J.³⁾, Ng, D. and Rippin, D.⁴⁾, Nisimura, Y. and Matsubara, S.^{5, 6)} and the authors⁹⁾. However, these models are based on adding one or more parameters to show the micromixing effect, and they do not always express physical phenomena in a tank. In this paper, the model of Weinstein et al. and that of the authors are applied to the analysis for each Reynolds number.

Next, it is emphasized that the degree of micromixing can be represented reasonably by modifying phenomenological models which have been proposed to express the residence time distribution in a stirred tank^{7, 8, 10)}.

1. Micromixing Model by Combining C.S. and M.M.

1.1 Explanation of the micromixing model

1.1.1. The model by Weinstein and Adler¹¹⁾ Defining the dimensionless time, age and life expectation, such as $\phi (= t/\theta)$, $\alpha^* (= \alpha/\theta)$ and $\lambda^* (= \lambda/\theta)$, in the consecutive model by Weinstein and Adler, the fluid elements with an age of $0 \leq \alpha^* < \phi_c$ flow out after experiencing only C.S., and the fluid elements with an age of $\alpha^* \geq \phi_c$ are affected by M.M. after experiencing C.S. during an age of ϕ_c . As shown in Fig. 1, the model agrees with the M.M. model of Zwietering for $\phi_c = 0$, and the C.S. model of Danckwerts for $\phi_c = \infty$. The concentration Γ_s of the fluid which flows out after experiencing only C.S. is

$$\Gamma_s = \int_0^{\phi_c} \Gamma(\phi) E(\phi) d\phi / \int_0^{\phi_c} E(\phi) d\phi \quad (1)$$

where $\Gamma(\phi)$ and $E(\phi)$ are defined as concentration in the segregated flow and residence time distribution function, respectively. The fraction of the fluid is represented by

$$\int_0^{\phi_c} E(\phi) d\phi$$

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** Dept. of Chem. Eng., Kyoto Univ., Kyoto

*** Dept. of Chem. Eng., Kanazawa Univ., Kanazawa

**** Central Research Lab., Asahi Glass Co.

Fig. 1 Consecutive reactor model by Weinstein and Adler

Fig. 2 Micromixing model of dividing the fluid into m equal parts

The concentration of the fluid leaving the part of M.M., $\Gamma_m(\phi_c)$ is calculated by

$$\frac{d\Gamma_m(\phi)}{d\phi} = -\frac{E(\phi)}{1-F(\phi)} \{ \Gamma(\phi_c) - \Gamma_m(\phi) \} + R[\Gamma_m(\phi)] \quad (2)$$

and the boundary condition of

$$\left. \frac{d\Gamma_m(\phi)}{d\phi} \right|_{\phi=\phi_{\infty}} = 0 \quad (3)$$

where $\Gamma(\phi_c)$ is the concentration of the input into the part of M.M., after staying at the part of C.S. for ϕ_c . The fraction of the fluid experiencing M.M. is represented by

$$\int_{\phi_c}^{\infty} E(\phi) d\phi$$

From the above, the average concentration $\bar{\Gamma}_e$ of the fluid leaving a tank reactor is

$$\bar{\Gamma}_e = \Gamma_s + \Gamma_m(\phi_c) \int_{\phi_c}^{\infty} E(\phi) d\phi \quad (4)$$

Fig. 3 Calculated values of the model by Weinstein et al. and the authors

1.1.2 Model of divided the fluid into m equal parts⁹⁾

Let us consider a micromixing model as shown in Fig. 2 (a).

