STUDY OF FLOW IN A POROUS TUBE WITH RADIAL MASS FLUX

TOKURO MIZUSHINA, SHUNJI TAKESHITA AND GENICHIRO UNNO

Department of Chemical Engineering, Kyoto University, Kyoto, Japan

In this paper, a fully developed turbulent flow of water in a porous-wall tube with suction or injection flow through the wall was experimentally studied. The experimental runs were executed at a fixed value of Reynolds number of inlet flow at about 25000 and with varying \( \nu_w/u \) from \(-1.78 \times 10^{-3}\) to \(4.16 \times 10^{-3}\).

The radial velocity component is almost linear, but it is found that the radial distribution of shear stress becomes non-linear as the mass flux through the wall becomes larger.

The mixing lengths in the flow with and without mass flux are correlated by a Reichardt-type equation and the normalized values of eddy diffusivity are also correlated with a single equation. The measurements of the local and average friction factor are in good agreement with predictions by the film theory.

Introduction

Drying and condensation from gas-vapor mixture have been studied as transport phenomena with mass flux at the wall by many investigators. In these phenomena, however, the mass flux is not too large. Since transport phenomena with high mass flux are applied to practical use, fundamental study of this subject is required. For this purpose, several investigations have been executed on the boundary layer developing on a porous flat plate from which the fluid is injected or sucked, but the experiments of injection and suction were studied separately.

In the previous paper, similar but comprehensive experimental studies in a circular tube for cases of both injection and suction were reported. In that paper, however, the average values of measurements over a whole length of porous tube were used to calculate the result, and it was assumed that the radial distribution of shear stress is linear as in the case of no radial mass flux.

In this paper, the above-mentioned weak points will be revised. Namely, the experimental apparatus was so made as to obtain axially local values of measurements, thus the radial distribution of shear stress was found to be non-linear in flow with high radial mass flux.

This fact will be taken into account, and the local radial distributions of axial velocity and local shear stress at the tube wall will be used in the calculation of the results.

Experimental Apparatus and Procedure

The experimental apparatus is shown schematically in Fig. 1. The process fluid, water, is maintained at \(20 \pm 0.05^\circ\)C in the reservoir tank (1) with a toluene expansion thermostat, and pumped into the test section (5) with a pump (2). The rate of this main flow is controlled by a valve (3) and measured by an orifice meter (4). In the case of sucked flow to the porous tube wall, the wall flux is controlled by a valve (6), and measured by an orifice meter (7). The water which is sucked out also returns to the reservoir.

In the case of injection flow, the water is pumped by a pump (8) and injected through the porous wall. The wall flux is controlled by a valve (9) and measured by an orifice meter (7). To check the material balance, the flow rate of the main stream at the exit of test tube is also measured. The test section which is shown in Fig. 2 consists of a sintered porous tube of bronze, 20 mm I.D., 25 mm O.D., 986 mm long, 30% porosity and 100 \(\mu\) diameter holes, and an outer tube of vinyl chloride polymer, 78 mm I.D.. As shown in Fig. 2, the active length of porous section is varied by wrapping vinyl chloride tape on the outer surface of porous tube in several lengths from the inlet end. The foreflow length before the test section is \(L_e/D = 80\).

At the exit end of the test tube, a pilot tube of 0.8 mm diameter is traversed to measure the radial distribution of axial velocity component. The axial change of static pressure is measured with pressure taps which are distributed axially on the porous tube wall.

The experimental runs for cases of both suction and injection were executed at fixed value of Reynolds number of the inlet flow at about 25,000 and with varying \(\nu_w/u\) from \(-1.78 \times 10^{-3}\) to \(4.16 \times 10^{-3}\).
The local values of radial velocity component, the local radial distribution of shear stress and mixing length and the local values of friction factor are calculated from the measurements of the radial distribution of axial velocity component and the static pressure at \( x/D = 40 \) and \( 49.3 \) which seem to be long enough to develop the flow under the effect of radial mass flux. In addition, the average values of friction factor are calculated from the momentum balance of the whole length of test section.

