THE BEHAVIOR OF SUSPENDED SOLID PARTICLES AND LIQUID IN BUBBLE COLUMNS*

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The longitudinal concentration distribution of solid particles and liquid mixing in 6.6, 12.2 and 21.4 cm i.d. bubble columns were measured and were analyzed by using diffusion model. The longitudinal dispersion coefficient of solid particles and liquid based on actual mean slurry velocity, the mean settling velocity of solid particles, and the concentration of solid particles at the top of the column, respectively, were correlated with dimensionless groups. From these correlations empirical equations were obtained. The values of concentration of solid particles calculated from the empirical equations agreed with the observed values within $\pm 20\%$.

Introduction

With the improved design of bubble columns containing suspended solid particles, a more satisfactory elucidation of the relation between the behavior of suspended solid particles and liquid in the column and operating conditions is necessary.

Suganuma *et al.*⁷⁾ measured the longitudinal concentration distribution of solid particles in batch and continuous operations for bubble columns of 6, 11.8 and 20.1 cm in diameter, and presented an empirical equation for the range where a linear relationship between the logarithmic concentration of solid particles and the axial height from the bottom of the column holds approximately. Goto *et al.*⁴⁾ and Farkas *et al.*³⁾ investigated the concentration distribution of solid particles in batch operations for 7.75, 20 and 90 cm columns and for a 3.81 cm column, respectively. Cova²⁾ investigated 4.57, 6.35 and 44.7 cm columns in cocurrent and countercurrent flow. He reported that the concentration distribution of solid particles in these columns was equal to that calculated by a sedimentation diffusion model, on the assumption that the dispersion coefficient of solid particles was the same as that of liquid and that the settling velocity of solid particles was equal to the terminal velocity of a single particle in stagnant liquid. Imafuku *et al.*⁵⁾ measured the concentration distribution of solid particles in batch operations for 5, 10 and 20 cm columns and in continuous operations for a 10 cm column.

They reported that from the analysis of their data the dispersion coefficient of solid particles had the same value as that of liquid in a gas-liquid system, and they also presented an empirical equation for the settling velocity of solid particles in a bubble column as a function of the terminal velocity of a single particle in stagnant liquid and a voidage function. According to Cova²⁾ and Imafuku *et al.*⁵⁾, in cocurrent flow the concentration of solid particles at the top of the column is equal to that in effluent (or feed) slurry, but the former is higher than the latter according to Suganuma *et al.*⁸⁾.

Although there are several papers as mentioned above, experimental data and their arrangement have not been sufficient to obtain an improved design of bubble columns containing suspended solid particles. Therefore, in this study the longitudinal concentration distribution of solid particles and liquid mixing in 6.6, 12.2 and 21.4 cm bubble columns were measured; and the correlation between operating conditions and the longitudinal dispersion coefficient of solid particles, the settling velocity of solid particles, the ratio of the concentration of solid particles at the top of the column to that in effluent (or feed) slurry, and the longitudinal dispersion coefficient of liquid are discussed.

1. Experimental Apparatus and Procedures

A schematic diagram of the experimental apparatus is shown in Fig. 1. Bubble columns of 6.6, 12.2 and 21.4 cm in diameter, having heights of 201, 196 and 405 cm, respectively, were used. The column, made of transparent acrylic plastic, was equipped with a gas distributor of the same diameter as that of the column. As gas distributors, three perforated plates were used

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Table 1 Experimental conditions for continuous operations

D_r [cm]	L [cm]	t [°C]	d_p [μ]	$\bar{C}$ [$\frac{\text{g-solid}}{\text{cm}^3\text{-slurry}}$]	$\bar{u}_l$ [cm/sec]	Key
6.6	201	19 23	63~88	0.101	0.67	□
			88~105	0.048	0.72	▽
				0.202	1.5	▽
					2.0	▽
			125~149	0.075	0.80	◇
				0.119	1.5	◇
			149~177		2.0	◇
				0.098	1.5	◇
				~ 0.108	2.2	◇
						◇
12.2	196	15 20	88~105	0.066	0.50	△
				0.116	0.86	△
					1.25	△
					2.07	△
			105~125	0.051	0.74	◇
				0.142	1.4	◇
					2.07	◇
					0.50	◇
			125~149	0.083	1.37	◇
				0.104	1.5	◇
					2.07	◇
						◇
21.4	405	11 19	88~105	0.096	1.0	△
				~ 0.106	1.5	△
			125~149	0.086	0.50	◇
				0.106	1.0	◇
					1.5	◇
					0.50	◇
			149~177	0.096	1.0	◇
				0.109	1.5	◇
						◇
						◇