The part below the curve of life expectation function $I(\phi) = 1 - F(\phi)$ is divided into m equal parts by horizontal lines, as shown in Fig. 2 (b). That is, the entering fluid is divided into m equal parts and flows through each divided section. They are never mixed with the fluid flowing through other sections. In each divided section, maximum mixing occurs only between fluid elements which have equal life expectation. The parameter of this model for representing a state between the cases of the M.M. model and the C.S. model is the number m of equally divided sections. This model agrees with the model of M.M. at $m = 1$ and with that of C.S. at $m = \infty$. The concentration Γ_j leaving the j -th section is calculated as follows. At first, as to the part A_j of M.M., the following equation can be written.

$$\frac{d\Gamma_m(\phi)}{d\phi} = -\frac{E(\phi)}{1-F(\phi)-\frac{j-1}{m}} \{ 1 - \Gamma_m(\phi) \} + R[\Gamma_m(\phi)] \quad (5)$$

The boundary condition at $\phi = \phi_j$ is

$$\Gamma_m(\phi)|_{\phi=\phi_j} = 1 \quad (6)$$

From the above, the concentration $\Gamma_m(\phi_{j+1})$ leaving the part A_j is obtained. Next, as to the part B_j of C.S., the following equation is written.

$$\frac{d\Gamma_j}{d\phi} = R[\Gamma_j] \quad (7)$$

The initial condition at $\phi = \phi_{j+1}$ is

$$\Gamma = \Gamma_m(\phi_{j+1}) \quad (8)$$

Assuming a second-order reaction, the reaction rate can be shown by the following relation

$$R[\Gamma_j] = K\Gamma_j^2 \quad (9)$$

In this case, the concentration Γ_j leaving the part B_j is obtained by

$$\Gamma_j = \frac{1}{\frac{1}{\Gamma_m(\phi_{j+1})} + K\phi_{j+1}} \quad (10)$$

The average concentration $\bar{\Gamma}_e$ leaving a tank reactor may be obtained by

$$\bar{F}_e = \sum_{j=1}^m \frac{\Gamma_j}{m} \quad (11)$$

In actual calculations, the value of $\Gamma_m(\phi)$ at $\phi = \infty$ of the first section may be numerically obtained by using the approximate value of $\Gamma_m(\phi)$ at $\phi \doteq 10$ without noticeable loss of accuracy.

1.2 An example of numerical calculation by the combined models of C.S. and M.M.

To compare the results of numerical calculations by two models of Weinstein et al. and the authors, Domköhler number $K = 5$ in reaction rate constant of second-order isothermal reaction is used as an example. The response of steady state to each value of parameter ϕ_e by the model of Weinstein et al. is shown for a 2-flat-blade paddle in Fig. 3. This calculation for each Reynolds number N_{Re} is made using the residence time distribution that is given by the value of the mixing parameters reported by the authors^{7,8)}, such that $\lambda_q = 8.1$, $\lambda_r = 4.2$ at $N_{Re} = 50$, $\lambda_q = 20.0$, $\lambda_r = 12.2$ at $N_{Re} = 200$, $\lambda_q = 11.0$, $\lambda_r = 9.2$ at $N_{Re} = 1000$ for $F_e = 4$. It is recognized that the value of $\bar{F}_e$ for $\phi_e = 0$ agrees with that of M.M. for each N_{Re} . The value decreases sharply in the region of $\phi_e = 0 \sim 0.5$ and approaches that of C.S. at each N_{Re} .

The curves of $\bar{F}_e$ vs. m calculated by the m equal parts model are of quite similar form to those of $\bar{F}_e$ vs. ϕ_e by Weinstein's model, if the abscissa of ϕ_e and m are taken properly. The values of parameters of these models, ϕ_e and m , can be determined by plotting the experimental values of $\bar{F}_e$ in Fig. 3. These values complete a model to predict conversion. And then, as the fluid elements in the neighborhood of the inlet seems to be in the condition of M.M., the model of the authors represents physical phenomena in a stirred tank fairly well.

2. Micromixing Model from the Viewpoint of Flow Pattern⁹⁾

2.1 Explanation of the micromixing model

Because of the micromixing effect, the conversion of a simple non-linear isothermal reaction may be between the values of C.S. and M.M. for each Reynolds number. To display actual values of non-linear reaction, the effect of parameters in micromixing models by Weinstein et al. and by the authors is calculated in the previous chapter. These parameters express whether it is near the state of C.S. or that of M.M., but these are not introduced directly from models based on physical phenomena in a tank reactor. Therefore, the values of these parameters must be evaluated by adding other information to that of macromixing, such as residence time distribution from tracer response. For example, we can determine the values of these parameters so that the calculated results by a model agree with the experimental values of tracer response and conversion for non-linear reaction. However, the phenomena of micromixing exist even if no reaction is occurring in

