HYDRAULIC CONVEYING OF SOLIDS THROUGH PIPE BENDS

MASAYUKI TODA, NORIO KOMORI, SHOZABURO SAITO
AND SIRO MAEDA
Department of Chemical Engineering, Tohoku University, Sendai, Japan

Hydraulic transport of solid materials through pipe bends was investigated experimentally. Four kinds of 90° pipe bends of which the radii of curvature were 0, 12, 24 and 48 cm, were made of polyacrylate pipe. The pressure drops were measured over sections of about 5 m, each including a pipe bend. The solid particles used in this experiment were glass beads (0.5~2.0 mm diameter) and polystyrene particles (1.0 mm diameter).

The behavior of particles in pipe bends was found to be very much complicated by the effect of gravitational and centrifugal forces and the secondary flow of fluid.

The results of the pressure-drop measurement were as follows. a) The horizontal pipe bend: In the case of polystyrene particles, even though delivered particle concentration exceeded about 20%, the effect of particle concentration on the pressure drop did not appear to be the same as in the case of a straight pipeline. On the other hand, in the case of glass particles the additional pressure drop, which was nearly constant regardless of flow rate, increased with increasing particle concentration except where R=0 and 12 cm. Moreover, the additional pressure drop was correlated by the dimensionless term $U_m^2/\rho g R (\rho_p/\rho_w - 1)$ and $m_w$. b) The vertical pipe bend: It was found from experiment that both polystyrene and glass particles showed additional pressure drop, which was nearly constant regardless of the flow rate, similar to the case of horizontal pipe bends.

Introduction

Hydraulic transport of solid materials through pipes has been used industrially for a long time as it has many advantages as compared with other traditional methods.

Therefore, in the many past investigations, attention has been directed to finding a correlation, necessary for designing equipment, between the pressure drop and the mean flow rate of a two-phase system in straight pipes.

However, where saltation or choking in hydraulic conveyers are concerned, pipe bends as well as straight pipes are an important element in the designing of a transport process. For the above reason, it is necessary to clarify the mechanism of flow of particles in pipe bends.

In solids-fluid two phase flow, there are many works on the subject of pipe bends for pneumatic conveyers, but for hydraulic conveyers only a few studies have been made, by Ayukawa. He measured pressure drop along pipe bends fixed in a vertical position. The pressure drop was analyzed by a simple model and expressed in terms of additional pressure drop coefficient $\delta_n$. But, its mechanism has not been clarified to a sufficient extent as yet.

The object of this work was to obtain data for the effects of radius of curvature on the head loss in both horizontal and vertical pipe bends over a wide range of velocities and concentrations, and to try to correlate the results.

1. Experimental Apparatus and Method

Fig. 1 shows the experimental apparatus schematically. A transparent polyacrylate pipe (30.2 mm inside diameter and 1 m in length) was used for flow path. Preliminary examination showed that the inside wall of the polyacrylate pipe was hydrodynamically smooth and that the connected parts of the pipeline did not affect pressure drop and flow.
Table 1 Physical properties of solid materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Particle diameter [mm]</th>
<th>Arithmetical standard deviation of particle diameter</th>
<th>Particle density [g/cm³]</th>
<th>Free falling velocity in water [cm/sec]</th>
<th>Particle Reynolds number in free fall</th>
<th>Drag coefficient [—]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>0.43</td>
<td>1.19</td>
<td>2.50</td>
<td>6.3</td>
<td>27</td>
<td>2.14</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.97</td>
<td>1.09</td>
<td>1.10</td>
<td>2.3</td>
<td>22</td>
<td>2.66</td>
</tr>
</tbody>
</table>

Fig. 1 Schematic diagram of experimental apparatus

Four kinds of 90° pipe bends of which the radii of curvature were 0°, 12°, 24° and 48 cm, were made of the above mentioned polyacrylate pipe.

The pressure-drop measurements were carried out over sections of about 5 m, each including a pipe bend.

Water-in-glass manometers were used for the pressure-drop measurement. The mean flow rate of the mixed-phase system and the supplied concentration of particles were evaluated by measuring the amount of the outlet directly.

The solid particles used in this experiment were glass beads and polystyrene particles. These particles were sieved with Tyler standard screens. Thus each sample of solid particles had a relatively uniform particle size. The physical properties of the materials are given in Table 1. Sizes were determined with a microscope. The arithmetic mean was taken as the representative value.