① bubble column ② gas distributor ③ sampling tap
④ air compressor ⑤ valve ⑥ orifice meter
⑦ slurry column ⑧ thermobaric device
⑨ slurry pump ⑩ Venturi meter ⑪ cock
⑫ electrical conductivity cell

Fig. 1 Experimental apparatus

(that for the 6.6 cm column had 31 holes of 0.1 cm, that for the 12.2 cm column had 97 holes of 0.1 cm and was covered with 200-mesh wire gauze, and that for the 21.4 cm column had 336 holes of 0.1 cm and was covered with 250-mesh wire gauze). The system in the column consisted of air, water and glass spheres. Five size ranges of glass spheres with a density of 2.52 g/cm³ were used: 63~88, 88~105, 105~125, 125~149 and 149~177 μ in diameter.

The longitudinal concentration distribution of solid particles was measured in batch and continuous (cocurrent) operation for slurry flow. In the batch operation, air from a compressor was first measured by an orifice flowmeter and blown into the column through the gas distributor. A definite amount of tap-water and solid particles was then fed into the column. After a steady concentration distribution of solid particles was established, samples of slurry were withdrawn through sampling taps into measuring cylinders. In the continuous operation, air was blown into the column through the orifice flowmeter and the gas distributor. The slurry from the slurry column was measured by a Venturi flowmeter and introduced into the bubble column at a point above the gas distributor. Then the slurry was allowed to flow upwards cocurrently with the air and to overflow from the top of the column into the slurry column. After the solid particle concentration in the overflowing slurry from the top of the column and the concentration distribution of solid particles in the column became steady, samples were taken from the sampling taps into measuring cylinders. The volume of each sample was measured and then solid particles

were separated from the liquid, dried and weighed. The concentration of solid particles was expressed in grams of solid particles per 1 cm³ of slurry. The liquid mixing was measured by the method of delta response⁹, using a KCl solution as the tracer. The gas holdup was calculated from the height of the column and the liquid level containing solid particles measured by stopping the feed of gas and slurry simultaneously. The experimental conditions in continuous operations are summarized in Table 1.

2. Analysis of Concentration Distribution of Solid Particles

According to a sedimentation diffusion model in which the longitudinal movement of suspended solid particles is assumed to be caused by longitudinal dispersion and settling of solid particles and liquid flow, the concentration distribution of solid particles in the bubble column in steady-state, continuous operation is expressed by the equation⁷⁾

$$C = \left(C_0 + \frac{u_l}{v_p - u_l} C^* \right) \exp \left(- \frac{v_p - u_l}{E_p} z \right) - \frac{u_l}{v_p - u_l} C^* \\ = \left(C_L + \frac{u_l}{v_p - u_l} C^* \right) \exp \left\{ \frac{v_p - u_l}{E_p} (L - z) \right\} - \frac{u_l}{v_p - u_l} C^* \quad (1)$$

By normalizing Eq.(1) with substitution of $X = C/C^*$, $X_1 = C_L/C^*$, $Z = z/L$, $P_p B = v_p L/E_p$ and $P_{pl} B = u_l L/E_p$, the following equation is obtained:

$$X = \left(X_1 + \frac{P_{pl} B}{P_p B - P_{pl} B} \right) \exp \{ (P_p B - P_{pl} B)(1 - Z) \} \\ - \frac{P_{pl} B}{P_p B - P_{pl} B} \quad (2)$$

Fig. 2 Gas holdup in bubble columns containing suspended solid particles

In the case of batch operation with slurry, Eq.(1) reduces to the following equation, and $P_p B$ can be calculated from the observed concentration of solid particles and the axial height from the bottom of the column.