Fig. 4 Mixing model of zone I of stirred tank reactor

Fig. 5 Mixing model in the case that each point of zone I is in the state of segregation

a vessel, and it is desirable that the degree of micro-mixing can be quantitatively estimated from information other than reaction in a vessel. From the viewpoint mentioned above, the authors suggest the expression of degree of micromixing by using a mixing model showing flow pattern in a stirred tank. 2.1.1 Mixing model of zone I⁷⁾ The mixing model of zone I ($N_{Re} \leq 50$, the range where the flow pattern is represented by the combination of dead space and piston flow) for low Reynolds number is shown in Fig. 4, in which point A is the confluence point of feed flow and feed back flow and C is the separating point of exit flow and feed back flow. When the feed flow and the feed back flow get together at point A, there may be two extreme cases. First, these streams may be mixed completely. Second, they aren't mixed at all and flow separately into the plug flow section. For the latter case the streams may or may not be mixed at points B, C and D. When mixing occurs at a point it is represented by the label "Mixedness (or M)". In this case, "Mixedness" means that all streams are mixed together. It should be noted that this is different from maximum mixedness.

The case where no mixing occurs is represented by "Segregation (or S)". This means that the streams aren't mixed and is different from complete segregation.

Table 1 Combinations of mixing condition at each point and section (zone I)

No.	Point A	B	C	D
1	S	S	S	S
2	S	S	S	M
3	S	S	M	M
4	S	M	M	M
5	M	M	M	M

Fig. 6 Mixing model of zone II of stirred tank reactor

When the streams are not mixed at all of the points A, B, C, and D, point A has N separated streams if the flow circulates $N-1$ times.

Let us consider each point concentration in the j -th circulation to express the concentration of separated streams shown in Fig. 5. As the conversion in piston flow relates to lag time,

$$\Gamma_2^j = \int_0^{\phi_L'} \delta(\phi - \phi_L') \Gamma(\phi) d\phi \quad (12)$$

$\Gamma(\phi)$ of Eq.(12) is

$$\frac{d\Gamma(\phi)}{d\phi} = R[\Gamma(\phi)] \quad (13)$$

The initial condition of Eq.(13) is

$$\Gamma(\phi) = \Gamma_1^j = \Gamma_4^{j-1} \quad (14)$$

$$\Gamma_4^{j-1} = \int_0^{\phi_L''} \delta(\phi - \phi_L'') \Gamma(\phi) d\phi \quad (15)$$

$\Gamma(\phi)$ of Eq.(15) is given by Eq.(13) at the initial condition

$$\Gamma(\phi) = \Gamma_2^{j-1} \quad (16)$$

where $j = 1, 2, 3, \dots, N$

$$\Gamma_1^1 \equiv \Gamma_4 \quad (17)$$

Therefore, Γ_2^N and Γ_4^{N-1} are given by calculating these equations one after another. The concentration Γ_e leaving the stirred tank reactor after circulation of $N-1$ times is

$$\Gamma_e = \frac{1}{1+\lambda_q} \left\{ \Gamma_1^1 + \frac{\lambda_q}{1+\lambda_q} \Gamma_2^2 + \left(\frac{\lambda_q}{1+\lambda_q} \right)^2 \Gamma_3^3 + \dots + \left(\frac{\lambda_q}{1+\lambda_q} \right)^{N-1} \Gamma_2^N \right\} \quad (18)$$

The average concentration $\bar{\Gamma}_e$ at the steady state is

$$\bar{\Gamma}_e = \lim_{N \rightarrow \infty} \Gamma_e \quad (19)$$

$$= \lim_{N \rightarrow \infty} \frac{1}{1+\lambda_q} \sum_{j=1}^N \left(\frac{\lambda_q}{1+\lambda_q} \right)^{j-1} \Gamma_2^j \quad (20)$$

If point A is M, point B is also M and if point C is M, point D is M, too. Let us consider each point concentration in such a case.