A 90° pipe bend with zero radius of curvature was made by connecting two pipes cut at an angle of 45° and used to observe flow patterns in the extreme case of pipe bend.

2. Experimental Results and Discussion

2-1) Flow state

The behavior of particles in pipe bends was very much complicated by the effect of the centrifugal force and the secondary flow of fluid.

In pneumatic conveyers, most of the particles are conveyed along the outside wall of pipe bends due to centrifugal force. Moreover, the phenomenon of back mixing of particles occurs due to irregular bouncing by collision with the wall of the pipe bends.

However, probably because the viscosity of fluid is much larger in hydraulic conveyers than in pneumatic conveyers, a different flow state from that for pneumatic conveyers was observed in hydraulic conveyers. It is difficult to take clear photographs of the flow state of glass particles. For better understanding of the transport of two-phase system in pipe bends, photographs were taken by using of sand particles, which have almost the same density and average particle diameter as those of glass particles. The flow patterns of sand-water system are shown in Photos. 1, 2, 3, and 4. The behavior of the glass
particles can be imagined from these photographs. (a) Horizontal pipe bend: At a low flow rate, particles were transported in the form of a sliding bed in a horizontal pipe. When they entered a pipe bend, they were lifted up along the inside wall as shown in Photo. 1. These phenomena are the characteristics of a low flow rate in pipe bends. Then, the particles fall slowly to the bottom of the pipe. This tendency is remarkable in abruptly curved tubes, especially in a pipe bend with zero radius of curvature. Even when a stationary bed was formed on the bottom of the straight pipe, almost all particles were floated in curved tubes. Therefore, a stationary bed was not formed in sharply bent pipes.

The above phenomena may be caused by the strong effect of the secondary flow of fluid with vortexes which are caused by sudden change in the direction of flow. But when the curvature of the pipe bend increases, the same flow state as in a straight
pipe is observed in the bend. At a high flow rate, particles are transported relatively more along the outside wall of the pipe bend because the centrifugal force acting on the particles becomes greater than the hydrodynamic force caused by the secondary flow, as shown in Photo. 2.

However, in the case of polystyrene particles, all the particles were transported in suspension in the pipe bend, as in a horizontal pipe.

(b) Vertical pipe bend: The effect of gravity appears noticeably in the case of a vertical pipe bend. Though almost all the particles were transported along the outside wall of the pipe bend, as shown in Photo. 3, the flow state in the downstream depends on the flow rate. That is, the distance from the pipe bend at which particles are fully dispersed increases with increasing flow rate. At low flow rates, the particles could be dispersed by secondary flow of fluid in outlet of pipe bends. But at high flow rates the centrifugal force acting on the particles is considered to become much larger than the force caused by secondary flow. Therefore, in this case the particles could not be dispersed by secondary flow in the outlet of pipe bends, as shown in Photo. 4.

2-2 Pressure drop

Definition of pressure drop: Figs. 2 and 3 show an example of the pressure drop gradient measured along a pipeline which involves a pipe bend. Fig. 4 shows schematically the pressure gradient along the pipe bend. In this figure, the line of A, B, C, D and E indicates the actual pressure gradient in a pipe bend and the line of A', B', C', D' and E' shows the pressure gradient of a straight pipe corresponding to the pipe bend.

The pressure drop is expected to coincide with the
line of A, B', C'', D'' and E'' if it is assumed that the pipe bend is removed. Therefore, the pressure drop caused by pipe bends will be defined by the EE'' which is expressed by $\Delta P_{\text{Bend}}$ and $\Delta P_{\text{BendV}}$ for horizontal and vertical pipe bends, respectively. The pressure drop due to pure water, $\Delta P_{\text{BW}}$, is also defined in the same way.

Generally, the contributions of fluid and solid to the pressure drop are not independent, since the particles in a mixed-phase system will almost certainly modify the flow pattern, but it is convenient as a first assumption to assume an additive relationship:

$$\Delta P_{\text{Bend}} = \Delta P_{\text{BW}} + \Delta P_{\text{BHS}}$$  \hspace{1cm} (1)

where $\Delta P_{\text{BHS}}$ is the additional pressure drop due to the particles.

(a) Pressure drop in horizontal pipe bends

Examples of the observed pressure drop are shown in Figs. 6 to 11.