$$C/C_0 = \exp(-v_p z/E_p) = \exp(-P_p B \cdot Z) \quad (3)$$

By substituting $X=X_{0.1}$ at $Z=0.1$ and $X=X_{0.76}$ at $Z=0.76$, respectively, into Eq.(2), the resulting equation is rearranged as follows:

$$\frac{P_p B}{P_p B - P_{pl} B} = 1 + \frac{X_{0.76} \exp \{ (P_p B - P_{pl} B)(0.76 - 0.1) \} - X_{0.1}}{1 - \exp \{ (P_p B - P_{pl} B)(0.76 - 0.1) \}} \quad (4)$$

Assuming that E_p and v_p are both independent of u_l (this was verified from experimental results) E_p and v_p can be separately obtained in the following manner. By substituting $P_p B$ obtained from batch operation, and $X_{0.1}$ and $X_{0.76}$ obtained from continuous operation into Eq.(4), the value of $P_{pl} B$ which satisfies Eq.(4) can be obtained, and then E_p can be obtained from $P_{pl} B$, u_l and L . Furthermore, this E_p , $P_p B$ and L give v_p . In the above calculations, the value of $P_p B$ in batch operation was corrected by using the correlation, $P_p B \propto v_l^{0.75}$, and Eq.(6) to a value under the liquid temperature and the average solid particle concentration in continuous operation, and the values of $X_{0.1}$ and $X_{0.76}$ used were obtained from the smoothing curve of the experimental values.

3. Experimental Results and Discussion

3.1 Gas holdup

Gas holdup data in bubble columns in which glass spheres were suspended are shown in Fig. 2. In the region of low gas velocity, the gas holdup of the air-water-glass sphere system is somewhat less than that of the air-water system; and the larger solid particles show a somewhat smaller gas holdup. According to visual observation, this is considered to be caused by the larger rising velocity of coalesced bubbles in the presence of solid particles. In the region of high gas velocity, where large coalesced bubbles rise frequently,

Fig. 3 Example data of concentration distribution of solid particles in batch operations

the effect of the concentration of solid particles on gas holdup becomes gradually smaller as gas velocity increases.

3.2 Concentration distribution of solid particles in batch operations

Example data of the concentration distribution of solid particles in batch operations with slurry are shown in Fig. 3. For an exact analysis of the concentration distribution of solid particles it is necessary to consider a voidage function at any local position, such as that used by Imafuku *et al.*⁵⁾ However, because the logarithm of the solid particle concentration is almost proportional to the axial height from the bottom of the column in this experimental region, we made an approximation of the concentration distribution of solid particles in batch operations by Eq.(3).

In the case of solid particles, 88~105 μ in diameter, $P_p B$ is proportional to the 0.75th power of the terminal velocity of a single solid particle v_l as the liquid temperature varies. This is shown in Fig. 4.

$$P_p B \propto v_l^{0.75} \quad (5)$$

Fig. 5 shows the relationship between $P_p B$ and the volume fraction of liquid in slurry $\varphi_l (=1 - \bar{C}/\rho_p)$. $P_p B$ decreases with a decrease in φ_l , or with an increase in $\bar{C}$ (this may be caused by a mutual interference between solid particles) and can be approximated by the equation

$$P_p B = P_p B^0 \varphi_l^{2.5} \quad (6)$$

where $P_p B^0$ is the value of $P_p B$ extrapolated to $\varphi_l = 1$.

Fig. 6 shows the effects of solid particle diameter, column diameter and gas velocity on the value of $P_p B$ corrected for temperature in batch operations. $P_p B$ decreases with an increase in $\bar{u}_g$ and D_T and a decrease

Fig. 4 Effect of v_t on $P_p B$ in batch operations ($d_p = 88 \sim 105 \mu$)

Fig. 5 Effect of ϕ_l on $P_p B$ in batch operations

in d_p (or a decrease in v_t). The interpolated value of $P_p B$ of solid particles of different sizes shown in Fig. 6 under the experimental conditions given in Fig. 4 ($D_T = 6.6$ cm, $\bar{u}_g = 10.0$ cm/sec and $\bar{C} = 0.1$ g-solid/cm³-slurry), is on a straight line in Fig. 4 and satisfies Eq.(5).

