$\bar{k}$ = mean times of disintegration	[—]
l^* = break-up distance from the nozzle exit	[cm]
m = slope of straight line in Fig. 5	[—]
n = number of disintegration chance	[—]
p = probability of disintegration	[—]
$q = 1 - p$	[—]
U = liquid velocity	[cm/sec]
v^* = detached wavy sheet volume	[cm ³]
x = diameter of liquid drop	[cm]
x^* = diameter determined from Eq.(5)	[cm]
$\bar{x}$ = geometric mean diameter	[cm]
$\bar{x}_v$ = volume mean diameter	[cm]
α = refraction angle	[degree]
λ = wavelength of light	[cm]
λ^* = break-up wavelength of liquid sheet	[cm]
θ = sheet angle at nozzle exit	[degree]
μ = refractive index of liquid	[—]
μ_L = viscosity of liquid	[poise]
ρ = density of liquid	[g/cm ³]
ϕ = incidence angle	[degree]
σ_L = logarithmic standard deviation	[—]
$\langle$ Dimensionless number $\rangle$	
Re = Reynolds number ($\rho U h^* / \mu_L$)	[—]

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ANALYSIS OF EXPRESSION OPERATIONS*

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The internal mechanisms of expression operations are analysed in view of the variations of flow rate through filter-cake and of the modified consolidation coefficient through consolidated cake, and basic equations for consolidation processes are presented in this paper.

Expression operations are classified into three types according to the relations of pressure and flow rate to time. The equations for flow variations in filter-cakes are numerically solved on the basis of the so-called modern filtration theory. The generalized basic equations for consolidation are solved by using the calculus of finite difference with variable coefficients. It has been demonstrated that the operational variables for expression operations can be more reasonably determined from calculations based upon these numerical solutions rather than predicted by the approximate methods formerly reported.

Favourable coincidence between theory and experiments are assured under constant pressure, constant rate and variable pressure-variable rate conditions.

Introduction

Expression is an operation of separating liquid from solid-liquid mixtures which has long been

used on a universal scale in widely divergent fields. In spite of the importance of the operation, logical attempts to develop expression theories have only been begun quite recently. In general, expression mechanisms of slurry materials consist of two flow phenomena, and the types of phenomena can be reasonably analysed first by filtration theory and second by consolidation theory.

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In previous investigations^{9,10)} of expressions under constant- and variable-pressure conditions, the concepts of Ruth's filtration resistance^{1,3)} and crude Terzaghi's procedure^{11,12)} have been used in order to simplify considerably the mathematical analyses.

In the concept of Ruth's resistance, however, it is assumed that internal flow rates through filter-cakes are constant, and in Terzaghi's consolidation theory the coefficient of consolidation is assumed to be constant during the consolidation process.

Recently, it has been demonstrated^{6,7,8,13,14)} that filtration mechanisms should be analysed in view of the variations of flow rates through filter-cakes and that the internal flow variations are especially important in the filtration of thick slurries as practiced in expression operations. In recent work⁵⁾ on settling of thick slurries due to consolidation, it has become evident that the consolidation coefficient of sediments varies enormously, e.g. a hundred times or more, during the settling processes.

The main objectives of this paper are to analyse expression processes in view of the variations of internal flow rates in filter-cakes and of consolidation coefficients in consolidated cakes, and to present a rigorous method for calculating expression operations.

Operational Variables for Filtration Period

According to the same arguments as reported in previous papers^{9,10)}, the filtration ratio U_f under constant-pressure conditions are written in the form⁹⁾

$$U_f \equiv \frac{L_0 - L}{L_0 - L_1} = \frac{i\sqrt{K_w}}{w_0} (\sqrt{\theta_f + \theta_m} - \sqrt{\theta_m}) \quad (1)$$

where L_0 is the initial thickness of original mixture, L the thickness of mixture at a filtration time θ_f , L_1 the thickness of mixture at the end of filtration period, w_0 the mass of solids in the original mixture per unit sectional area, θ_m the fictitious filtration time, i the number of drainage surface, and K_w the modified filtration coefficient defined by

$$K_w = \frac{2g_c \rho s p}{\mu \alpha (1 - ms)} \quad (2)$$

In due consideration of the variations of average specific filtration resistance α and the ratio m of wet to dry cake mass with increasing applied pressure p , the filtration ratio U_f under variable pressure-variable rate conditions is given by¹⁰⁾