$$\Gamma_2 = \int_0^{\phi_L'} \delta(\phi - \phi_L') \Gamma(\phi) d\phi \quad (21)$$

$\Gamma(\phi)$ of Eq.(21) is given by Eq.(13) at the initial condition

$$\Gamma(\phi) = \Gamma_1 \quad (22)$$

$$\Gamma_4 = \int_0^{\phi_L''} \delta(\phi - \phi_L'') \Gamma(\phi) d\phi \quad (23)$$

$\Gamma(\phi)$ of Eq.(23) is given by Eq.(13) at the initial condition

$$\Gamma(\phi) = \Gamma_2 = \Gamma_3 \quad (24)$$

At a point A, Γ_1 is

$$\Gamma_1 = \frac{\lambda_q}{1+\lambda_q} \Gamma_4 + \frac{1}{1+\lambda_q} \Gamma_5 \quad (25)$$

Γ_2 or Γ_3 is given by one of these equations. And then the average concentration $\bar{\Gamma}_e$ leaving the stirred tank reactor is obtained.

The combinations of mixing conditions at each point are shown in Table 1.

2.1.2 Mixing model of zone II⁽⁷⁾ The mixing model of zone II ($50 < N_{Re} \leq 10^3$, the range where the flow pattern is represented by a combination of piston flow and perfect mixing) is shown in Fig. 6, and the mixing conditions at points A, B, C and D are assumed to be the same as those for zone I, but section E of perfect mixing should be added. The state of perfect mixing region E is C.S. as one extreme and M.M. as another. If point A is M, point B is also M and if point C is M, point D is M, too, as in zone I.

Let us consider about the case where A and B are S, C and D are M, and E is C.S. as in Fig. 6. At a point A the inflowing fluid of concentration Γ_1' and feed back of concentration Γ_4'' are in separated piston flows. These two streams are mixed at point C to give a concentration of Γ_4 .

$$\Gamma_2' = \int_0^{\phi_L'} \delta(\phi - \phi_L') \Gamma(\phi) d\phi \quad (26)$$

$$\Gamma_2'' = \int_0^{\phi_L''} \delta(\phi - \phi_L'') \Gamma(\phi) d\phi \quad (27)$$

$$\Gamma_3 = \int_0^{\infty} e^{-\phi} \Gamma(\phi) d\phi \quad (28)$$

$$\Gamma_5 = \int_0^{\phi_L''} \delta(\phi - \phi_L'') \Gamma(\phi) d\phi \quad (29)$$

$\Gamma(\phi)$ of Eqs.(26)~(29) are given by Eq.(13) at each initial condition

$$\Gamma(\phi) = \Gamma_1' \equiv \Gamma_4 \quad (30)$$

$$\Gamma(\phi) = \Gamma_1'' \equiv \Gamma_5 \quad (31)$$

$$\Gamma(\phi) = \Gamma_2 \quad (32)$$

$$\Gamma(\phi) = \Gamma_4 \quad (33)$$

And then

$$\Gamma_2 = \frac{1}{1+\lambda_q} \Gamma_2' + \frac{\lambda_q}{1+\lambda_q} \Gamma_2'' \quad (34)$$

$$\bar{\Gamma}_e = \frac{1+\lambda_q-\lambda_r}{1+\lambda_q} \Gamma_2 + \frac{\lambda_r}{1+\lambda_q} \Gamma_3 \quad (35)$$

Therefore, $\bar{\Gamma}_e$ is obtained by these equations. If

region E is M.M., in place of Eq.(28) and Eq.(32)

$$\Gamma_3 = \Gamma_2 + \theta R[\Gamma(\phi)] \quad (36)$$

is used.

In case A and C are S, B and D are M, and E is C.S..

$$\Gamma'_5 = \int_0^{\phi''} \delta(\phi - \phi''_L) \Gamma(\phi) d\phi \quad (37)$$

$$\Gamma''_5 = \int_0^{\phi''} \delta(\phi - \phi''_L) \Gamma(\phi) d\phi \quad (38)$$

should be used in place of Eq.(29).