3.3 Concentration distribution of solid particles in continuous operations

When the concentration distribution of solid particles in continuous operations is plotted, with the axial height from the bottom of the column as the abscissa, against the logarithm of solid particle concentration as the ordinate, curves depending upon the operating conditions are obtained as shown in Fig. 7. The solid lines in Fig. 7 are the concentration distribution curves of solid particles calculated as follows: Eq.(2) is transformed to have the term, $X_{0.1}$. The value of $P_{p1}B$ obtained by the method mentioned in section 2, together with $P_p B$ and $X_{0.1}$, is substituted in the transformed equation, and the values of X corresponding to Z are calculated. The values of C obtained by multiplying X by C^* and Z give the concentration distribution curve of solid particles. All of the calculated curves fully satisfy the observed values; and, therefore, this sedimentation diffusion model

Fig. 6 Effects of particle diameter, column diameter and gas velocity on $P_p B$ in batch operations

Fig. 7 Example data of concentration distribution of solid particles in continuous operations

is considered to be applicable to express the concentration distribution of solid particles in continuous operations.

3.4 Longitudinal dispersion coefficient of suspended solid particles and liquid

The effects of particle diameter d_p , column diameter D_T , average solid particle concentration $\bar{C}$, slurry velocity $\bar{u}_l$ and gas velocity $\bar{u}_g$ on the longitudinal dispersion coefficient of solid particles E_p are shown in Fig. 8. The effects of d_p , D_T , $\bar{C}$, $\bar{u}_l$ and $\bar{u}_g$ on the longitudinal dispersion coefficient of liquid in slurry E_{sl} are shown in Fig. 9. In Fig. 8 the data of E_p obtained by Imafuku *et al.*⁵⁾ are also plotted, and the data of E_{sl} of this experiment are shown by broken lines. In Fig. 9 the data of E_l ⁶⁾ in the air-water system are shown by broken lines. E_p , E_{sl} and E_l

Fig. 8 Effects of particle diameter, column diameter, average solid particle concentration, slurry velocity and gas velocity on longitudinal dispersion coefficient of solid particles

Fig. 9 Effects of particle diameter, column diameter, average solid particle concentration, slurry velocity and gas velocity on longitudinal dispersion coefficient of liquid in slurry

increase with an increase in $\bar{u}_g$ and are proportional to the 1~1.5th power of D_T . E_p and E_{sl} are independent of $\bar{u}_l$ as is the case of E_l ; and E_p is almost independent of the average solid particle concentration. In the case of large d_p (or large v_l), at low $\bar{u}_g$, E_p is less than E_{sl} ; but as $\bar{u}_g$ increases, E_p gradually approaches E_{sl} . In the case of small d_p (or small v_l), E_p is nearly equal to E_{sl} , including the range of low $\bar{u}_g$. When the column diameter becomes as small as 6.6 cm, the effect of d_p on E_p becomes very small.

In the same manner as the arrangement concerning E_l in a previous paper⁶⁾, Peclet number $(Pe)_p$ including E_p , and $(Pe)_{sl}$ including E_{sl} are plotted as the ordinate against Froude number Fr as the abscissa in Fig. 10 and Fig. 11, respectively. The relationship between $(Pe)_{sl}$ and Fr and the relationship

Fig. 10 Relationship between $(Pe)_p$ and Fr

Fig. 11 Relationship between $(Pe)_{sl}$ and Fr

between $(Pe)_p$ in the case of small d_p and Fr are shown approximately by the solid line and are expressed by the following equation:

$$(Pe)_{sl} = (Pe)_p = 13Fr / (1 + 8Fr^{0.85}) \quad (7)$$

The broken lines in Fig. 10 and Fig. 11 show the values calculated from the following empirical equation, which was reported previously concerning E_l for the gas-liquid system in a bubble column with diameter larger than 12.2 cm, using a perforated plate of hole diameter more than 0.2 cm or a nozzle as a gas distributor⁶⁾:

$$\bar{u}_g D_T / E_l = 13Fr / (1 + 6.5Fr^{0.8}) \quad (8)$$

In Fig. 10, the value of $(Pe)_p$ of large d_p is larger than that of small d_p at low Fr , but gradually approaches that of small d_p as Fr increases. This tendency is pronounced for the 21.4 cm column. Led by the examination of the correlation between $(Pe)_p$ and Reynolds number $Re_p (= \bar{d}_p v_l / \nu)$ at $Fr=0.02$ and $Fr=0.05$ in this column, the relation is expressed by $(Pe)_p = (Pe)_p^0 (1 + a Re_p)$, where $(Pe)_p^0$ is the value of $(Pe)_p$ calculated from Eq. (7). The relation between a and Fr has been approximated by $a = 0.009Fr^{-0.8}$. Assuming that these relations are applicable to the 12.2 and 6.6 cm columns, all data are plotted with $(Pe)_p / (1 + 0.009 Re_p Fr^{-0.8})$ as the ordinate and Fr as the abscissa, as shown in Fig. 12, and then the following