$$U_f^2 = \frac{2i^2 g_c \rho s}{\mu w_0^2 (1 - m_1 s)^2} \int_0^{\theta_f} \frac{1 - ms}{\alpha} (p - p_m) d\theta_f \quad (3)$$

and the relation of p vs. θ_f under constant rate conditions is given by¹⁰⁾

$$\theta_f \equiv \frac{g_c (1 - ms)}{\mu \rho s \alpha q_1^2} (p - p_m) \quad (4)$$

where q_1 represents the expression rate, s the fraction of solids in original mixture, m_1 the m -value at the end of filtration period, p_m the hydraulic pressure required to overcome the medium resistance, ρ the liquid density, and μ the liquid viscosity.

In the previous mathematical procedures^{9,10)} for

filtration period in expression processes, an approximate method for predicting U_f and p was developed on the presumption of constant flow rates through filter-cakes. To obtain rigorous calculations for expression operations, U_f and p should be evaluated in view of the variations of flow rates through filter-cakes, because the assumption of constant flow rates in filter-cakes is not valid for filtration processes involving thick slurries as practiced in expression operations.

Recently, on the basis of an analysis of internal flow mechanisms in filter-cakes, the equations for the flow variations q_x/q_1 , the hydraulic pressure distributions p_i/p , and the correction factor J_R for Ruth's average specific resistance α_R have been derived as following^{6,7,8)}:

$$\frac{q_x}{q_1} = 1 - \frac{s\rho(\varepsilon_x - \varepsilon_{avx})}{\rho_s(1 - \varepsilon_{avx})(1 - ms)} \cdot \frac{iw_x}{w_L} \quad (5)$$

$$\frac{p_i}{p} = 1 - \frac{p_s}{p} = \frac{\int_0^{iw_x/w_L} \left[\frac{q_x}{q_1} (1 + e_x) - e_x \right] \alpha_x d\left(\frac{iw_x}{w_L}\right)}{\int_0^1 \left[\frac{q_x}{q_1} (1 + e_x) - e_x \right] \alpha_x d\left(\frac{iw_x}{w_L}\right)} \quad (6)$$

$$\alpha \equiv J_R \cdot \alpha_R = \int_0^1 \left[\frac{q_x}{q_1} (1 + e_x) - e_x \right] d\left(\frac{iw_x}{w_L}\right) \cdot \frac{p - p_m}{\int_0^{p-p_m} \frac{dp_s}{\alpha_x}} \quad (7)$$

where w_L is the mass of solids in filter-cake per unit sectional area at a time θ_f , w_x the mass of solids per unit area in distance x from the medium, q_x the flow rate of liquid in cake at x , ε_x the local porosity, ε_{avx} the average value of porosity between the distance from the drainage surface zero and x , e_x the local void ratio defined by $e_x \equiv \varepsilon_x/(1 - \varepsilon_x)$, α_x the local specific resistance, and p_s is the local solid compressive pressure.

Compression-permeability data of α_x , ε_x and e_x have been conventionally represented in the following forms^{6,7,8)} as tabulated in **Table 1**.

$$\left. \begin{aligned} \alpha_x &= \alpha_0 + \alpha_1 p_s^n \\ \varepsilon_x &= \varepsilon_1 p_s^{-\beta} \\ e_x &= e_0 - C_e \cdot \ln p_s \end{aligned} \right\} \quad (8)$$

By using numerical procedures^{6,7,8)} based on Eqs. (5), (6), (7) and (8), values of m and α at a constant pressure p can be obtained. m and α being known, the relation of U_f vs. θ_f under a constant pressure can be calculated from Eqs. (1) and (2). On the assumption that m and α of the variable-pressure filter-cake will have the same values as those of the constant-pressure filter-cake provided the pressure drop across the cakes is the same, the relations of U_f vs. θ_f and p vs. θ_f under variable pressure conditions can be analytically determined by Eqs. (3) and (4), respectively. The predicted values of U_f under a constant pressure and a variable pressure are illustrated in **Figs. 1** and **2**, respectively. In the same figures, experimental values and the conventional calculated values^{9,10)} based on the concept of the crude Ruth's filtration resistance are also plotted.