### Analyses and Results

To simplify the problem, it is assumed that the pressure is constant over a cross section because of comparatively small radial component of flow, and that the flux of injection or suction is constant over the whole tube surface. In addition, it is reasonable to consider that the fluid is incompressible and that the flow is axisymmetrical and isothermal.

The equations of continuity and motion are written in cylindrical coordinates as Eqs.(1) and (2) respectively.

### Boundary conditions are

at \( r = 0 : \ u = u_{w, ax}, \nu = 0, \tau = 0 \)

at \( r = R : \ u = 0, \ v = v_w, \tau = \tau_w \)

#### 1) Radial velocity component and radial distribution of shear stress

Integrating Eq.(1) from \( r = 0 \) to \( r = r \), one obtains

\[
\nu = \frac{1}{r} \int_0^r \left( \frac{\partial u}{\partial r} \right) \, dr
\]

From the assumptions we obtain

\[
\frac{\partial \nu}{\partial r} = 0
\]

and

\[
\frac{\partial \nu}{\partial x} \frac{\partial \nu}{\partial x}
\]

Substituting Eqs.(5) and (6) into Eq.(2), and multiplying \( r \) by the resultant, one obtains

\[
u u + \nu v = \frac{1}{\rho} \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr + \frac{1}{\rho} \frac{\partial (r \nu)}{\partial r} \]

Substituting Eq.(4) into Eq.(7) and integrating the resultant from \( r = 0 \) to \( r = r \) give

\[
\left[ \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr - \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr \right] + \int_0^r \left( \frac{\partial \nu}{\partial x} \right) \, dr = \frac{1}{\rho} \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr
\]

From the measurements of axial component of velocity at \( x/D = 40 \) and \( 49.3 \), \( \Delta u/\Delta x \) is calculated and regarded approximately as \( \partial u/\partial x \) in Eq.(4). The values of \( \nu \) thus calculated are plotted in Fig. 3 for flow with mass flux to the wall, and in Fig. 4 for flow with mass flux from the wall, respectively. From these diagrams, it is seen that the following usual assumption is almost valid in both cases.

\[
v/\nu_w = r/R
\]

The radial distribution of shear stress will be calculated from Eq.(9).

At \( r = R, \tau = \tau_w \), therefore Eq.(9) gives

\[
\frac{\partial}{\partial x} \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr = \frac{2}{\rho} \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr + \frac{1}{\rho} \frac{\partial (r \nu)}{\partial r}
\]

From the measurements of axial component of velocity at \( x/D = 40 \) and \( 49.3 \), \( \Delta u/\Delta x \) is calculated and regarded approximately as \( \partial u/\partial x \) in Eq.(4). The values of \( \nu \) thus calculated are plotted in Fig. 3 for flow with mass flux to the wall, and in Fig. 4 for flow with mass flux from the wall, respectively. From these diagrams, it is seen that the following usual assumption is almost valid in both cases.

\[
\frac{\partial (\Delta u/\Delta x)}{\partial x} \int_0^r \left( \frac{\partial u}{\partial x} \right) \, dr = \frac{r}{1} \frac{\partial (r \nu)}{\partial r}
\]

In Fig. 5, the axial distribution of static pressure is shown. With small mass flux, the curve is almost linear, but the larger the mass flux, the larger the curvature becomes.

Replacing \( \partial u, \Delta p \) and \( \Delta x \) in Eq.(9) with the measured values of \( \Delta u, \Delta p \) and \( \Delta x \) between \( x/D = 40 \) and \( 49.3 \), one can calculate the distribution of shear stress at \( x/D = 45 \). The results are plotted in Fig. 6, which indicates that the radial change of shear stress is almost linear between \( r/R = 0 \) and 0.95 but that there are very sharp changes near the wall.
Fig. 3 Radial velocity components in flow with suction mass flux to porous wall

Fig. 4 Radial velocity components in flow with injection mass flux from porous wall

Fig. 5 Axial static pressure distributions in tube flow with suction or injection mass flux

Fig. 6 Radial distributions of shear stress in tube flow with suction or injection mass flux

Fig. 7 Comparison of predictions with measurements of eddy diffusivity in tube flow with suction or injection mass flux
2) Profile of axial velocity component and mixing length