Fig. 12 Relationship between $(Pe)_p / (1 + 0.009 Re_p Fr^{-0.8})$ and Fr

Fig. 13 Relationship between $v_p(\varphi_i^*/\varphi_l)^{2.5}/v_t$ and $\bar{u}_g/v_t$

empirical equation can be obtained:

$$(Pe)_p = 13Fr(1 + 0.009Re_p Fr^{-0.8}) / (1 + 8Fr^{0.85}) \quad (9)$$

3.5 Mean settling velocity of solid particles

As E_p is almost independent of the average solid particle concentration $\bar{C}$ in the range of $\bar{C} = 0.048 \sim 0.20$ g-solid/cm³-slurry, the following equation between v_p and φ_l is obtained from Eq.(6):

$$v_p = v_p^0 \varphi_l^{2.5} \quad (10)$$

where v_p^0 is the value of v_p extrapolated to $\varphi_l = 1$. As most of these experiments were carried out under the condition of $\bar{C} = 0.1$ g-solid/cm³-slurry, denoting the value of φ_l at $\bar{C} = 0.1$ g-solid/cm³-slurry by φ_l^* , the mean settling velocity of solid particles corrected to the value at $\bar{C} = 0.1$ is expressed as $v_p(\varphi_l^*/\varphi_l)^{2.5}$. The ratio of $v_p(\varphi_l^*/\varphi_l)^{2.5}$ to the terminal velocity of a single particle v_t , namely $v_p(\varphi_l^*/\varphi_l)^{2.5}/v_t$, is plotted as the ordinate against $\bar{u}_g/v_t$ as the abscissa as in Fig. 13. v_p is almost independent of D_T and $\bar{u}_l$ and can be approximated by the following equation:

$$v_p = 1.20v_t(\bar{u}_g/v_t)^{0.25}(\varphi_l/\varphi_l^*)^{2.5} \quad (11)$$

where $\varphi_l^* = 0.96$. Substituting this value into Eq.(11), it reduces to the equation

$$v_p = 1.33v_t(\bar{u}_g/v_t)^{0.25}\varphi_l^{2.5} \quad (12)$$

3.6 The ratio of the solid particle concentration in slurry at the top of the column to that in effluent (or feed) slurry

In the case of cocurrent flow, the solid particle concentration in the effluent (or feed) slurry C^* is less than that in slurry at $z = L$, C_L , as shown in Fig. 7.

Fig. 14 Relationship between X_1 and $\bar{u}_g/v_t$

Fig. 15 Comparison of calculated and observed effluent (or feed) concentration of solid particles

Suganuma *et al.*⁸⁾ recognized this relationship qualitatively, but they did not present the empirical equation. In this experiment, $X_1 = C_L/C^*$ is almost independent of D_T , $\bar{C}$ and $\bar{u}_l$, but it decreases with an increase in $\bar{u}_g$ and with a decrease in v_t . Then X_1 was plotted as the ordinate against $(\bar{u}_g/v_t)$ as the abscissa in Fig. 14, and is expressed as a function of $(\bar{u}_g/v_t)$. As X_1 should reduce to unity when $\bar{u}_g$ is infinity and v_t is infinitesimal, the following empirical equation was obtained:

$$X_1 = 1 + 0.5(\bar{u}_g/v_t)^{-0.4} \quad (13)$$

3.7 Comparison of observed values with calculated values

The concentration distribution of solid particles and the concentration of solid particles in effluent (or feed) slurry can be calculated from empirical Eqs.(9), (12) and (13), the basic Eq.(2) and the following Eq.(14).

$$\bar{X} = \left(X_1 - \frac{P_{pl}B}{P_pB - P_{pl}B} \right) \left(\frac{\exp(P_pB - P_{pl}B) - 1}{P_pB - P_{pl}B} \right) - \frac{P_{pl}B}{P_pB - P_{pl}B} \quad (14)$$

The broken lines in Fig. 7 show the values calculated from the above equations and agree satisfactorily with the observed values. The largest difference between observed and calculated concentration of solid particles seems to occur in the concentration of effluent slurry. This comparison of calculated and observed effluent concentration of solid particles is shown in Fig. 15. The calculated values agree with the observed ones within $\pm 20\%$.