Table 1 Compression-permeability cell data of materials used				
Material	True density of solids ρ_s [g/cm ³]	Specific resistance [cm/g]	Porosity [—]	Void ratio [—]
Mitsukuri-Gairome clay	2.824	$1.25 \times 10^{10} + 1.36 \times 10^{10} p_s^{0.681}$	$0.925 p_s^{-0.0618}$	$3.747 - 0.307 \cdot \ln p_s$
Korean Kaolin	2.579	$1.20 \times 10^{10} + 1.06 \times 10^{10} p_s^{0.559}$	$0.845 p_s^{-0.0720}$	$2.240 - 0.162 \cdot \ln p_s$
Cement material	2.720	$8.08 \times 10^9 p_s^{0.385}$	$0.693 p_s^{-0.0765}$	$1.199 - 0.0734 \cdot \ln p_s$

Fig. 1 U_f vs. $i(\sqrt{\theta_f + \theta_m} - \sqrt{\theta_m})$ (constant pressure)

Fig. 2 U_f vs. θ_f (variable pressure-variable rate)

Fig. 3 Schematic diagram of consolidated cake (single drainage, $i=1$)

It can be seen from the figures that the predictions based on the so-called modern filtration theory^{6,7,8)} are more accurate than the conventional predictions^{9,10)}.

Basic Equations for Consolidation Period

Dehydration mechanisms due to consolidation can be analysed on the basis of the consolidation theory^{11,12)} originally developed by K. Terzaghi. In Terzaghi's theory, however, it is assumed that the thickness of consolidated cake and the coefficient of consolidation remain constant and that the void ratio-solid compressive pressure relation is linear. In the previous mathematical procedures^{9,10)} for the consolidation processes in expression operations, the assumption of constant cake thickness was corrected by using a new variable w , i.e. mass of solids per unit sectional area lying

Fig. 4 Distribution of consolidation coefficient at various time factors

above the medium, for representing an arbitrary position in consolidated cake. In dealing with the rigorous basic equation of consolidation processes, the variation of consolidation coefficient and the non-linear relation of void ratio-solid compressive pressure should be taken into account in addition to the change in cake thickness.

To derive a generalized basic equation for consolidation period, begin with the flow equation in the form

$$u_w = \frac{g_c}{\mu \alpha_w} \cdot \frac{\partial p_i}{\partial w} = - \frac{g_c}{\mu \alpha_w} \cdot \frac{\partial p_s}{\partial w} \quad (9)$$

and with the definition equation for the coefficient of volume change m_v in the form

$$m_v = - \frac{1}{(1 + e_w) g_c} \frac{de_w}{dp_s} \quad (10)$$

where u_w denotes the apparent relative velocity of liquid to solids at an arbitrary position w in cake, e_w the local void ratio and α_w the local specific resistance.* The effect of acceleration of both liquid

* The apparent relative velocity, the void ratio and the specific resistance at an arbitrary position w in consolidated cake were previously^{9,10)} denoted by u_x , e_x and α_x in accordance with the conventional definitions in filtration theory. To make a clear distinction between the values at w and the values at a distance x , the subscript w is used in place of x .

and solids being negligibly small, the relation of $p = p_t + p_s$ is used in Eq.(9).

In reference to differential element in a consolidated cake shown in Fig. 3, the continuity equation of flow relating the change in u_w to the time rate of change in e_w can be written as

$$\frac{\partial u_w}{\partial w} = \frac{1}{\rho_s} \cdot \frac{\partial e_w}{\partial \theta_c} \quad (11)$$

where ρ_s is the true density of solids and θ_c the consolidation time. Differentiating Eq.(9) with respect to w gives

$$\frac{\partial u_w}{\partial w} = -\frac{g_c}{\mu} \cdot \frac{\partial}{\partial w} \left(\frac{1}{\alpha_w} \cdot \frac{\partial p_s}{\partial w} \right) \quad (12)$$

and substituting Eq.(10) into Eq.(11) gives

$$\frac{\partial u_w}{\partial w} = -\frac{g_c m_v (1 + e_w)}{\rho_s} \cdot \frac{\partial p_s}{\partial \theta_c} \quad (13)$$

where α_w is a function of p_s only as written by Eq.(8). Combining Eq.(12) with Eq.(13) and rearranging yields

$$\frac{\partial p_s}{\partial \theta_c} = C_e \left[\frac{\partial^2 p_s}{\partial w^2} - \frac{1}{\alpha_w} \left(\frac{d\alpha_w}{dp_s} \right) \left(\frac{\partial p_s}{\partial w} \right)^2 \right] \quad (14)$$

$$C_e \equiv \frac{\rho_s}{\mu \alpha_w m_v (1 + e_w)} = \frac{g_c \rho_s p_s}{\mu C_e (\alpha_0 + \alpha_1 p_s^n)} \quad (15)$$

The modified consolidation coefficient C_e defined by Eq.(15) varies enormously with location and time during consolidation processes as illustrated in Fig. 4.