Since the value of $\partial v/\partial x$ is negligibly small, the following equation is valid in this case.

$$\frac{\tau}{\rho} = - (\nu + \varepsilon_M) \frac{du}{dr}$$  \hspace{1cm} (12)

At the turbulent core, $\nu$ may be neglected compared with $\varepsilon_M$. Hence, Eq.(12) is simplified to

$$\frac{\tau}{\rho} = - \varepsilon_M \frac{du}{dr}$$  \hspace{1cm} (13)

From Eq.(13) and the definition of Prandtl's mixing length,

$$\frac{\tau}{\rho} = - l \frac{du}{dr} \left( \frac{du}{dr} \right)$$  \hspace{1cm} (14)

the following equation is obtained.

Eqs.(13) and (14) enable us to calculate the values of $\varepsilon_M$ and $l$, because $\tau/\rho$ and $du/dr$ are already known. The values of $l$ are plotted in Fig. 7, which shows that the results for various values of $v_w/u$ are correlated by a single equation of Reichardt type\textsuperscript{10}, i.e.

$$l = \frac{\varepsilon_M}{\nu} \frac{\sqrt{\tau}}{\rho}$$  \hspace{1cm} (15)

However, the values of $\kappa$ and $n$ are different from those of Reichardt, i.e. $\kappa = 0.36$, $n = 3.35$ whereas Reichardt gives $\kappa = 0.4$, $n = 2$. This difference may be due to the effect of surface roughness in this experiment. Fig. 7 the equations of Reichardt\textsuperscript{10}, Prandtl\textsuperscript{9} and Nikuradse\textsuperscript{7} are also plotted for comparison.

Substituting Eq.(11) into Eqs.(1) and (2) and integrating the resultants from $r = 0 \sim r$ with the boundary conditions (3) give

$$v_w \left( \frac{r}{R} \right) \left(u - \int_0^r \frac{ru}{s} \, dr \right) = - \frac{r}{2p} \frac{dp}{dx} - \frac{\tau}{\rho}$$

Since $2 \int_0^r \frac{ru}{s} \, dr = \bar{u}$ at the turbulent core, the above equation is simplified to

$$v_w \left( \frac{r}{R} \right) (u - 2\bar{u}) = - \frac{r}{2p} \frac{dp}{dx} - \frac{\tau}{\rho}$$  \hspace{1cm} (17)

Substituting Eqs.(16) and (17) into Eq.(13), one obtains

$$\nu = \frac{3}{\varepsilon(n+1)} \left[ u_k^2 + v_w (2\bar{u} - u_{max}) \right]^{3/2}$$

$$\times \left( \frac{1 + ny}{1 - \eta^2} \right) + \frac{gr}{4x^2(n+1)^2} \left( \frac{1 + ny}{1 - \eta^2} \right)$$

$$\times \left( \frac{1 + ny^2}{1 - \eta^2} \right)$$

where $\eta = r/R$, $u_k^2 = - \frac{R \, dp}{2\rho \, dx}$

Eq. (18) is the equation of axial component of velocity of the flow in a circular tube with radial mass flux. The measurements and the calculations of the velocity...
profile at \(x/D = 49.3\) are compared in Fig. 8 to show their agreement.

From Eqs. (15) and (16), the following equation for normalized values of eddy diffusivity is obtained

\[
\frac{\xi_y}{\nu} R^+ = \frac{2}{3} \left[ 1 + n \left( \frac{r}{R} \right) \right] \sqrt{\frac{\tau}{\rho \nu^2}} \left( \frac{r}{R} \right)
\]

(19)

where \( R^+ = R/\nu \).

Fig. 9 shows that the experimental data of normalized values of eddy diffusivity in the tube flow with and without mass flux are correlated fairly well with Eq. (19). This result differs from that of Olson and Eckert, who indicated that the deviation of the normalized values of eddy diffusivity in the flow with injection mass flux from those without mass flux increased with increase of mass flux ratio \(v_w/u\).