In the case of polystyrene particles ($\rho_s = 1.1 \text{g/cm}^3$), even though delivered particle concentration exceeds about 20%, the effect of particle concentration $m_c$ on the pressure drop $\Delta P_{\text{Bend}}$ does not appear, as shown in Fig. 6. The observed values agree well with the pressure drop for the flow of pure water, $\Delta P_{\text{BW}}$, just as in the case of a straight pipeline. It is assumed that collisions and/or friction between particles and the pipe wall are very small because the particles are dispersed almost uniformly in the pipe. Therefore, in this section, the experimental results for glass particles alone will be discussed. It is clearly seen from Fig. 5 that in the straight pipeline additional pressure drop caused by the particles decreases when the flow rate increases. On the other hand, additional pressure drop in the pipe bend is nearly constant regardless of flow rate, as shown in Fig. 7. When the radius of curvature becomes small, however, there are two kinds of phenomena. One concerns cases in which no additional pressure drops are observed.
and the total pressure drops agree with $\Delta P_{\text{BW}}$, as shown in Figs. 8, 9 and 10. The other concerns cases in which additional pressure drops are observed for each particle concentration, as shown in Fig. 11. As for the former, the friction loss between the particles and the pipe wall is thought to be offset by the drag reduction of water in a mixed-phase system. In the latter, it may be considered that the drag reduction of water in a mixed phase of $d_s=0.043\text{cm}$ is not so large as the case of $d_s=0.099\text{cm}$ and $0.189\text{cm}$. Moreover, Fig. 12 shows examples of the comparison of the pressure gradient of the pipeline involving a pipe bend with that of the straight pipe corresponding to a bent pipe. Though the pressure drop of a pipe bend is almost identical for each particle concentration, there are two kinds of phenomena when the particle concentration $m_c$ is increased. One concerns cases in which the pressure drop of the pipeline involving a pipe bend is smaller than that of a straight pipe corresponding to a pipe bend.

The other concerns cases in which the pressure drop of the pipeline involving a pipe bend is almost equal to that of a straight pipe. Thus the effects of particle size, particle concentration and radius of curvature on the pressure drop are very complicated. Accordingly, at present these phenomena cannot be explained satisfactorily.

Fig. 11 shows the effect of the ratio of $R$ to $r$. It is seen from the figure that the curves of $\Delta P_{\text{BW}}$ vs. the ratio of $R$ to $r$ have a minimum value for every flow rate. This tendency agrees with Ito's experimental results.

(a)—2 Empirical equation of the pressure drop

The practical problem is to estimate the energy required to convey material between two points at a specified rate.

For coarse particles Durand has shown that the additional pressure drop of horizontal pipeline is a function of $m_c$ and $u^2_{m}/gD[(\rho_s/\rho_w-1)$, where $D$ is the pipe diameter. In the pipe bends, by considering mainly the effect of centrifugal force on the particles, $D$ in Durand’s equation was replaced by the radius of curvature, $R$, and the experimental data are correlated as follows.

$$\Delta P_{\text{BW}} - \Delta P_{\text{BW}} = 4.4 \left( \frac{U_{m}^2}{gD(\rho_s/\rho_w-1)} \right)^{-0.45} m_c^{1.41}$$

The limit of this equation is: $R>24\text{cm}$ because when radius of curvature becomes smaller than $12\text{cm}$ additional pressure drop, with exception of Fig. 11, comes to zero.

The observed values are compared with the calculated ones in Fig. 14. It is seen from the figure that they almost agree with each other.

(b) Pressure drop in vertical pipe bends

Some results of the pressure drop measurement are shown in Figs. 15 to 18. Generally, the effect of particles for the pressure drop in vertical pipe bends is larger than that for horizontal ones because the effect of gravitational force becomes very large when the particles enter the vertical section from the horizontal section.
Fig. 12 Pressure drop gradient along the pipeline with a horizontal pipe bend

Fig. 13 Relationship between R/r and pressure drop of horizontal pipe bend

Fig. 14 Comparison of experimental formula with the observed values of pressure drop for horizontal pipe bend

Fig. 15 Relationship between pressure drop and mean velocity of slurry in vertical pipe bend

Fig. 16 Relationship between pressure drop and mean velocity of slurry in vertical pipe bend
In the case of glass particles of 1 mm, the pressure drop increases with increasing particle concentration even when the radii of curvature of pipe bends are 0 and 12 cm, unlike in horizontal pipe bends.