Operational Variables for Consolidation Period

Sato and Atsumi⁴⁾ presented a numerical solution of consolidation equation under constant pressure conditions by using the method of moments on the presumption of simple functions for consolidation coefficients. To obtain numerical computations for both constant and variable pressure conditions, however, it is convenient to use Runge-Kutta-Gill's method²⁾ as attempted in the recent work on settling due to consolidation⁵⁾.

By using the following definitions of

$$\left. \begin{aligned} \phi &\equiv p_s/p \text{ (for constant pressure conditions) or} \\ &\quad p_s/p_2 \text{ (for variable pressure conditions),} \\ \tau_c &\equiv i^2 C_e^* \theta_c / w_0^2, \\ C &\equiv (C_e p / \alpha_w) \cdot (d\alpha_w / dp_s), \\ \text{and } \omega &\equiv iw / w_0 \end{aligned} \right\} \quad (16)$$

Eq.(14) can be rewritten in a dimensionless form as

$$\frac{\partial \phi}{\partial \tau_c} = \frac{C_e}{C_e^*} \frac{\partial^2 \phi}{\partial \omega^2} - \frac{C}{C_e^*} \left(\frac{\partial \phi}{\partial \omega} \right)^2 \quad (17)$$

where p_2 is a maximum applied pressure under variable-pressure conditions, and C_e^* represents the C_e -value for the bottom of consolidated cake ($\omega=0$) under a constant applied pressure p or under the final maximum pressure p_2 of variable-pressure processes. In solving Eq.(17), the initial and boundary conditions for constant-pressure conditions are

$$\left. \begin{aligned} \phi &= \phi(\omega, 0) & \text{at } \tau_c &= 0 \\ \phi &= 1 & \text{at } \omega &= 0 \\ \partial \phi / \partial \omega &= 0 & \text{at } \omega &= 1 \end{aligned} \right\} \quad (18)$$

and the conditions for variable pressure are

$$\left. \begin{aligned} \phi &= \phi(\omega, 0) & \text{at } \tau_c &= 0 \\ \phi &= \phi(0, \tau_c) & \text{at } \omega &= 0 \\ \partial \phi / \partial \omega &= 0 & \text{at } \omega &= 1 \end{aligned} \right\} \quad (19)$$

where the initial condition $\phi(\omega, 0)$ of consolidation period in expression operations of slurry materials can be determined by Eq.(6).

To obtain the numerical solutions, Eq.(17) is transformed to the following Eq.(20) by using a finite difference in place of the derivatives²⁾.

$$\left. \begin{aligned} \frac{d\phi_i}{d\tau_c} &= N^2 \left(\frac{C_{ei}}{C_e^*} \right) (\phi_{i+1} - 2\phi_i + \phi_{i-1}) \\ &\quad - \frac{N^2}{4} \left(\frac{C_i}{C_e^*} \right) (\phi_{i+1} - \phi_{i-1})^2 \end{aligned} \right\} \quad (20)$$

In Eq.(20), N denotes the number of partitions of a consolidated cake and the subscript i represents the i -th value of the variables counted from the medium as shown in Fig. 3. The values of C_{ei} and C_i depend on ϕ_i , and can be determined by Eqs.(8), (15) and (16).