3) Friction factor

From the definition, the following equation is written.

\[
\psi = \frac{\tau_w}{(1/2) \rho u^2}
\]

(20)

Substitution of Eq. (10) into Eq. (20) gives

\[
\psi = -\frac{R}{\rho \nu^2} \frac{d p}{d x} - \frac{2}{R} \frac{d}{d x} \left[ \left( \frac{r}{R} \right)^{1/2} \left( \frac{u}{\bar{u}} \right) \right] d r
\]

(21)

Using Eq. (21), the authors calculated the local friction factor at \(x/D = 45\) as follows.

\[
\psi\left|_{x/D=45} = \frac{1}{49.3-40} \int_{40}^{49.3} \frac{r}{R} \frac{d}{d r} \left( \frac{u}{\bar{u}} \right) d r \right.
\]

(21a)

From the balance of momentum in \(x\) direction between inlet and exit of the porous tube, the following equation is obtained.

\[
\pi R^2 [\rho (\bar{u}_2^2 - \bar{u}_1^2)] + (p_o - p_L) - F = 0
\]

(22)

Approximately,

\[
\bar{u}^2 = \bar{u}_1^2
\]

Hence,

\[
\pi R^2 [\rho (\bar{u}_2^2 - \bar{u}_1^2)] + (p_o - p_L) = F
\]

(23)

On the other hand, the force \(dF\) on the tube wall surface between \(x\) and \(x + dx\) is expressed as

\[
dF = 2 \pi R \ dx \left( \frac{\bar{u}^2}{2} \right) \psi
\]

(24)

Accordingly, the average friction factor over the wall surface \(\psi\) may be defined as follows.

\[
\psi = \frac{\pi R \bar{u}^2 dx}{(1/2) \rho u^2}
\]

(25)

From Eqs. (23) and (25), one obtains

\[
\psi = \frac{R [\rho (\bar{u}_2^2 - \bar{u}_1^2)] + (p_o - p_L)]}{(1/2) \rho \bar{u}_1^2}
\]

(26)

The material balance gives

\[
\bar{u} = \bar{u}_o - \frac{2}{R} v_w x
\]

\[
\bar{u}_L = \bar{u}_o - \frac{2}{R} v_w L
\]

(27)

By using Eq. (27), the integration in Eq. (26) is performed to give

\[
\int_0^L \bar{u}^2 dx = \frac{R}{6 \bar{u}_o} (\bar{u}_2^2 - \bar{u}_1^2)
\]

(28)

Consequently

\[
\bar{u}_L = \frac{6 \rho \bar{u}_o [\rho (\bar{u}_2^2 - \bar{u}_1^2)] + (p_o - p_L)]}{\rho (\bar{u}_2^2 - \bar{u}_1^2)}
\]

(29)

The experimental results of the local and average friction factors are plotted in Fig. 10 and compared with the prediction from the film theory by Stewart. The experiments and predictions are in good agreement. In this diagram \(\theta_o = \bar{c}_f/c_{f_o}\) or \(\bar{c}_f/c_{f_o}\), \(R_e = 2 \rho v_w/\mu \bar{u}_1\), and \(c_{f_o}\) is the friction factor of the flow in the porous tube with no mass flux. For plotting the average friction factors, \(\bar{u}\) in \(R_o\) for the experimental run for \(c_{f_o}\) is the arithmetic mean of \(\bar{u}_o\) and \(\bar{u}_L\).

Conclusion

1. The assumption that \(v/v_w = r/R\) is valid in both cases of flow with mass flux to and from the wall.
2. The radial change of shear stress is almost linear between \(r/R = 0\) and 0.95, but there is a very sharp change near the wall.
3. The mixing length in turbulent flow with radial mass flux is correlated by a single equation of Reichardt type.
4. The normalized values of eddy diffusivity in turbulent flow in a circular tube with and without radial mass flux is correlated by a single equation.
5. The equation of axial component of velocity of flow in a circular tube with radial mass flux is presented.
6. The experimental results of the local and average friction factors are in good agreement with the predictions of the film theory.