$\Gamma(\phi)$ of Eq.(37) and Eq.(38) are given by Eq.(13) at each initial condition

$$\Gamma(\phi) = \Gamma'_4 \quad (39)$$

$$\Gamma(\phi) = \Gamma''_4 \quad (40)$$

Moreover,

$$\Gamma_5 = \frac{1+\lambda_q-\lambda_r}{1+\lambda_q} \Gamma'_5 + \frac{\lambda_r}{1+\lambda_q} \Gamma''_5 \quad (41)$$

Therefore, $\bar{\Gamma}_e$ is obtained by these equations and

$$\Gamma_5 = \Gamma''_1 \quad (42)$$

If region E is M.M., $\bar{\Gamma}_e$ is obtained by Eq.(36) in place of Eqs.(28) and (32).

The combinations of mixing condition in each point and section E are shown in **Table 2**.

2.1.3 Mixing model of zone III⁷⁾ The mixing model of zone III ($N_{Re} > 10^3$, the range where the flow pattern is represented by the combination of perfect mixing, compound vortex and short-circuiting) can be classified to only two states of C.S. and M.M. for perfect mixing region E.

2.2 Analysis and applicability of the mixing model

As to the mixing model of zone I, the effect of the combinations of mixing condition at each point, such as Nos. 1~5 of Table 1, can be numerically calculated. No. 1 is similar to the case of C.S. and No. 5 to M.M..

The reaction rate of second-order with 2-flat-blade paddle in a tank reactor will be considered as an example^{9,10)}.

The conversion for each Reynolds number is calculated from the residence time distribution of zone I that is given by the values of the parameters which were reported previously by the authors^{7,8)}, such that $\lambda_q = 4.2$ at $N_{Re} = 20$, $\lambda_q = 8.1$ at $N_{Re} = 50$ for Domköhler number $K = 5$. The results calculated for $N_{Re} = 20, 50$ shows that No. 1 in Table 1 corresponds to C.S., No. 5 to M.M. and Nos. 2, 3, 4, are between C.S. and M.M.. As a result, the range of $N_{Re} < 30$ in zone I is represented by the case of No. 1 or C.S. and the range of $30 \leq N_{Re} < 80$ is represented by the case of Nos. 2, 3, as in **Fig. 7**. This indicates that the mixing condition of point A and B, and that of C and D are almost equivalent. This fact might correspond to the actual phenomena.

As to the mixing model of zone II, the effect of the combinations of mixing condition at each point, such as Nos. 1~18 of Table 2, can be calculated by the values in Fig. 3. Taking into account that the mixing conditions of point A and B are nearly

Fig. 7 Conversion of the 2nd-order reaction in steady state

Table 2 Combinations of mixing condition at each point and section (zone II)

Point					
No.	A	B	C	D	E
1	S	S	S	S	C.S.
2	S	S	S	M	C.S.
3	S	S	M	M	C.S.
4	S	S	S	S	M.M.
5	S	S	S	M	M.M.
6	S	S	M	M	M.M.
7	S	M	S	S	C.S.
8	S	M	S	M	C.S.
9	S	M	M	M	C.S.
10	S	M	S	S	M.M.
11	S	M	S	M	M.M.
12	S	M	M	M	M.M.
13	M	M	S	S	C.S.
14	M	M	S	M	C.S.
15	M	M	M	M	C.S.
16	M	M	S	S	M.M.
17	M	M	S	M	M.M.
18	M	M	M	M	M.M.

Table 3 The results of combination for Reynolds number

Zone	I			II			III	
	No. 1	2	3	2	3	6	200~2000	2000~
Point	A	S	S	S	S	S	S	
B	S	S	S	S	S	S	S	
C	S	S	M	S	M	M	M	
D	S	M	M	M	M	M	M	
E				C.S.	C.S.	M.M.	M.M.	M.M.

equal, as also are those of point C and D as in zone I, it is found that the experimental values of the range of $80 \leq N_{Re} < 200$ in zone II are represented by Nos. 2, 3, 6 and those of the range of $200 \leq N_{Re} < 2000$ by No. 6 in Table 2.

As to the mixing model of zone III, the experimental values are represented by the case of M.M..