(1) Consolidation ratio U_c vs. time θ_c

Solving Eq.(20) by using Runge-Kutta-Gill's method, compression-permeability data gives the p_s -distribution for a given system. In the numerical computations, the number of N was set to 8 and the interval of the time factor ($d\tau_c$) was set to 0.01 or 0.05 if necessary. The values of p_s being known, the consolidation ratio U_c can be numerically calculated by

$$\begin{aligned} U_c &\equiv \frac{L_1 - L}{L_1 - L_\infty} = \frac{\int_0^1 (e_{w1} - e_w) d\left(\frac{iw}{w_0}\right)}{\int_0^1 (e_{w1} - e_{w\infty}) d\left(\frac{iw}{w_0}\right)} \\ &= \frac{\int_0^1 [(e_0 - C_e \cdot \ln p_{s1}) - (e_0 - C_e \cdot \ln p_s)] d\left(\frac{iw}{w_0}\right)}{\int_0^1 [(e_0 - C_e \cdot \ln p_{s1}) - (e_0 - C_e \cdot \ln p)] d\left(\frac{iw}{w_0}\right)} \\ &= \frac{\int_0^1 \ln(p_s / p_{s1}) d\left(\frac{iw}{w_0}\right)}{\int_0^1 \ln(p / p_{s1}) d\left(\frac{iw}{w_0}\right)} \end{aligned} \quad (21)$$

for constant-pressure conditions, and by

$$U_c = \frac{\int_0^1 \ln(p_s / p_{s1}) d\left(\frac{iw}{w_0}\right)}{\int_0^1 \ln(p_2 / p_{s1}) d\left(\frac{iw}{w_0}\right)} \quad (22)$$

for increasing-pressure conditions. In Eqs.(21) and (22), p_{s1} denotes the p_s -value at the beginning of consolidation processes.

The values of U_c calculated by Eqs.(21) and (22) are illustrated in Figs. 5 and 6. In later periods of consolidation processes, the predicted curves tend to approach the final equilibrium ($U_c = 1$) faster than the experimental results, as may be seen from the figures. This may be primarily due to so-called secondary consolidation¹¹⁾, which is neglected in deriving Eq.(14). On the basis of U_f vs. τ_f and U_c vs. τ_c thus calculated, the relation L vs. θ during the whole expression processes can be obtained as illustrated in Fig. 7.

Fig. 5 U_c vs. $i/\sqrt{\theta_c}/w_0$ (constant pressure)

Fig. 6 U_c vs. θ_c (variable pressure-variable rate)

Fig. 7 L vs. θ (constant pressure and variable pressure-variable rate conditions)

(2) P vs. θ_c under constant rate

To determine the relation of p vs. θ_c under constant-rate expression, Runge-Kutta-Gill's method and an iterative method were used simultaneously. The value of $\phi(0, \tau_c)$ in Eq.(19) being assumed, the p_s -distribution can be obtained by Eq.(20) and the cake thickness L can be calculated by

$$L = \frac{w_0}{\rho_s} (1 + e_{av.}) \quad (23)$$

Provided the value of L thus calculated satisfies the constant-rate condition in the form

$$L_1 - L = q_1 \theta_c \quad (24)$$

the applied pressure p at a time θ_c can be obtained from the assumed value of $\phi(0, \tau_c)$.

In Fig. 8, the calculated results of p vs. θ_c are illustrated in addition to the results calculated from the method formerly reported¹⁰⁾. In the same figure, the results for filtration period are also plotted. It can be seen from the figures that the new predictions are more favourable than the former predictions^{9,10)}, which do not account for the variable flow rates through filter-cakes and for the variable consolidation coefficients in consolidated cakes.

Conclusion

Expression mechanisms of slurry materials are analysed in view of the variations of flow rates in

Fig. 8 P vs. θ (constant rate condition)

filter-cakes and of consolidation coefficients in consolidated cakes, and the numerical solutions of expression processes are compared with experimental results.

1) Operational variables for filtration period, i.e. filtration ratio U_f , and applied pressure p , can be reasonably calculated in view of the internal flow variations through filter-cakes on the basis of the modern filtration theory.

2) The consolidation coefficient C_e varies during consolidation period. By using a basic consolidation Eq.(14) derived in this paper, consolidation ratio U_c and applied pressure p can be predicted in view of the variations in C_e . Reasonable coincidence between theories and experiments are assured for both constant-pressure and variable-pressure conditions. To execute the rigorous calculations of expression, the processes should be analysed on the basis of the so-called modern filtration theory^{6,7,8)} and of a modified Terzaghi's theory.