Moreover, even in the case of polystyrene particles the effect of addition of particles on the pressure drop appears at high concentrations such as 20%, as shown in Fig. 18. On the other hand, additional pressure drop is constant regardless of the flow rate. This tendency is the same as in horizontal pipe bends.

The experimental values are shown in Fig. 19 using dimensionless term \( \frac{U^2}{\rho g R (\rho_s/\rho_w - 1)} \) and \( m_e \), as well as horizontal pipe bends.

In the case of vertical pipe bends, it was found from Fig. 19 that \( (\Delta P_{mv} - \Delta P_{bw})/m_e \Delta P_{bw} \) is proportional to \( m_e \) and \( \frac{U^2}{\rho g R (\rho_s/\rho_w - 1)} \) on logarithmic graph paper as well as horizontal pipe bends. But logarithmic plots gave a different line for each size of particle and each radius of curvature. This shows that the effect of these factors on the pressure drop of vertical pipe bends is very complicated. Therefore, the experimental values did not yet result in giving the form of an empirical equation.

The effect of \( (R/r) \) against \( \Delta P_{mv} \), as shown in Fig. 20, is similar to the results obtained in horizontal pipe bends.

(c) Comparison of \( \Delta P_{mvH} \) and \( \Delta P_{mvV} \)

For example, the comparison of \( \Delta P_{mvH} \) with \( \Delta P_{mvV} \) is shown in Fig. 21. It is observed from the figure that \( \Delta P_{mvV} \) is greater than \( \Delta P_{mvH} \) and \( \Delta P_{mvV}/\Delta P_{mvH} \) tends to decrease with an increase of flow rate.

(d) The pressure drop of pipeline involving a pipe bend

In the previous section, only the pressure drops of pipe bends were discussed. But, for designing a pipeline involving bend it is a most interesting prob-
3.0 \leq \rho_d < 2.0 \leq \rho_e < 1.0

I = \text{distance from the pipe bend [cm]}

\Delta P_{AB} = \text{pressure drop between A and D [cmH}_2\text{O]}

\Delta P_{BC} = \text{pressure drop between B and C [cmH}_2\text{O]}

1) The flow state in pipe bends
(a) Horizontal pipe bend
At low flow rates, particles are affected strongly by secondary flow of fluid and lifted up along the inside wall. At high flow rates, particles are transported along the outside wall because the centrifugal force acting on the particles becomes greater than the hydrodynamic force caused by secondary flow.
(b) Vertical pipe bend
The effect of gravity appears in the case of a vertical pipe bend. Almost all the particles are transported along the outside wall of a pipe bend, and the flow state in the downstream depends on the flow rate.

2) Pressure drop
(a) Horizontal pipe bend
In the case of polystyrene particles ($\rho_d=1.1$ g/cm$^3$), even though delivered particle concentration exceeds about 20%, the effect of particle concentration $m_c$ on the pressure drop, $\Delta P_{inlet}$, does not appear to be the same as in the case of a straight pipeline.
On the other hand, in the case of glass particles ($\rho_d=2.5$ g/cm$^3$) the additional pressure drop, which is nearly constant regardless of the flow rate, increases with increasing particle concentration except where $R=0$ and 12 cm. Moreover, the additional pressure drop is correlated by the dimensionless term $\overline{U}^4_m/\rho_d g R (\rho_d/\rho_w)^{-1}$ and $m_c$.
(b) Vertical pipe bend
Both polystyrene particles and glass particles show additional pressure drops, which are nearly constant regardless of the flow rate, similar to the case of horizontal pipe bend.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
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<tr>
<td>$D$</td>
<td>pipe diameter</td>
<td>cm</td>
</tr>
<tr>
<td>$d_s$</td>
<td>particle diameter</td>
<td>cm</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
<td>m/sec$^2$</td>
</tr>
<tr>
<td>$l$</td>
<td>distance from the pipe bend</td>
<td>cm</td>
</tr>
<tr>
<td>$m_c$</td>
<td>delivered particle concentration</td>
<td>%</td>
</tr>
<tr>
<td>$\Delta P_{AB}$</td>
<td>pressure drop between A and D</td>
<td>cmH$_2$O</td>
</tr>
<tr>
<td>$\Delta P_{BC}$</td>
<td>pressure drop between B and C</td>
<td>cmH$_2$O</td>
</tr>
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</table>

Fig. 21 Comparison of $\Delta P_{inlet}$ with $\Delta P_{AB}$

Fig. 22 Relationship between $R/r$ and pressure drop of pipeline with the horizontal pipe bend

Fig. 23 Relationship between $R/r$ and pressure drop of pipeline with the vertical pipe bend

3. Conclusion

The flow state in pipe bends has been observed for a solid-liquid mixed phase flow. The pressure drop for pipe bends was determined. The following results were obtained.