The results for each zone are shown in **Table 3**, which shows that the experimental values from extremely low Reynolds number to very high Reynolds number are easily expressed by the model of physical phenomena in the stirred tank. From them conversely the state of micromixing in the tank reactor can be imagined. There is little influence of micromixing at $N_{Re} < 30$, and all points may be in the state of "Segregation". At $30 \leq N_{Re} < 80$ points C and D are in the state of "Mixedness". The range of

$N_{Re} \geq 80$ has the perfect mixing section and it is in the state of C.S. at $80 \leq N_{Re} < 200$. As N_{Re} approaches about 2000, it becomes the state of M.M.. At $N_{Re} > 2000$ the effect of micromixing becomes largest and the state of the whole fluid in a tank reactor becomes nearly M.M..

Conclusion

In the case of non-linear reaction process in a continuous stirred tank reactor, the conversion varies according to micromixing effect. Weinstein et al. proposed models combining C.S. and M.M. to represent micromixing effect. The authors similarly proposed models and analyzed them with the experimental results of the same reaction conditions. It is confirmed from the numerical calculations that the values between C.S. and M.M. are expressed by the parameter ϕ_e of the model of Weinstein et al. and m of the authors. Various models are possible, assuming some parameters are introduced to express the degree of micromixing. However, it is usually difficult to evaluate the values of these parameters in advance without experimental data. Moreover, the experiments should be repeated with change of tank geometry.

Next, as it might be thought more proper to represent micromixing effect directly by a mixing model which takes account of flow pattern, such a representation was tried with the mixing model proposed by the authors. As a result, it is known that these models represent experimental values fairly well at any Reynolds number. The effect of micromixing at each Reynolds number, conversely, will be predicted by these models, and they can be used in scaleup.

In this paper, the isothermal second-order reaction is used as an example. If the equations of heat balance are used, the results of this paper may be applicable to non-isothermal reaction.

Nomenclature

c	= concentration	[g-mol/cm ³]
$E(\phi)$	= residence time distribution	[—]
F_e	= feed rate	[cm ³ /sec]
$F(\phi)$	= $\int_0^\phi E(\phi) d\phi$	[—]
$I(\phi)$	= life expectation function (= $1 - F(\phi)$)	[—]
K	= dimensionless rate constant of reaction (Damköhler number)	[—]
m	= number of section	[—]
N	= number of circulation	[—]
N_{Re}	= Reynolds number	[—]
q	= circulating rate	[cm ³ /sec]
R	= rate of reaction	[—]
R_b	= region of perfect mixing	[—]
R_p	= region of piston flow	[—]

r	= interchanging rate	[cm ³ /sec]
T_L'	= lag time of piston flow region of influent flow (= $V_p/2(F_e + q)$)	[sec]
T_L''	= lag time of piston flow region of feed back flow (= $V_p/2q$)	[sec]
V	= volume of reactor	[cm ³]
V_b	= volume of perfect mixing	[cm ³]
V_p	= volume of piston flow	[cm ³]
α	= age of a fluid element	[sec]
α^*	= α/θ	[—]
Γ	= dimensionless concentration (= c/c_i)	[—]
Γ_j	= dimensionless concentration leaving the j -th section	[—]
$\Gamma_m(\phi)$	= dimensionless concentration of maximum mixedness model	[—]
Γ_s	= dimensionless concentration leaving complete segregation model	[—]
θ	= mean residence time	[sec]
λ	= life expectation of a fluid element	[sec]
λ_q	= dimensionless circulating rate	[—]
λ_r	= dimensionless interchanging rate	[—]
λ^*	= λ/θ	[—]
ϕ	= t/θ	[—]
ϕ_e	= parameter of micromixing	[—]
ϕ_j	= time of the j -th section	[—]
ϕ_L'	= T_L'/θ	[—]
ϕ_L''	= T_L''/θ	[—]
ϕ_∞	= long time ($\phi_\infty \geq 10$)	[—]
Superscript		
1,2,3, ..., N	= number of circulation	
—	= average value	
Subscript		
1,2,3,4,5	= each point	
e	= output value	
i	= input value	

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