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Nomenclature

C	= modified coefficient defined by Eq.(16)	$[(\text{cm}^2/\text{sec})(\text{g-mass}/\text{cm}^3)^2]$
C_c	= empirical constant in Eq.(8)	[—]
C_e	= modified consolidation coefficient defined by Eq.(15)	$[(\text{cm}^2/\text{sec})(\text{g-mass}/\text{cm}^3)^2]$
C_e^*	= C_e -value for the bottom of consolidated cake under a constant pressure p or under the maximum pressure p_2 of variable-pressure condition	$[(\text{cm}^2/\text{sec})(\text{g-mass}/\text{cm}^3)^2]$
e_{av}	= average void ratio	[—]
e_w, e_x	= local void ratio	[—]
e_{w1}	= e_w -value at the beginning of consolidation period	[—]
$e_{w\infty}$	= e_w -value at $\theta_c = \infty$	[—]
g_c	= conversion factor	[dyne/g-force]
i	= number of drainage surface	[—]
J_R	= correction factor for Ruth's filtration resistance	[—]
K_w	= modified filtration coefficient, defined by Eq.(2)	$[(\text{cm}^2/\text{sec})(\text{g-mass}/\text{cm}^3)^2]$
L	= thickness of mixture at time θ	[cm]
L_0	= initial thickness of original mixture	[cm]
L_1	= thickness of mixture at the end of filtration period, i.e. at the beginning ($\theta_c = 0$) of consolidation period	[cm]
L_∞	= thickness of mixture at $\theta_c = \infty$	[cm]
m	= ratio of wet to dry cake mass	[—]
m_1	= ratio of wet to dry cake mass at the end of filtration period	[—]
m_v	= coefficient of volume change, defined by Eq.(10)	$[\text{cm}^2/\text{dyne}]$
N	= number of equally spaced intervals	[—]
n	= compressibility coefficient of cake	[—]
p	= applied pressure	$[\text{g-force}/\text{cm}^2]$
p_l	= local hydraulic pressure	$[\text{g-force}/\text{cm}^2]$
p_m	= hydraulic pressure required to overcome medium resistance	$[\text{g-force}/\text{cm}^2]$
p_s	= local solid compressive pressure	$[\text{g-force}/\text{cm}^2]$
p_{s1}	= p_s -value at the end of filtration period, i.e. at the beginning of consolidation period	$[\text{g-force}/\text{cm}^2]$
q_1	= expression rate	$[\text{cm}/\text{sec}]$
s	= fraction of solids in original mixture	[—]
U_c	= average consolidation ratio, defined by Eq.(21)	[—]
U_f	= filtration ratio, defined by Eq.(1)	[—]
u_w	= apparent relative velocity of liquid to solids at w from the medium	$[\text{cm}/\text{sec}]$
w	= variable for representing an arbitrary position in consolidated cake, i.e. mass of solids per unit sectional area	$[\text{g-mass}/\text{cm}^2]$
w_L	= solids mass per unit sectional area in filter-cake	$[\text{g-mass}/\text{cm}^2]$

w_0	= total solids mass in original mixture per unit sectional area	$[\text{g-mass}/\text{cm}^2]$
w_x	= mass of solids per unit sectional area in distance x from medium surface	$[\text{g-mass}/\text{cm}^2]$
<Greek letters>		
α	= average specific resistance	$[\text{cm}/\text{g-mass}]$
α_R	= Ruth's average specific resistance, defined by Eq.(7)	$[\text{cm}/\text{g-mass}]$
α_w, α_x	= local specific resistance	$[\text{cm}/\text{g-mass}]$
ϵ_x	= local porosity at distance x from the medium	[—]
ϵ_{avx}	= average porosity for filter-cake lying between medium and a distance x	[—]
θ	= time	[sec]
θ_c	= time for consolidation period	[sec]
θ_f	= time for filtration period	[sec]
θ_m	= fictitious filtration time	[sec]
μ	= viscosity of liquid	$[\text{g-mass}/\text{sec} \cdot \text{cm}]$
ρ	= density of liquid	$[\text{g-mass}/\text{cm}^3]$
ρ_s	= true density of solids	$[\text{g-mass}/\text{cm}^3]$
τ_c	= time factor for consolidation period defined by Eq.(16)	[—]
τ_f	= $(i^2 K_w \theta_f / w_0^2)$ = time factor for filtration period	[—]
ϕ	= local value of reduced solid compressive pressure defined by Eq.(16)	[—]
ω	= $i w / w_0$ = modified dimensionless distance defined by Eq.(16)	[—]

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