Problem how the pressure drop is changed by the pipe bend and whether an optimum radius of curvature exists or not.

Figs. 22 and 23 show the relationship between the ratio of $R$ to $r$ and the pressure drop of a pipeline involving a pipe bend. As for the pressure drop for pipe bends, it seems that an optimum radius of curvature will exist, as shown in Figs. 13 and 20. However, the effect of $R$ for the total pressure drop of the pipeline involving a pipe bend is very small, as shown in Figs. 22 and 23. Accordingly, the optimum radius of curvature will be decided by other factors.
\( \Delta P_{BHs} \) = additional pressure drop of the pipe bend due to the particles [cmH2O]
\( \Delta P_{mH} \) = total pressure drop of horizontal pipe bend [cmH2O]
\( \Delta P_{mV} \) = total pressure drop of vertical pipe bend [cmH2O]
\( \Delta P_{Bw} \) = pressure drop of the pipe bend due to pure water [cmH2O]
\( \Delta P_a/L \) = total pressure drop in the straight pipeline per unit length [cmH2O/m]
\( R \) = radius of curvature of the pipe bend [cm]
\( r \) = pipe radius [cm]
\( U_m \) = mean velocity of slurry [m/sec]
\( \rho_s \) = density of particle [g/cm^3]
\( \rho_w \) = density of water [g/cm^3]

Literature Cited
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COLUMN EFFICIENCIES OF RECTIFYING COLUMN WITH LIQUID MIXING ON PLATE*

KAKUSABURO ONDA, HIROSHI TAKEUCHI
AND KATSUROKU TAKAHASHI
Department of Chemical Engineering, Nagoya University, Nagoya, Japan

Relation between column efficiency and plate efficiency in a rectifying column is investigated by considering the liquid mixing on the plate. Under the assumption that the vapor on the plate is in equilibrium with the liquid at the center of the plate, the column efficiency is derived as a function of the plate efficiency, \( \lambda (=mV/L) \) and the mixing parameter \( F \) for the case where the equilibrium curve is given as a straight line. It is shown that the results can be applied by using the mean value of \( \lambda \) in case the equilibrium curve is not a straight line.

Introduction
The column efficiency \( E_T \), a ratio of the theoretical plate numbers to the practical plate numbers, is very important for the design of a rectifying column. However, there is no strict method to calculate column efficiency. Lewis\(^9\) gave the column efficiency as a function of Murphree’s plate efficiency \( (E_M Y)_{1/2} \) and \( \lambda (=mV/L) \) when the equilibrium curve is a straight line. Murphree’s plate efficiency \( (E_M Y)_{1/2} \), defined with the vapor composition in equilibrium with the liquid leaving the plate, depends upon \( \lambda \).\(^9\) Also, Lewis’s relation does not show clearly the dependence of \( E_T \) on \( \lambda \), which is a function of reflux ratio. Onda\(^5\) and Kobayashi\(^13\) defined the modified plate efficiency \( (E_M Y)_{1/2} \) based on the vapor composition in equilibrium with the liquid at the center of the plate, and showed that \( (E_M Y)_{1/2} \) is reasonable and depends little upon \( \lambda \).

In our previous paper\(^4\), the relation between column efficiency and modified plate efficiency \( (E_M Y)_{1/2} \) was derived under the assumption that the liquid flows on the plate in plug flow. In this paper, column efficiency is discussed as a function of the reflux ratio. Further, taking into account liquid mixing on the plate, the column efficiency \( E_T \) has been derived as a function of \( (E_M Y)_{1/2} \), \( \lambda \) and the mixing parameter \( F \). The plate efficiency \( (E_M L)_{1/2} \) based on the composition of liquid phase has been also expressed as a function of \( (E_M Y)_{1/2} \), \( \lambda \) and \( F \), because the plate efficiency based on the composition of liquid phase is more often used.

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