Conclusion

The longitudinal concentration distribution of solid particles and liquid mixing have been studied. The concentration distribution of solid particles was analyzed by using a sedimentation diffusion model.

Empirical equations (9), (12) and (13) were obtained for the longitudinal dispersion coefficient of solid particles, the mean settling velocity of solid particles and the ratio of the solid particle concentration in slurry at the top of the column to that in effluent slurry, respectively. The longitudinal dispersion coefficient of liquid in slurry was correlated by Eq.(7). Furthermore, the values of solid particles concentration calculated from Eqs.(9), (12) and (13) and Eqs.(2) and (14) agree with the observed ones within $\pm 20\%$.

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Nomenclature

C	= concentration of solid particles in slurry	[g-solid/cm ³ -slurry]
C_0	= concentration of solid particles in slurry at $z=0$	[g-solid/cm ³ -slurry]
C_L	= concentration of solid particles in slurry at $z=L$	[g-solid/cm ³ -slurry]
C^*	= concentration of solid particles in feed or effluent	[g-solid/cm ³ -slurry]
$\bar{C}$	= average concentration of solid particles in slurry	[g-solid/cm ³ -slurry]
d_p	= diameter of a solid particle	[μ]
$\bar{d}_p$	= mean diameter of solid particles	[cm]
D_T	= diameter of column	[cm]
E_l	= longitudinal dispersion coefficient of liquid based on actual mean liquid velocity for gas-liquid system	[cm ² /sec]
E_p	= longitudinal dispersion coefficient of solid particles based on actual mean slurry velocity	[cm ² /sec]
E_{sl}	= longitudinal dispersion coefficient of liquid in slurry based on actual mean slurry velocity	[cm ² /sec]
Fr	= $\bar{u}_g/\sqrt{gD_T}$, Froude number	[—]
g	= gravitational acceleration	[cm/sec ²]

L	= height of column	[cm]
$(Pe)_p$	= $\bar{u}_g D_T / E_p$, Peclet number based on E_p	[—]
$(Pe)_{sl}$	= $\bar{u}_g D_T / E_{sl}$, Peclet number based on E_{sl}	[—]
$P_p B$	= $v_p L / E_p$	[—]
$P_p B^0$	= value of $P_p B$ extrapolated to $\phi_l = 1$	[—]
$P_{pl} B$	= $u_l L / E_p$	[—]
Re_p	= $\bar{d}_p v_l / \nu$, Reynolds number	[—]
t	= temperature of slurry	[°C]
$\bar{u}_g$	= superficial velocity of gas based on empty column	[cm/sec]
$\bar{u}_l$	= superficial velocity of slurry based on empty column	[cm/sec]
u_l	= $\bar{u}_l / (1 - \epsilon_g)$, actual mean velocity of slurry	[cm/sec]
v_p	= mean settling velocity of solid particles	[cm/sec]
v_p^0	= value of v_p extrapolated to $\phi_l = 1$	[cm/sec]
v_t	= terminal velocity of a single solid particle in stagnant liquid	[cm/sec]
X	= $C/\bar{C}^*$, normalized concentration of solid particles	[—]
$\bar{X}$	= $\bar{C}/\bar{C}^*$, normalized average concentration of solid particles	[—]
$X_{0.1}$	= value of X at $Z=0.1$	[—]
$X_{0.76}$	= value of X at $Z=0.76$	[—]
X_1	= $C_L/\bar{C}^*$, value of X at $Z=1$	[—]
z	= axial height from the bottom of column	[cm]
Z	= z/L , normalized axial height from the bottom of column	[—]
ϵ_g	= gas holdup	[—]
ν	= kinematic viscosity of liquid	[cm ² /sec]
ρ_p	= density of solid particle	[g/cm ³]
ϕ_l	= volume fraction of liquid in slurry = 1 - (volume fraction of solid particles in slurry)	[—]
ϕ_l^*	= value of ϕ_l at $\bar{C}=0.1$	[—]

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