Nomenclature

- \(D\) = diameter of tube
- \(F\) = force worked by fluid
- \(L\) = length of porous tube
- \(L_s\) = fore flow length before test section
- \(l\) = mixing length
- \(n\) = coefficient in Eq. (16)
- \(p\) = static pressure
- \(R\) = radius of tube
- \(r\) = radial distance from axis
- \(R^+\) = nondimensionalized radius of tube
- \(Re\) = Reynolds number (= \(D\bar{u}/\nu\))
- \(R_o\) = mass flux ratio (= \(2 \rho v_w/\mu \bar{u}_1\) or \(2 \rho v_w/\mu \bar{u}_2\))
- \(S\) = cross sectional area of tube
- \(\bar{u}\) = average value of \(u\) over a cross section
- \(u_{max}\) = maximum axial component of velocity
- \(v\) = velocity component in radial direction
- \(v_w\) = value of \(v\) at surface of porous pipe wall
- \(x\) = axial distance from inlet of porous tube
- \(x_m\) = eddy diffusivity of momentum
- \(\gamma\) = non-dimensional distance from axis (= \(r/R\))
- \(\theta_o\) = \(\bar{c}_f/c_{f_o}\) or \(\bar{c}_f/c_{f_o}\)
- \(\mu\) = viscosity of fluid

Here, \(\bar{u}^2 = \bar{u}_1^2\)
\( \nu \) = kinematic viscosity of fluid  
\( \rho \) = density of fluid  
\( \tau \) = shear stress  
\( \tau_w \) = shear stress at wall  

(Subscripts)  
\( o \) = refers to inlet of porous pipe  
\( L \) = refers to outlet of porous pipe  
\( 0 \) = refers to no mass flux at wall  

\( \text{[cm}^2/\text{sec]} \)  
\( \text{[g/cm}^3] \)  
\( \text{[g/cm}^2 \cdot \text{sec}] \)  

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**EXTENSION OF SCHUMANN'S THEORY TO THE CASE OF LOW THERMAL DIFFUSIVITY OF SOLID PARTICLES**

**SATOSHI MURATA**

Department of Agricultural Engineering, Kyushu University, Fukuoka, Japan

Two simultaneous partial integro-differential equations which determine the transient heat transfer in a packed bed are introduced instead of Schumann's equations which assume high thermal diffusivity and uniform temperature within the solid particles.  

These equations are useful for cases of both high and low thermal diffusivity of the solid.  

An approximate analytical solution for low Biot number is derived from the equations by neglecting all terms after the first in one of the equations. The analytical solution is similar to Furnas' except for the definition of variables, and it is mathematically proved that the variables are also identical in the case of infinite thermal diffusivity of the solid.  

Procedure of numerical calculation for rigorous solution is given for a set of operating conditions, and computed solution for Biot number = 1, 2, 3, 4, 5, are graphically presented. The theoretical results agree well with experimental results in the cooling of a column of eggs.

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**Introduction**

Furnas' solution\(^4\) of Schumann's equations\(^5\) of heat transfer from fluid to solid in a packed bed is used for estimating the cooling time and temperature distribution of piled farm products for cold storage.  

However, owing to the fact that farm products often have large size and low thermal diffusivity, Furnas' solution is limited in its usefulness by Schumann's assumption that the particles comprising the bed are so small or have such high thermal diffusivity that any given particle may be considered as being at a uniform temperature at any instant.  

Sugiyama and Nagasaka\(^8\) introduced approximate equations for this problem assuming that the temperature gradient in the solid piece was expressed by the power series of \( r \) of polar coordinate in the spherical solid particles, and gave a numerical method of solution. Owing to the assumption of \( r \) power series, it seems to be rather complicated for calculation.

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The basic equations presented in this paper, which are two simultaneous partial integro-differential equations, precisely express the transient heat transfer in a packed bed for low and high thermal diffusivity of solid.

**Assumption**

The theory assumes that  
1) Compared to the transfer of heat from fluid to solid, the transfer of heat by conduction in the fluid to solid, the transfer of heat by conduction in the fluid itself or in the solid itself is small and may be neglected;  
2) The rate of heat transfer from fluid to solid at any point is proportional to the average difference in temperature between fluid and surface of solid at that point;  
3) Change in volume of fluid and solid due to change in temperature may be neglected;  
4) The thermal constants are independent of the temperature; and  
5) Solid particles have spherical shape, and